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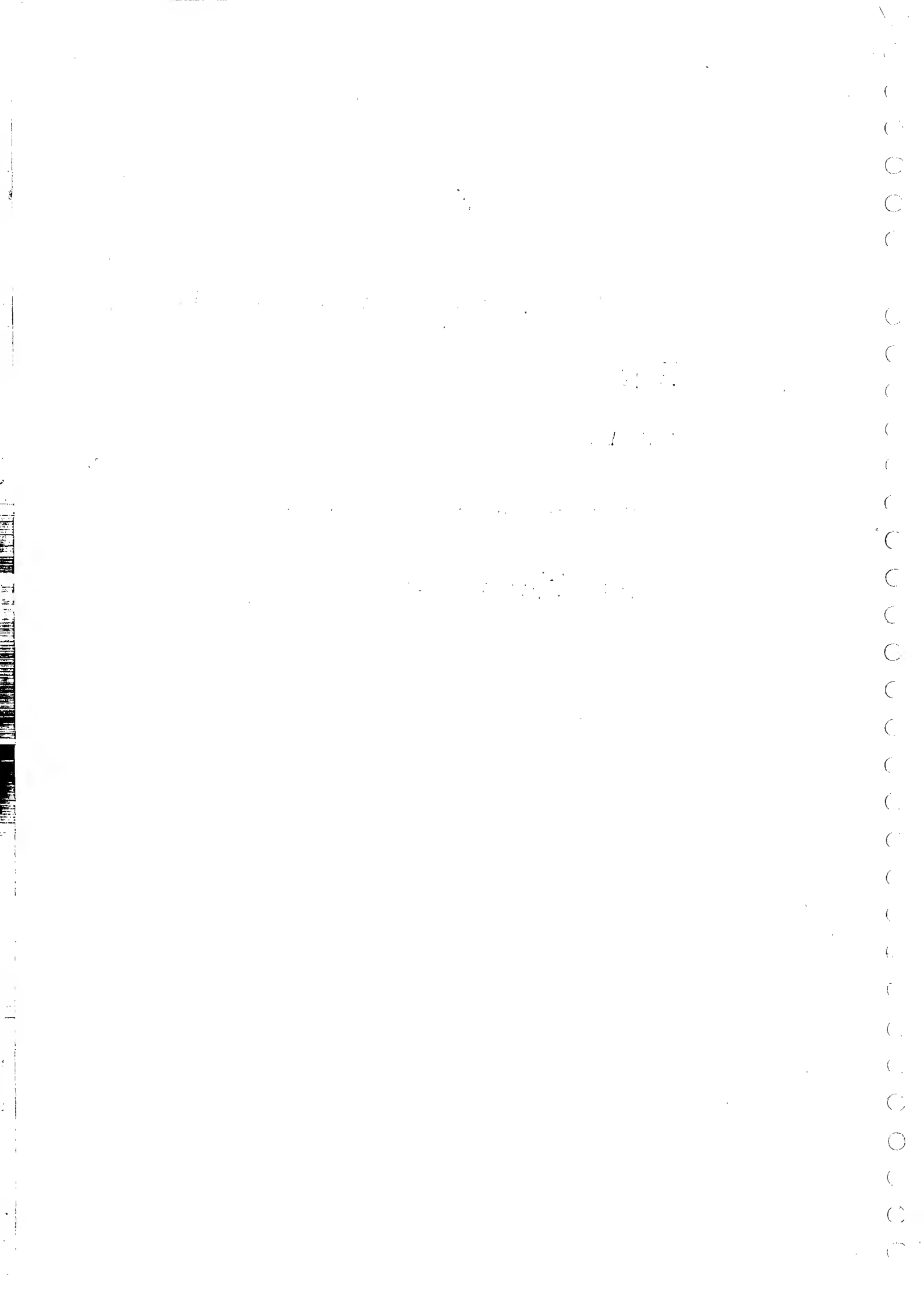
ECE

PM1(B)

Communication System.

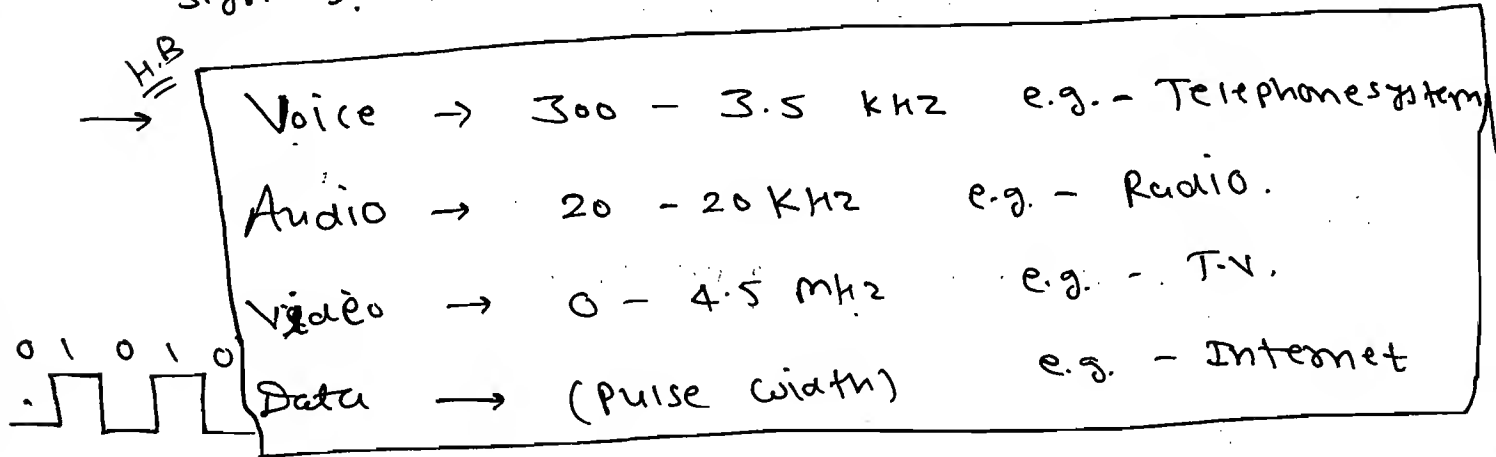
Sir: Muoli Sir

All the best  
propose

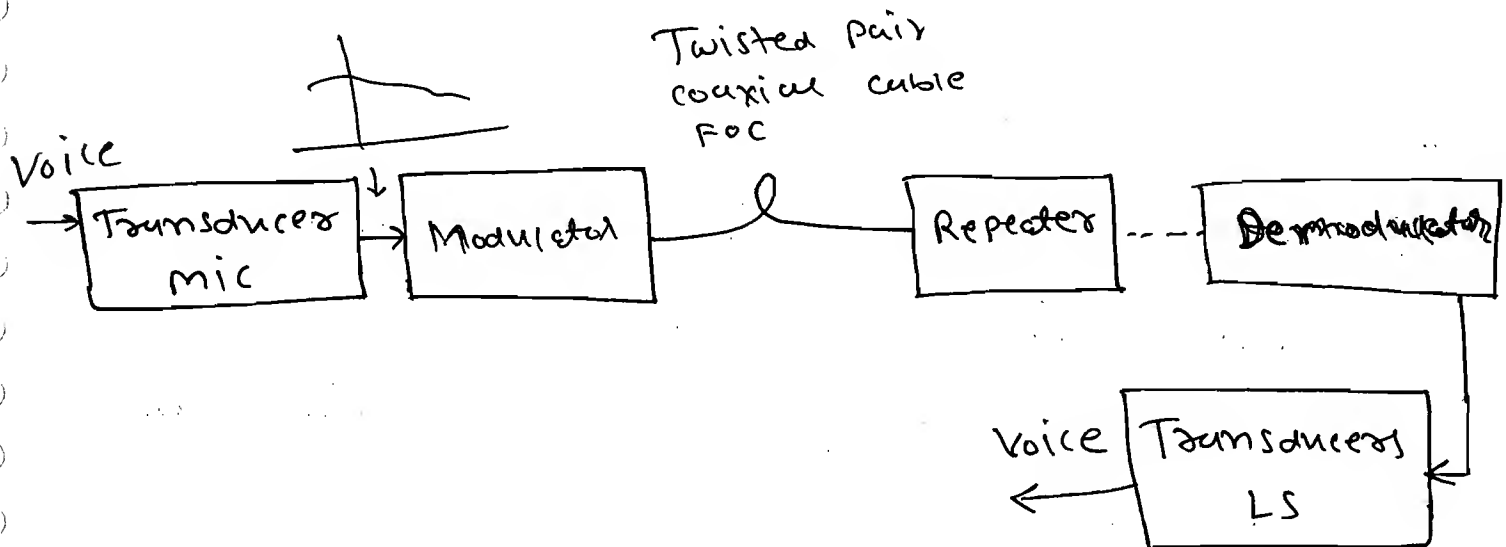


## \* Objective of Electronic Communication System:

→ objective of Electronic Communication System is to transfer information from one place to another place using electrical signals.



## \* Block Diagram of Communication System:



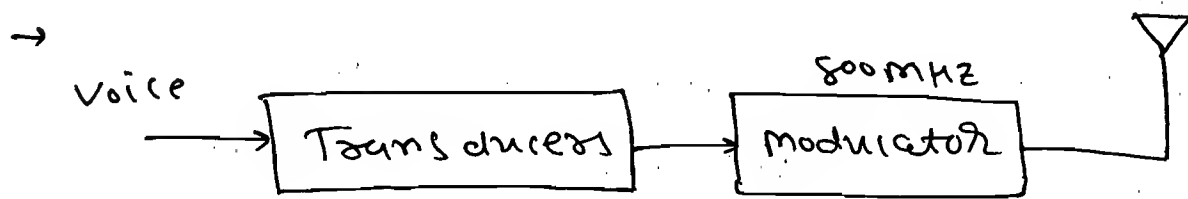
$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{3 \times 10^8}{300} = \text{very large}$$

→ Therefore, Antenna size may be very

Very large because antenna length <sup>is</sup> ~~may~~ to be define in terms of its wavelength  $(\lambda)$  which is practically not possible.

→ In order to avoid this problem we should use modulator which convert ~~a~~ Low freq. signal into high freq. signal.



$$\therefore \lambda = \frac{3 \times 10^8}{800 \times 10^6}$$

So, Antenna size reduced very much.

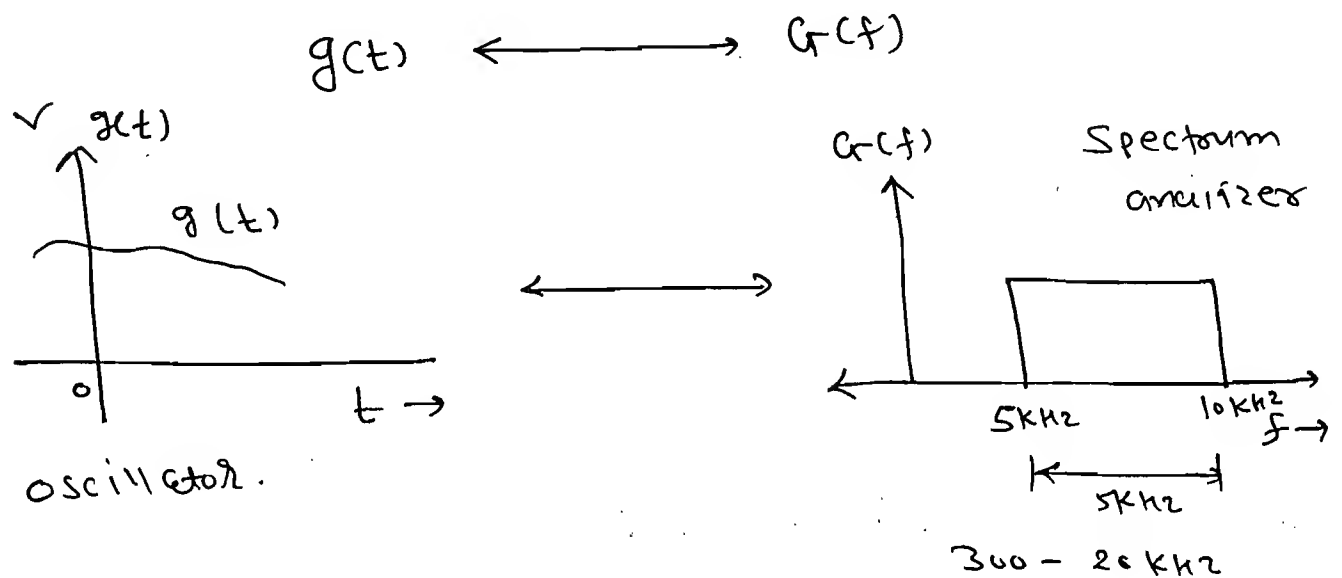
### \* Advantages:

- ① To reduce the size of antenna.
- ② Multiplexing is possible.
- ③ To reduce the effect of Noise and interference.
- ④ For narrow banding of signals.

## \* Review of Fourier Transform:

→ Fourier transform concept is used to determine the frequency present in the signal.

→ Fourier transform converts time domain signals into freq. domain signal.



### NOTE:

→ The oscillator will display time domain signal.

~~spectrum analyzer~~

→ The Spectrum analyzer will display freq. domain spectrum.

## \* Bandwidth:

→ Range of the frequencies occupied by the signal is called as the BW.

→ Mathematically,

$$BW = f_H - f_L$$

$$BW = 10 - 5$$

$$BW = 5 \text{ kHz}$$

\* Spectrum:

→ It is a graphical representation of signal in freq. domain.

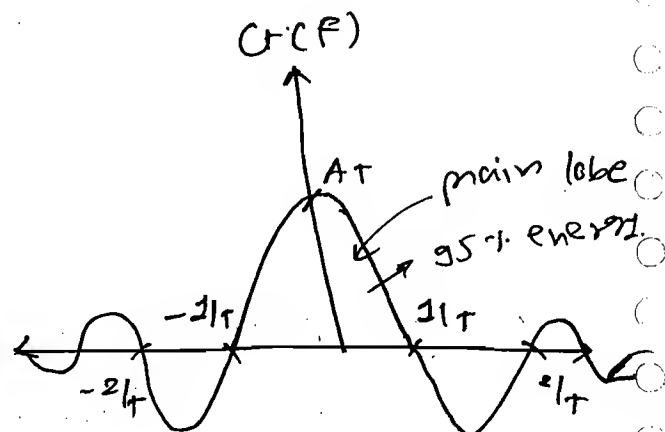
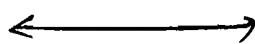
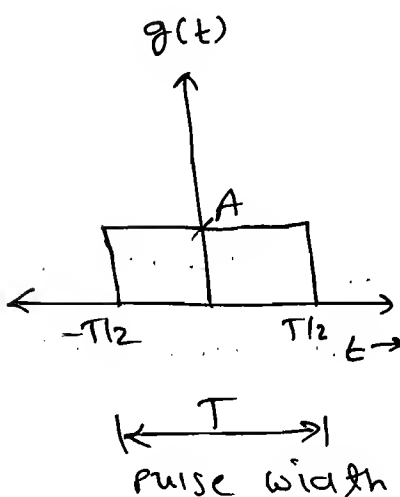
NOTE:

→ Practically the Bandwidth of signal should be low as possible and channel BW should be as high as possible.

$$\Rightarrow g(t) \xleftrightarrow{FT} G(f).$$

$$G(f) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi ft} \cdot dt.$$

⇒



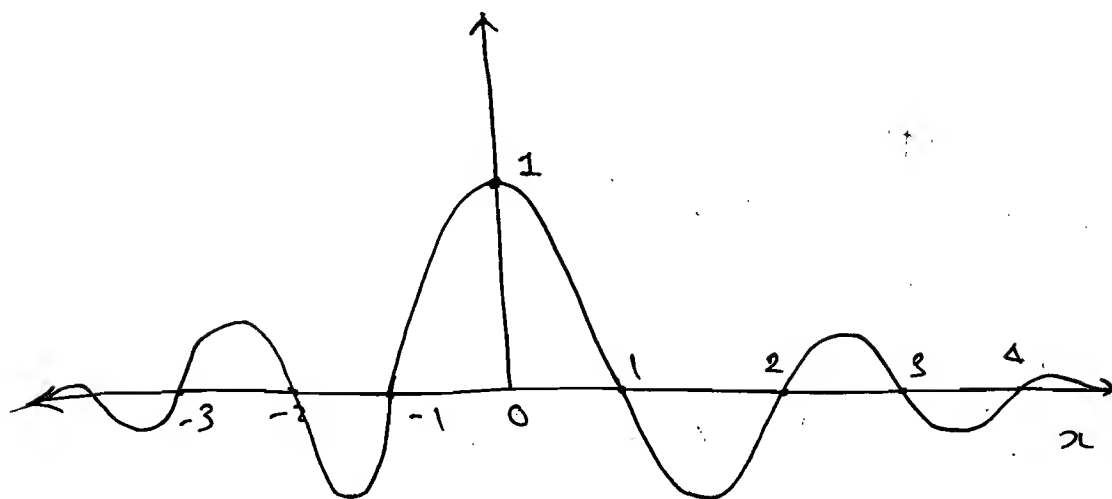
$$G(f) = \int_{-T/2}^{T/2} A \cdot e^{-j2\pi ft} dt.$$

$$\therefore G(f) = \frac{AT \sin(\pi fT)}{\pi fT}$$

$$\therefore \boxed{G(f) = AT \operatorname{sinc}(fT)}$$

$$* \quad \operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$= 0, \quad x = \pm 1, \pm 2, \pm 3, \pm 4, \dots$$



$$\Rightarrow G(f) = AT \operatorname{sinc}(fT)$$

$$\therefore f = 0, \pm \frac{1}{T}, \pm \frac{2}{T}, \pm \frac{3}{T}, \dots$$

Practically (-ve) freq. do not exist.

$\Rightarrow$  BW of a signal is:

$$\Rightarrow BW = f_H - f_L$$

$$= \infty - 0$$

$$\boxed{BW = \infty} \quad \text{Theoretical}$$

$\Rightarrow$  Signal which is having finite duration is called Energy signal.

$\Rightarrow$  Signal which is having Infinite duration is called Power signal.

$$\rightarrow E = \int_{-\infty}^{\infty} g^2(t) dt = A^2 T \text{ Joules.}$$

$$E = \int_{-\infty}^{\infty} |G(f)|^2 df = A^2 T \text{ Joules.}$$

$$\therefore E = \int_{-1/T}^{1/T} |G(f)|^2 df.$$

$$\therefore \boxed{E = 0.95 A^2 T} \text{ J.}$$

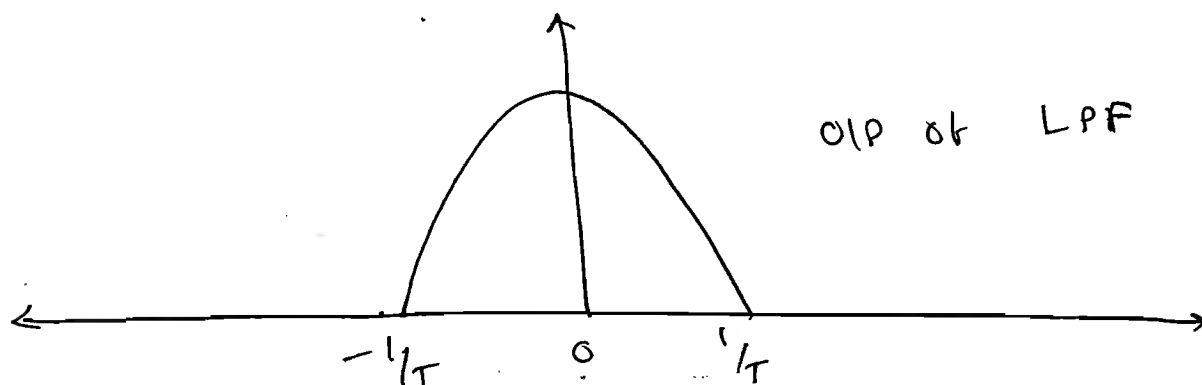
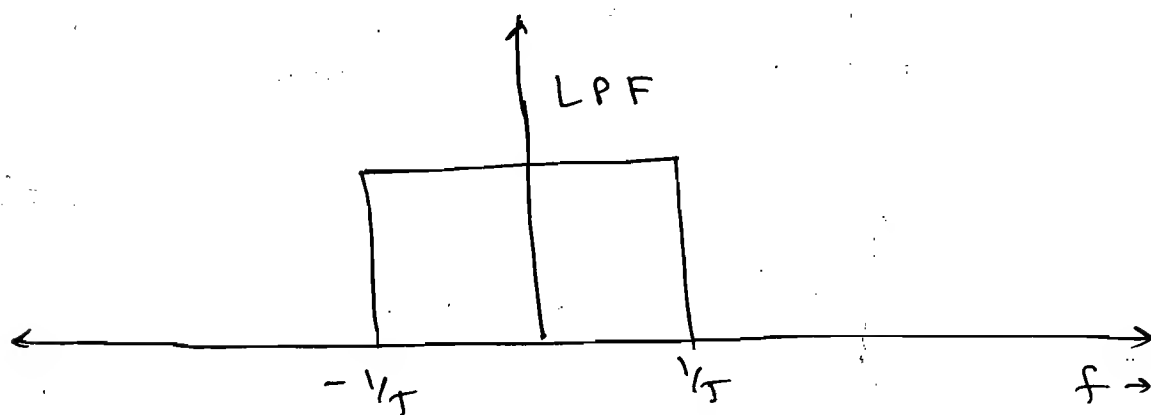
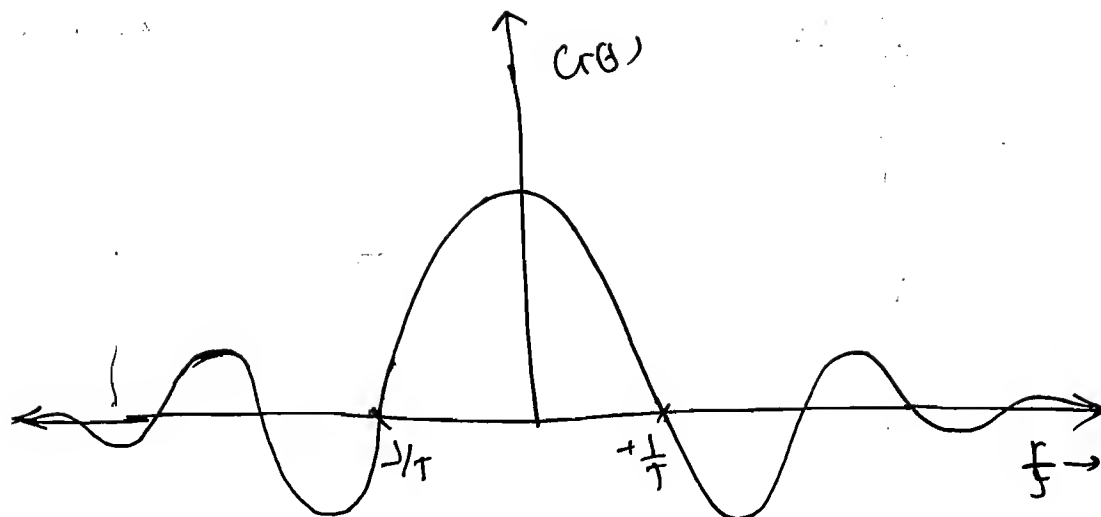
$\Rightarrow$  So, Energy present in main lobe,

$$\boxed{E = 0.95 A^2 T \text{ J}}$$

$\Rightarrow$  95% of energy is contained by only main lobe.



\*



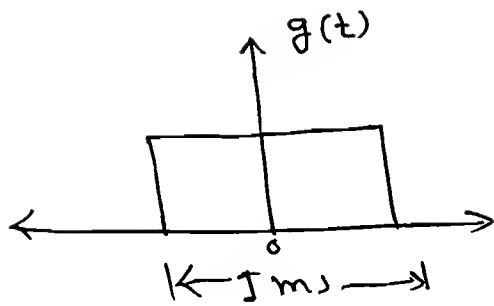
$$\Rightarrow BW = f_H - f_L$$

$$= \frac{1}{T} - 0$$

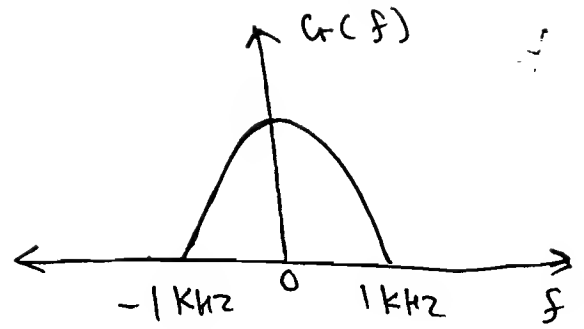
$$\boxed{BW = \frac{1}{T}} \quad (\text{Practical BW})$$

$\Rightarrow$  Practical BW of Rectangular pulse is  
inversely proportional to the pulse width.

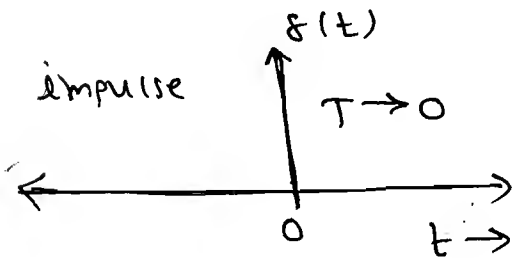
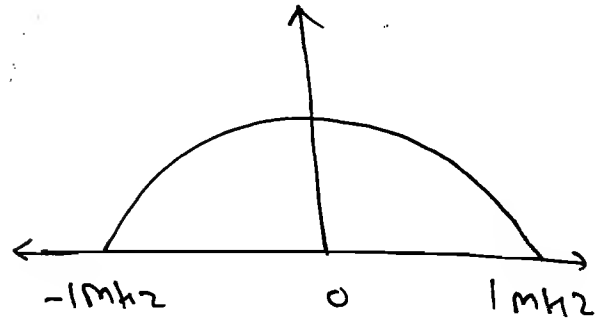
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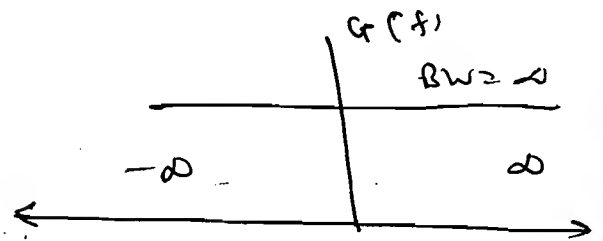
$\longleftrightarrow$



$\longleftrightarrow$



$\longleftrightarrow$



$\Rightarrow$  Bw of impulse is infinite.

\* Freq. Shifting Property: [Modulation property]:

$$g(t) \longleftrightarrow G(f)$$

H.B.

$$g(t) \cdot \cos 2\pi f_c t \longleftrightarrow \frac{G(f - f_c) + G(f + f_c)}{2}$$

$\swarrow$   
 Carrier  
 $f_c$ : Carrier freq.

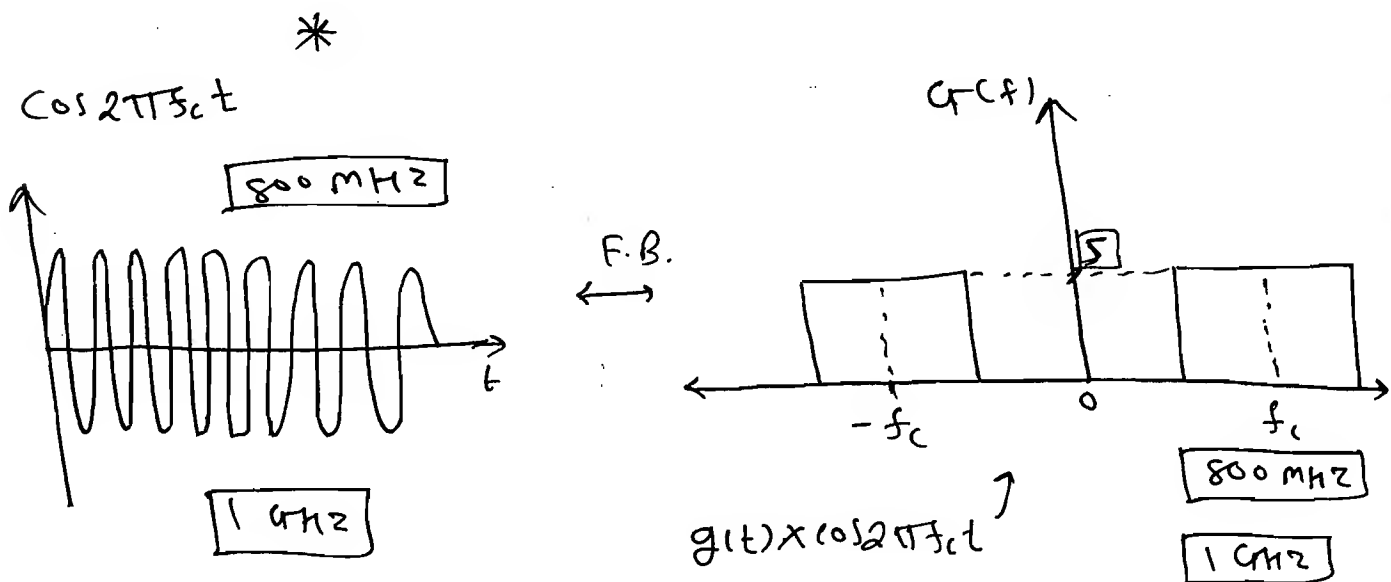
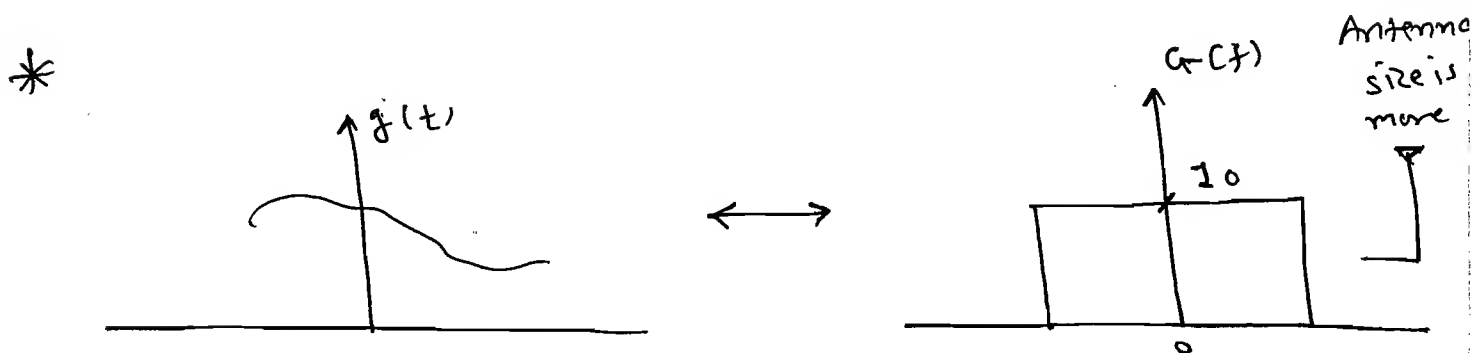
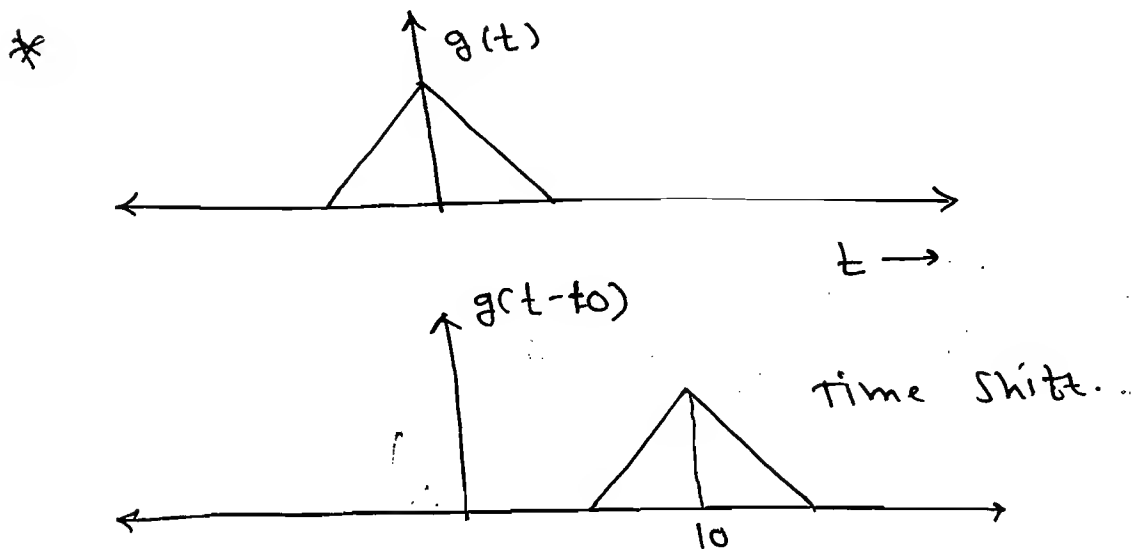
mobile: 800 MHz

FM: 88-108 MHz

Radio: order of GHz

$\Rightarrow$  <sup>HB</sup> If a signal is multiplied with carrier, the spectrum is shifted to the left and right side, by  $f_c$ . The amplitude decreases by a factor of 2!

\* Time shift:

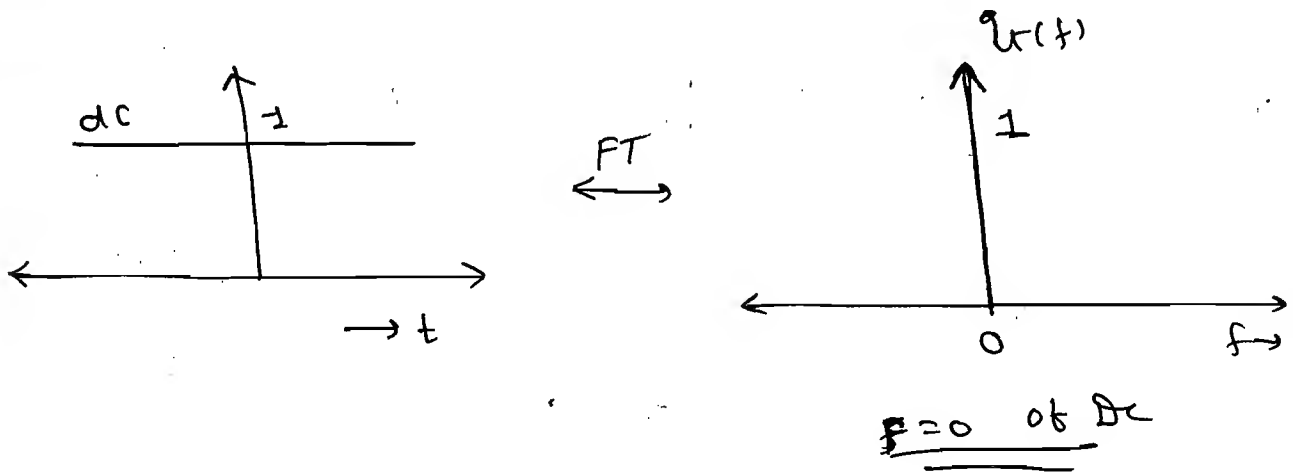


## \* Modulation :-

→ Modulation is defined as a process or freq. translation by modulating a signal. The spectrum is shifted from low freq. region to the high freq. region. In order to modulate a signal it should be multiplied with a carrier (high freq. sinusoidal signal).

$$\Rightarrow \delta(t) \xleftrightarrow{FT} 1.$$

$$1 \xleftrightarrow{FT} \delta(f) \Rightarrow 2\pi \delta(\omega).$$

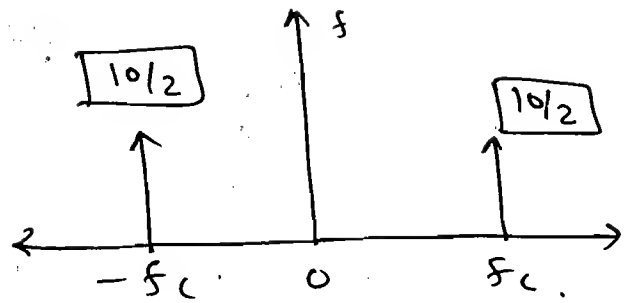
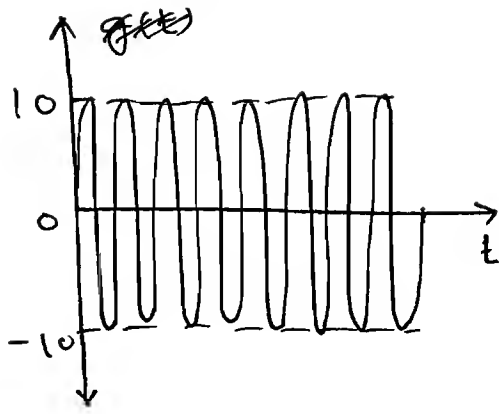


## Impulse:

$$\text{H.B. } 1. \cos 2\pi f_c t \xleftrightarrow{FT} \frac{\delta(f-f_c) + \delta(f+f_c)}{2}$$

$$= 2\pi \left[ \frac{\delta(\omega-\omega_c) + \delta(\omega+\omega_c)}{2} \right]$$

$$= \pi [\delta(\omega-\omega_c) + \delta(\omega+\omega_c)].$$



## \* Concept of Modulation and

### Demodulation:

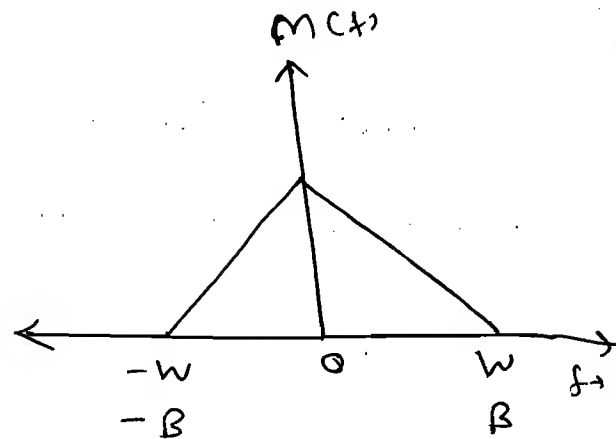
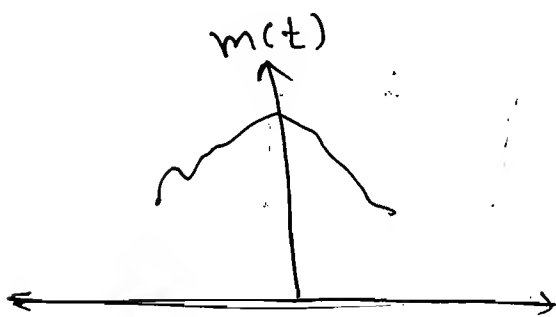
⇒  $m(t)$  = message signal.

= modulating signal

= Base-band signal.

→ A signal which is having significant low freq. is called base-band signal.

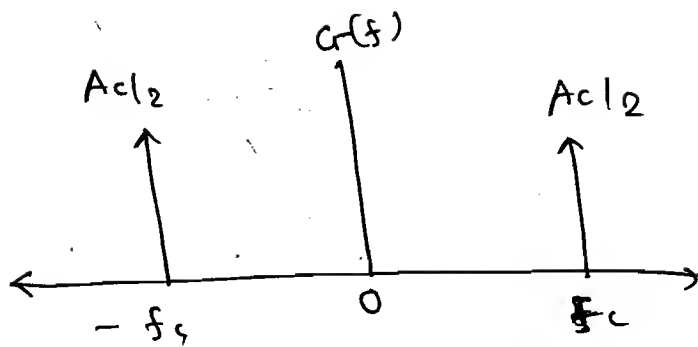
$$m(t) \xleftrightarrow{FT} M(f)$$



$W$  = highest freq. of signal  $m(t)$ .

⇒  $c(t)$  = Carrier

$$= A_c \cos \underline{2\pi f_c t} \longleftrightarrow \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)]$$



$\Rightarrow S(t) = \text{modulated signal.}$

H.B.

$$S(t) = m(t) \cdot c(t).$$

$\Rightarrow$   $m(t)$   $\longleftrightarrow$

$T_x$   
Modulation  $\downarrow$   $X \cos 2\pi f_c t$

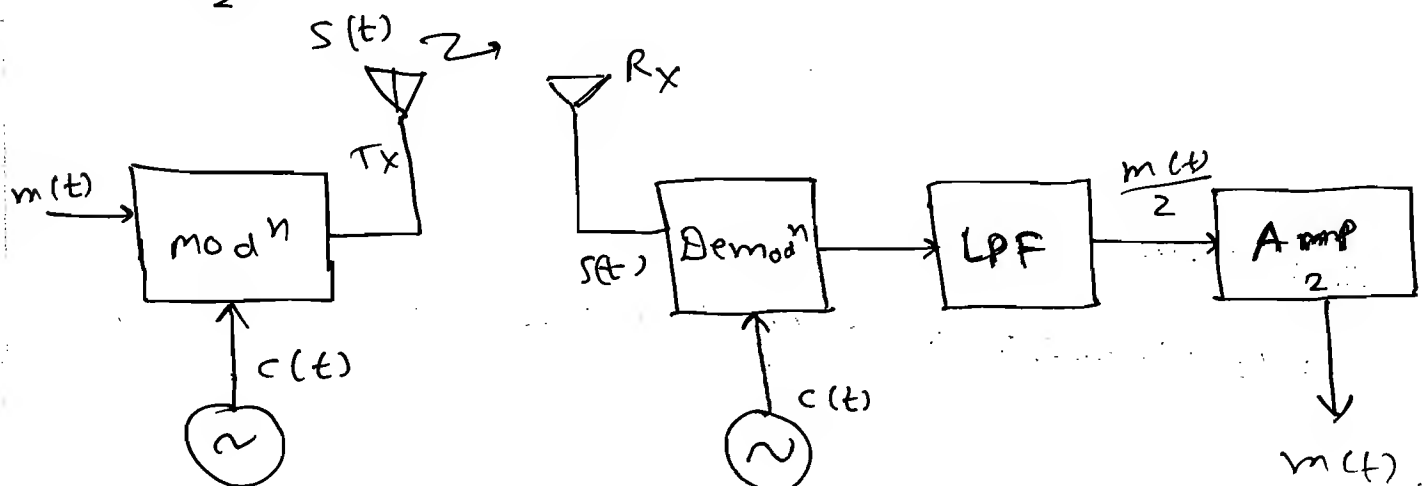
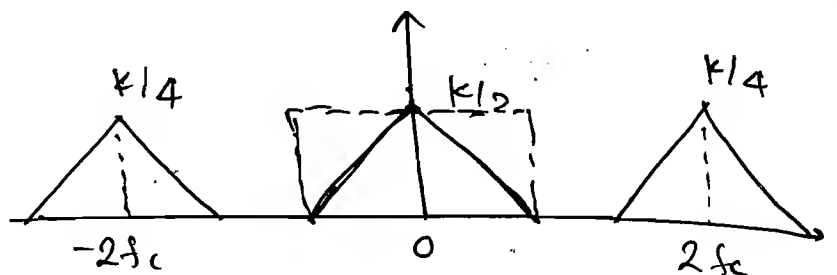
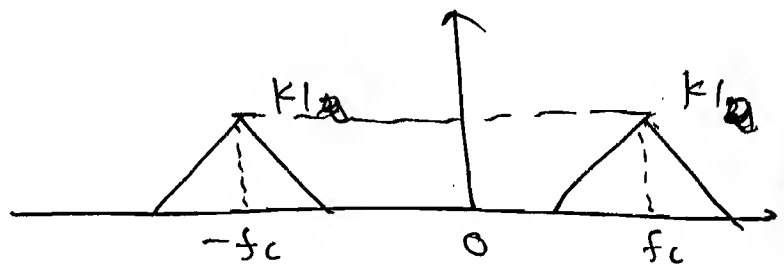
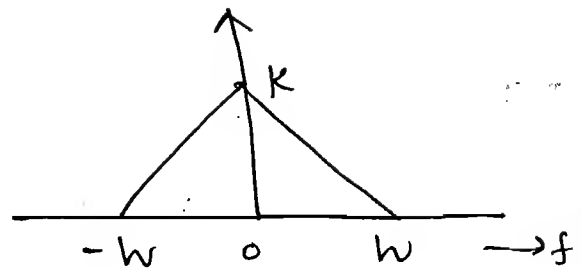
$$S(t) = m(t) \cdot \cos 2\pi f_c t$$

$R_x$   
Demodulation  $\downarrow$   $X \cos 2\pi f_c t$

$$= m(t) \cdot \cos^2 2\pi f_c t$$

$$= \frac{m(t)}{2} + \frac{m(t) \cos 4\pi f_c t}{2}$$

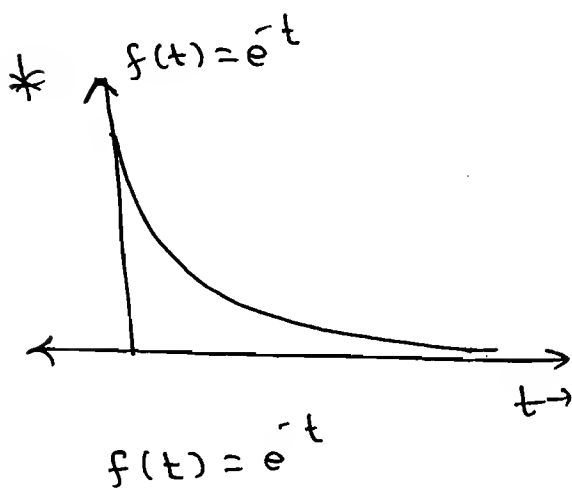
$$= \frac{m(t)}{2}$$



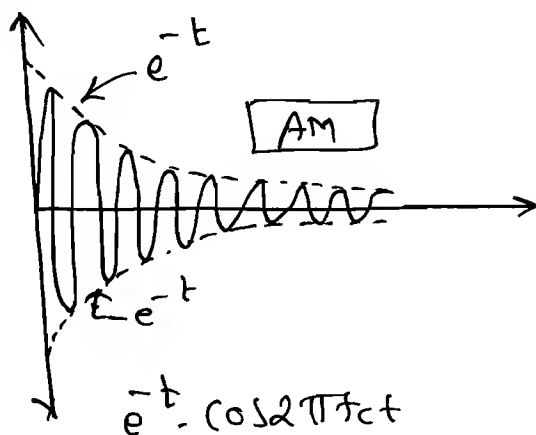
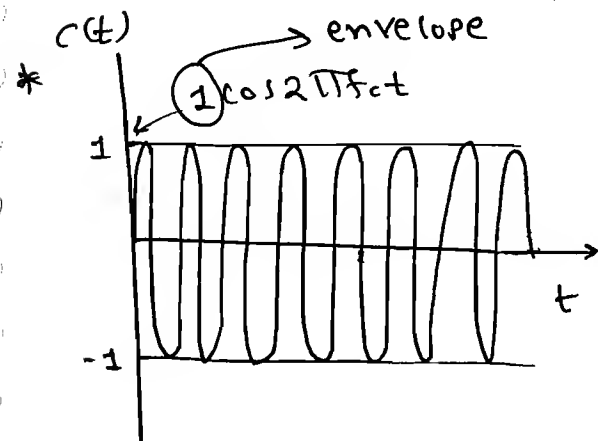
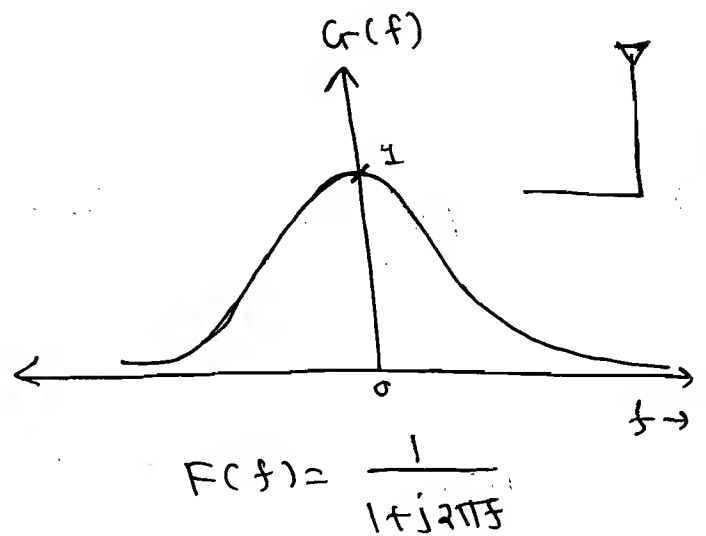
# Analog Communication

## \* Modulation (time domain):

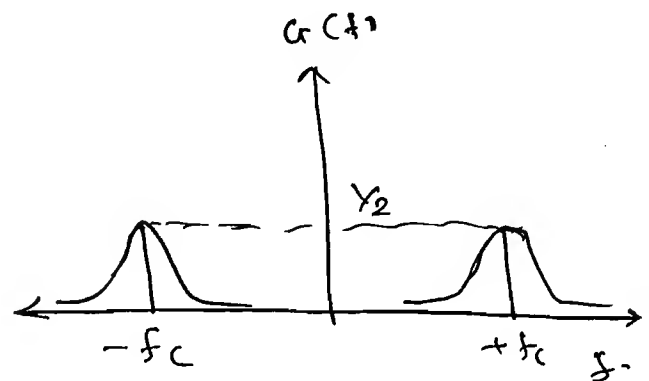
→ Modulation is defined as the process in which the peak amplitude of carrier is varied according to the message signal.



$\longleftrightarrow FT$



$\longleftrightarrow FT$



⇒ Three Parameters of the Carrier which can be varied according to the message signal are Amplitude, Freq. and Phase.

→ So, three analog Communication techniques are, AM, FM & PM.

\*  $\underline{H.B}$

$$\underline{A_c} \cos(2\pi \underline{f_c} t + \underline{\phi}).$$

$\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$   
AM                      FM                      PM

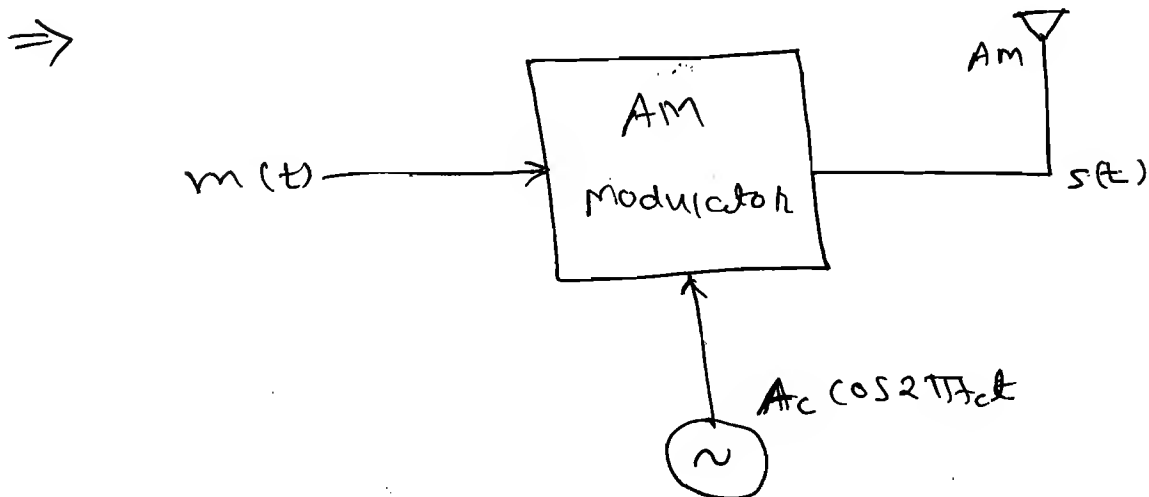
- |       |        |         |
|-------|--------|---------|
| ① AM  | ① NBFM | ① NBPM  |
| ② DSB | ② WBFM | ② WBPM. |
| ③ SSB |        |         |
| ④ VSB |        |         |



## (1) Amplitude Modulation (AM):

→ Amplitude Modulation is defined as the process in which amplitude of the carrier varies according to the message signal.

→ Assume that the message signal  $m(t)$  and carrier signal is applied to the AM modulator.



→ The time domain eq<sup>n</sup> of the AM wave is

$$S(t) = A_c \cos 2\pi f_c t + K_a m(t) \cdot A_c \cos 2\pi f_c t$$

$K_a$  = Amplitude sensitivity of the modulator.

H.B.

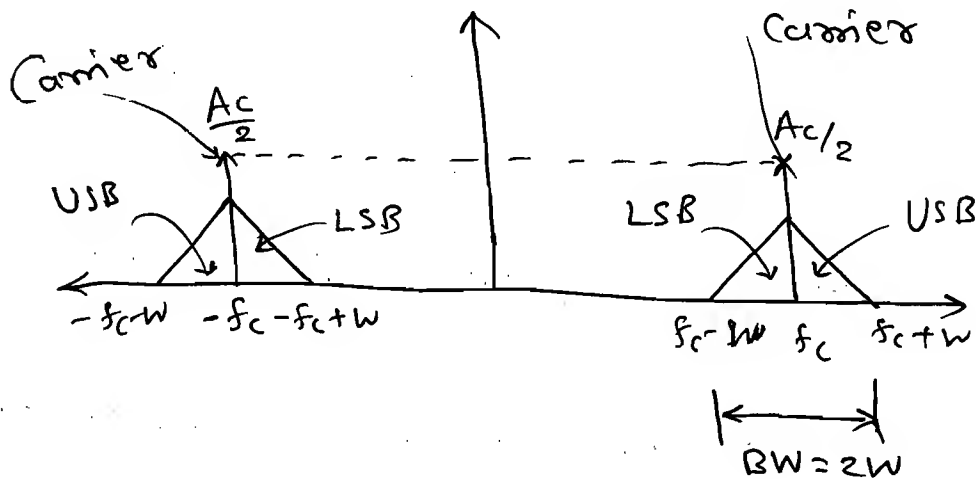
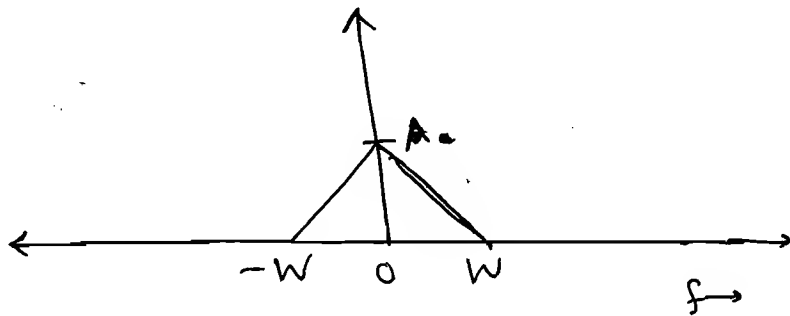
$$\therefore S(t) = A_c [1 + K_a m(t)] \cos 2\pi f_c t$$

This is the standard form.

→ The Peak amplitude of the Carrier modulation is  $A_c$  and after modulation  $A_c[1 + K_m m(t)]$ .

→ Now, Freq. domain eq<sup>n</sup> is,

$$S(f) = \frac{A_c}{2} [s(f-f_c) + s(f+f_c)] + \frac{A_c K_u}{2} [M(f-f_c) + M(f+f_c)].$$



→  $BW = f_H - f_L$

$BW = f_c + W - (f_c - W).$

$\therefore BW = 2W.$

$\therefore BW = 2 \times \text{highest freq. of } m(t).$

→ AM Spectrum consist of the following freqs.

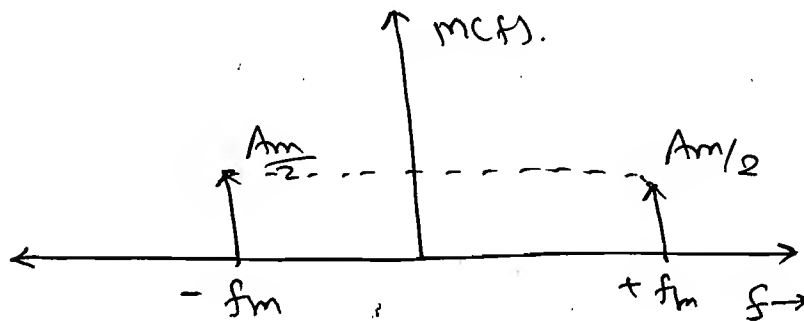
- ① Carrier frequency ' $f_c$ '. ✓
- ② Upper sideband freq. ' $f_c$ ' to ' $f_c + W$ '. ✓
- ③ Lower sideband freq. ' $f_c - W$ ' to ' $f_c$ '. ✓

\* Single tone Modulation of Am:-

→ In the single tone Modulation message is the sinusoidal signal.

$$m(t) = A_m \cos 2\pi f_m t$$

$$M(f) = \frac{A_m}{2} [\delta(f - f_m) + \delta(f + f_m)]$$



⇒ Time domain eq<sup>n</sup> of the Am signal for single tone modulation is.

H.B.

$$S(t) = A_m \cos 2\pi f_c t + A_c \underline{K_a A_m} \cos 2\pi f_m t \cdot \cos 2\pi f_c t$$

$$K_a A_m = \mu = m$$

= modulation index

= depth of modulation.

⇒ The modulation index concept is required for the demodulation of Am.

→ Practically the modulation index lies bet<sup>n</sup> 0 to 1.

$$0 < \mu, m \leq 1.$$

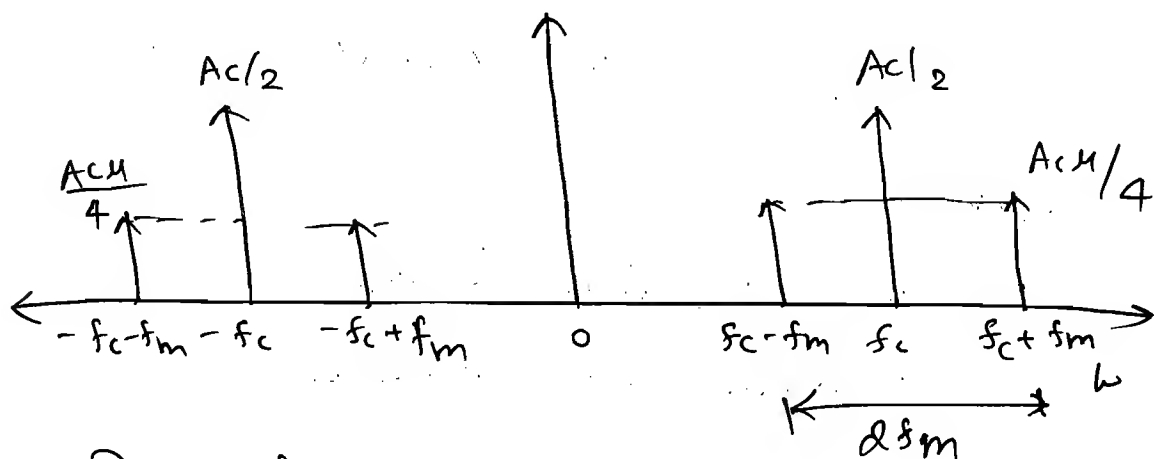
→ When  $\mu = 0$  the carrier is not modulated.

→ When  $\mu > 1$  the ~~carrier~~ Am signal is said to be over modulated.

H.B.

$$s(t) = A_c [1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t.$$

$$\therefore s(t) = \underset{\substack{\uparrow \\ \text{Carrier}}}{A_c \cos 2\pi f_c t} + \underset{\substack{\uparrow \\ \text{USB}}}{\frac{A_c \mu}{2} \cos 2\pi (f_c + f_m)t} + \underset{\substack{\uparrow \\ \text{LSB}}}{\frac{A_c \mu}{2} \cos 2\pi (f_c - f_m)t}$$



$$Bw = 2 f_m.$$

## \* Power Calculation:

$$\rightarrow P_t = P_c + P_{USB} + P_{LSB} \leftarrow \text{H.B.}$$

$$\rightarrow P_c = \frac{V_{rms}^2}{R} = \frac{\left(\frac{V_m}{\sqrt{2}}\right)^2}{R}$$

$$\therefore P_c = \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R}$$

$$\text{H.B.} \Rightarrow \boxed{P_c = \frac{A_c^2}{2R}}$$

$R$  = Antenna Resistance ( $\Omega$ )  
if not given then take  
 $R = 1 \Omega$ .

$$\rightarrow P_{USB} = \frac{\left(\frac{A_c \mu}{2\sqrt{2}}\right)^2}{R}$$

$$\therefore P_{USB} = \frac{A_c^2 \mu^2}{8R} \leftarrow \text{H.B.}$$

$$\therefore P_{LSB} = \frac{A_c^2 \mu^2}{8R} \leftarrow \text{H.B.}$$

$$\therefore P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \mu^2}{8R} + \frac{A_c^2 \mu^2}{8R}$$

$$\therefore P_t = \frac{A_c^2}{2R} \left[ 1 + \frac{\mu^2}{2} \right]$$

$$\therefore \boxed{P_t = P_c \left[ 1 + \frac{\mu^2}{2} \right]} \leftarrow \text{H.B.}$$

$$\rightarrow P_t = \underbrace{P_c}_{\text{Carrier power}} + \underbrace{\frac{P_c \mu^2}{2}}_{\text{Side Band Power}}$$

→ Total Power  $P_t = P_c + P_{SB}$

→ Practically SideBand power is more important than the Carrier power because ~~signal~~ signal to noise ratio depends only on the SideBand power.

→ To determine the sideband power w.r.t. to the total power modulation efficiency (or) power efficiency is used.

$$\eta = \frac{P_{SB}}{P_t} \leftarrow \underline{H.B.}$$

$$\therefore \eta = \frac{\frac{P_c \mu^2}{2}}{P_c \left(1 + \frac{\mu^2}{2}\right)}$$

$$\therefore \eta = \frac{\mu^2}{2 + \mu^2} \leftarrow \underline{H.B.}$$

When  $\mu = 1 \Rightarrow \eta = \frac{1}{3}$  or 33.33%

$$\rightarrow P_{SB} = \frac{1}{3} P_t$$

$$(or) \boxed{P_{SB} = 33.33\% P_t}$$

$$\rightarrow P_t = P_c + P_{sb}$$

$$\mu=1 \Rightarrow P_t = \frac{2}{3} P_t + \frac{1}{3} P_t$$

$$P_t = 66.67\% P_t + 33.33\% P_t$$

$$\mu = 0.707 \Rightarrow \mu = 1/\sqrt{2}$$

$$\therefore P_t = 80\% P_t + 20\% P_t$$

→ In AM Carrier takes more power than the sideband power. The maximum efficiency possible in the case of single tone modulation is 33.33%.

→ The maximum efficiency possible in AM is 50%. (When a message is a square wave).

Ex-1 Consider an AM signal

$$s(t) = 20 [1 + 0.9 \cos 2\pi 10^4 t] \cos 2\pi 10^6 t$$

The signal is radiated into free space using an antenna having a resistance of  $5\Omega$ . Sketch the spectrum and calculate the B.W., power and Modulation efficiency.

Sol<sup>n</sup>: Here, A

$$s(t) = A_c [1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t.$$

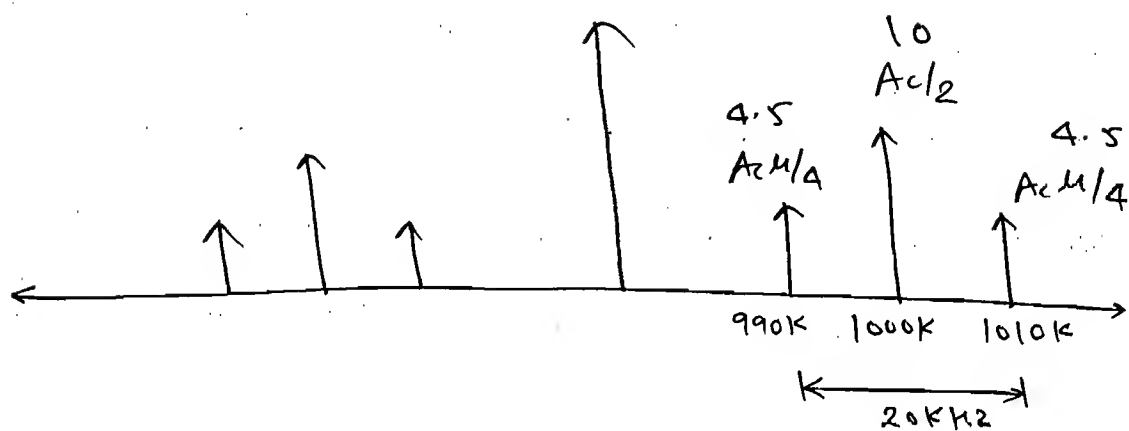
$$\therefore s(t) = 20 [1 + 0.9 \cos 2\pi 10^4 t] \cos 2\pi 10^6 t.$$

$$\therefore \mu = 0.9$$

$$A_c = 20.$$

$$f_m = 10 \text{ kHz}$$

$$f_c = 1000 \text{ kHz}.$$



$$\therefore B_w = 1010k - 990k$$

$$B_w = 20 \text{ kHz}$$

(Or)

$$B_w = 2 f_m.$$

$$= 2 \times 10$$

$$B_w = 20 \text{ kHz}$$

$$P_c = \frac{A_c^2}{2R}.$$

$$\therefore P_c = \frac{400}{2 \times 5}$$

$$P_c = 40 \text{ Watts}$$

$$\therefore P_t = P_c \left[ 1 + \frac{\mu^2}{2} \right].$$

$$\therefore P_t = 40 \left[ 1 + \frac{0.81}{2} \right]$$

$$P_t = 56.2 \text{ Watts.}$$



$$P_t = P_c + P_{SB}$$

$$\therefore P_{SB} = 56.2 - 40$$

$$\therefore P_{SB} = 16.2 \text{ Watts.}$$

$$\therefore P_{USB} = P_{LSB} = 8.1 \text{ Watts.}$$

$$\therefore \eta = \frac{P_{SB}}{P_t}$$

$$(\text{or}) \eta = \frac{\mu^2}{2 + \mu^2}$$

$$\therefore \eta = \frac{16.2}{56.2}$$

$$\therefore \eta = \frac{0.81}{2.81}$$

$$\boxed{\eta = 28.8 \%}$$

$$\therefore \boxed{\eta = 28.8 \%}$$

Ex-2 Consider an AM signal

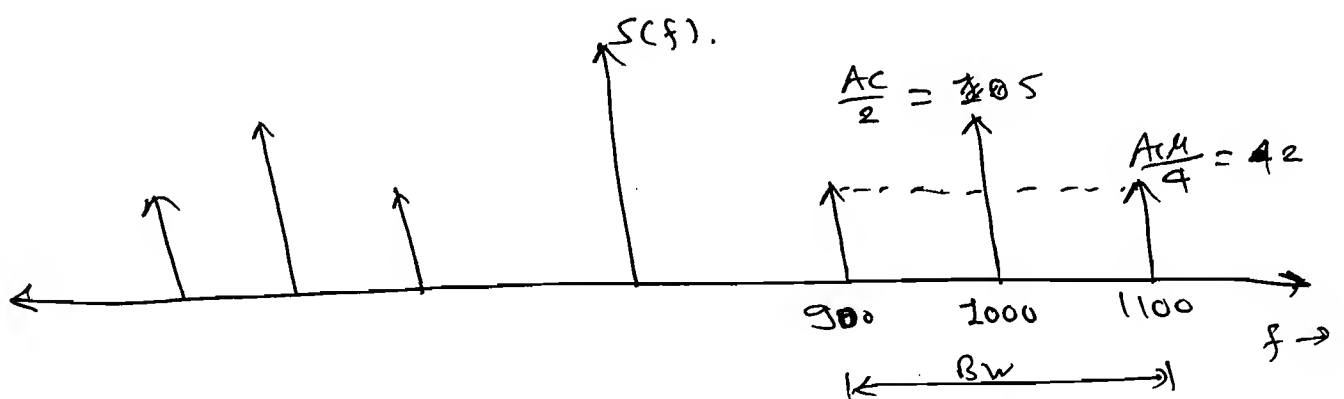
$$s(t) = 4 \cos 1800\pi t + 10 \cos 2000\pi t + 4 \cos 2200\pi t$$

Sketch the spectrum and calculate the B.W., power & Modulation efficiency.

Soln:

$$s(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2} \cos 2\pi (f_c + f_m) + \frac{A_c \mu}{2} \cos 2\pi (f_c - f_m)$$

$$\rightarrow s(t) = 10 \cos 2000\pi t + 4 \cos 1800\pi t + 4 \cos 2200\pi t$$



$$\rightarrow B_w = 1100 - 900$$

$$B_w = 200 \text{ Hz}$$

$$P_c = \frac{A_c^2}{2R}$$

$$\rightarrow \frac{A_c}{2} = 10$$

$$A_c = 10$$

$$R = 1 \Omega$$

$$\therefore P_c = \frac{100}{2}$$

$$\frac{A_c \mu}{2} = 4$$

$$P_c = 50 \text{ Watts}$$

$$\therefore \mu = \frac{4 \times 2}{10}$$

$$\mu = 0.8$$

$$\Rightarrow P_t = P_c \left[ 1 + \frac{\mu^2}{2} \right]$$

$$\therefore P_t = 50 \left[ 1 + \frac{0.64}{2} \right]$$

$$\therefore P_t = 66 \text{ W}$$

$$\therefore \eta = \frac{\mu^2}{2 + \mu^2} = \frac{0.64}{2.64}$$

$$\eta = 24.24 \%$$

Ex-3 A Carrier signal  $c(t) = 5 \cos 2\pi 10^6 t$  is modulated by message signal  $m(t) = 4 \cos 8\pi \times 10^3 t$  to generate an AM signal. Calculate the B.W. and power.

Ans:  $c(t) = 5 \cos 2\pi \times 10^6 t$

$$A_c = 5 \text{ V}$$

$$f_c = 1000 \text{ kHz}$$

$$m(t) = 4 \cos 8\pi 10^3 t.$$

$$\therefore A_m = 4, \quad f_m = 4 \text{ kHz.}$$

$$\therefore B_w = 2 f_m.$$

$$B_w = 2 \times 4 \text{ K}$$

$$\boxed{B_w = 8 \text{ K Hz}}$$

$$\rightarrow P_c = \frac{A_c^2}{2R} = \frac{25}{2 \times 1} = 12.5 \text{ W.}$$

$$\therefore \boxed{P_c = 12.5 \text{ W.}}$$

\* When  $\mu$  is not given take

H.B

$$K_a = \frac{1}{A_c} \Rightarrow \boxed{\mu = \frac{A_m}{A_c}}$$

$$\therefore \mu = A_m / A_c$$

$$\therefore \mu = \frac{4}{5} = 0.8.$$

$$\therefore P_t = P_c \left[ 1 + \frac{\mu^2}{2} \right].$$

$$\therefore P_t = 12.5 \left[ 1 + \frac{0.64}{2} \right].$$

$$\therefore \boxed{P_t = 16.5 \text{ Watts.}}$$

$$\eta = \frac{\mu^2}{2 + \mu^2}.$$

$$= \frac{0.64}{2.64}$$

$$\therefore \boxed{\eta = 24.24 \%}$$

Ex-4 A carrier signal  $c(t) = 5 \cos 2\pi 10^6 t$  is modulated by a message signal  $m(t) = \cos 4\pi 10^3 t$  to generate an AM signal with an modulation index of 0.5.

① Calculate B.W. and power.

② Determine the quantity  $(P_{SB}/P_c)$

Gate: 2003

Ans:

$$\rightarrow A_c = 5V, \quad f_c = 1000 \text{ kHz}$$

$$A_m = 1V, \quad f_m = 2 \text{ kHz}$$

$$\mu = 0.5.$$

$$\therefore B.W. = 2 f_m.$$

$$\therefore \boxed{B.W. = 4 \text{ kHz.}}$$

$$\therefore P_t = P_c \left[ 1 + \frac{\mu^2}{2} \right].$$

$$\therefore P_t = 12.5 \left[ 1 + \frac{0.25}{2} \right]$$

$$\therefore \boxed{P_t = 14.0625 \text{ Watts}}$$

$$\therefore P_t = P_c + P_{SB}.$$

$$\therefore P_{SB} = 14.0625 - 12.5$$

$$\therefore P_{SB} = 1.5625$$

$$\therefore P_{SB}/P_c = \frac{1.5625}{12.5} = 0.125.$$

$$\therefore \boxed{P_{SB} = 12.5 \cdot P_c}$$

$$P_c = \frac{A_c^2}{2R}$$

$$\therefore P_c = \frac{25}{2}$$

$$\boxed{P_c = 12.5 \text{ W}}$$

(Q.2)

→ quantity  $\frac{P_{SB}}{P_c} = \frac{P_c \mu^2 / 2}{P_c}$

$$\therefore \frac{P_{SB}}{P_c} = \frac{\mu^2}{2} = 0.125$$

$$\therefore \boxed{\frac{P_{SB}}{P_c} = 12.5 \%}$$

[expressing SB Power w.r.t. (Carrier Power)]

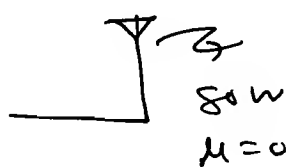
Ex-5 An Am transmitter radiates 80 watts

when the carrier is not modulated.

Determine the total power radiated

if the carrier is modulated and modulated index is 1.

Ans:



$$P_t = P_c \left[ 1 + \frac{\mu^2}{2} \right]$$

When carrier is not modulated

then  $\mu = 0$ . H.B

$$\therefore P_t = P_c [1 + 0]$$

$$\therefore \boxed{P_t = P_c}$$

$$\therefore P_t = P_c = 80 \text{ Watts.}$$

→ When  $\mu = 1$ ,

$$\therefore P_t = 80 \left[ 1 + \frac{1}{2} \right]$$

$$\therefore P_t = 80 \times 1.5 = 120 \text{ W}$$

$$\therefore P_t = 120 \text{ W} = 80 \text{ W} + 40 \text{ W}.$$

$$\therefore P_t = P_c + (0.5 P_c)$$

$$P_t = P_c + (50 + 0.5 P_c)$$

$$50 \text{ W} \rightarrow 75 \text{ W}$$

$$200 \text{ W} \rightarrow 300 \text{ W}.$$

NOTE: H.B.

→ when modulation index is changed from 0 to 1 the power increases by 50%.

$$\rightarrow P_t = P_c \left[ 1 + \frac{\mu^2}{2} \right].$$

$$\left[ \begin{array}{l} \mu = 0 \rightarrow P_t = P_c \\ \mu = 1 \rightarrow P_t = 1.5 P_c. \end{array} \right] \leftarrow \text{H.B.}$$

Ex-6 An Am transmitter radiates 50 watts when the carrier is modulated by sinusoidal signal and the modulation index 0.707.

- ① Determine the modulation efficiency, carrier power & sideband power.
- ② Determine the peak Amplitude of the carrier before modulation and after modulation.

$$\Rightarrow \int \Rightarrow P_t = 50W ; \mu = 0.707.$$

$$\therefore \eta = \frac{\mu^2}{2 + \mu^2}$$

$$\therefore \eta = \frac{1/2}{2 + 1/2} = 1/5$$

$$\therefore \boxed{\eta = 20\%}$$

$$\eta = \frac{P_{SB}}{P_t}$$

$$\therefore P_c = 80\% \cdot P_t$$

$$P_{SB} = 20\% \cdot P_t$$

$$\therefore P_c = 0.8 \times 50 = 40W$$

$$P_t = 0.2 \times 50 = 10W.$$

→ Peak ~~modulation~~ <sup>Amp.</sup> before modulation =  $A_c$ .

$$\therefore P_c = \frac{A_c^2}{2 \times 1} = A_c^2$$

$$\therefore A_c^2 = 80$$

$$\therefore \boxed{A_c = 8.944V}$$

→ Peak Amp. after modulation =  $A_c [1 + \mu \cos 2\pi f_m t]$

← HB

$$\therefore \begin{array}{|l} V_{\max} = A_c [1 + \mu] \quad (\theta = 0) \\ V_{\min} = A_c [1 - \mu] \quad (\theta = \pi) \end{array}$$

$$\Rightarrow V_{\max} = 8.944 [1 + 0.707]$$

$$\boxed{V_{\max} = 15.27V}$$

$$V_{\min} = 8.944 [1 - 0.707] \Rightarrow \boxed{V_{\min} = 6.323V}$$

$$\Rightarrow \frac{V_{\max}}{V_{\min}} = \frac{1+\mu}{1-\mu}$$

$$\therefore V_{\max} [1-\mu] = [1+\mu] V_{\min}$$

$$\therefore \boxed{\mu = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}} \leftarrow \mu = \frac{m}{B}$$

→ Formula to determine  $\mu$  practically.

$$\therefore V_{\max} + V_{\min} = 2A_c$$

$$\therefore \boxed{A_c = \frac{V_{\max} + V_{\min}}{2}}$$

Ex-1 An amplitude of an Am signal varies from 5V to 15V. Determine the modulation index, carrier power, sideband power & total power.

Ans:  $V_{\min} = 5V$ ,  $V_{\max} = 15V$ .

$$\therefore \mu = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}$$

$$P_c = \frac{A_c^2}{2R}$$

$$\therefore \mu = \frac{15-5}{20}$$

$$A_c = \frac{V_{\max} + V_{\min}}{2}$$

$$\mu = \frac{1}{2}$$

$$A_c = \frac{20}{2}$$

$$\therefore \boxed{\mu = 0.5}$$

$$\boxed{A_c = 10V}$$



$$\therefore P_c = \frac{100}{2}$$

$$\boxed{P_c = 50 \text{ W}}$$

$$\therefore P_{SB} = \frac{P_c \mu^2}{2}$$

$$\therefore P_{SB} = \frac{50 \times 0.25}{2}$$

$$\therefore \boxed{P_{SB} = 6.25 \text{ W}}$$

$$\therefore P_t = P_c + P_{SB}$$

$$\therefore \boxed{P_t = 56.25 \text{ W}}$$

### \* Antenna Current:-

→ Consider an antenna having a resistance of  $R_a$  and ' $I_t$ ' is the antenna current when Am signal is radiated into free space.

$$P_t = P_c \left[ 1 + \frac{\mu^2}{2} \right]$$

$$\therefore I_t^2 \cdot R = I_c^2 R \left[ 1 + \frac{\mu^2}{2} \right]$$

$$\therefore I_t^2 = I_c^2 \left[ 1 + \frac{\mu^2}{2} \right]$$

$$\therefore \boxed{I_t = I_c \sqrt{1 + \frac{\mu^2}{2}}}$$

← H.B.

→  $I_c$  is the antenna current before modulation &  
 $I_t$  is the antenna current after modulation.

Ex-2 The antenna current of an AM transmitter is 8A before modulation and 8.5 A after modulation. Calculate the modulation index and modulation efficiency.

Ans:  $I_c = 8A$ ,  $I_t = 8.5A$ .

$$\therefore I_t^2 = I_c^2 \left[ 1 + \frac{\mu^2}{2} \right].$$

$$\therefore (8.5)^2 = (8)^2 \left[ 1 + \frac{\mu^2}{2} \right].$$

$$\therefore 1.128 = 1 + \frac{\mu^2}{2}.$$

$$\therefore \boxed{\mu = 0.5}$$

$$\therefore \eta = \frac{0.25}{2.25}.$$

$$\therefore \boxed{\eta = 11.11\%}$$

## \* Multitone Modulation :-

⇒ Generalized Am signal is,

$$\Rightarrow S(t) = A_c \cos 2\pi f_c t + A_c k_a \boxed{m(t)} \cos 2\pi f_c t.$$

⇒  $m(t)$  is whatever sin or cosine ⇒ no change in B.W. & power.

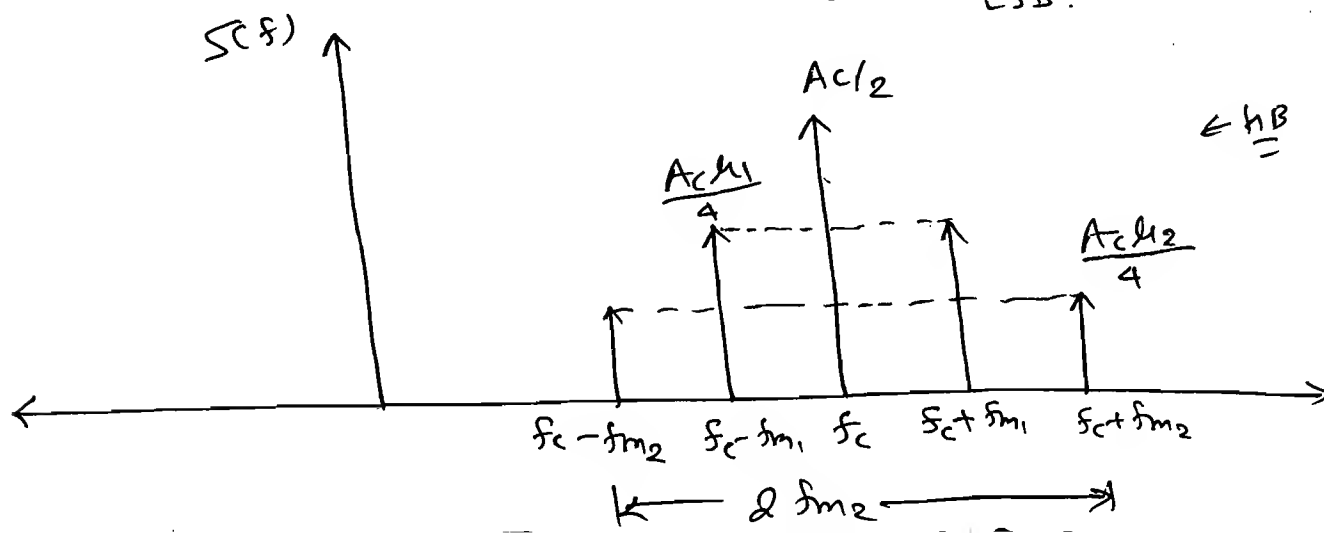
$$\text{Let, } m(t) = A_{m1} \cos 2\pi f_{m1} t + A_{m2} \cos 2\pi f_{m2} t$$

$f_{m2} > f_{m1}$  with -ve sign no change in B.W. & power.

$$\Rightarrow S(t) = A_c \cos 2\pi f_c t + A_c k_a [A_{m1} \cos 2\pi f_{m1} t + A_{m2} \cos 2\pi f_{m2} t] \cos 2\pi f_c t.$$

$$\therefore S(t) = \underset{\text{Carrier}}{A_c \cos 2\pi f_c t} + \underset{\text{USB}}{\frac{A_c \mu_1}{2} \cos 2\pi (f_c + f_{m1}) t}$$

$$+ \underset{\text{LSB}}{\frac{A_c \mu_1}{2} \cos 2\pi (f_c - f_{m1}) t} + \underset{\text{USB}}{\frac{A_c \mu_2}{2} \cos 2\pi (f_c + f_{m2}) t} + \underset{\text{LSB}}{\frac{A_c \mu_2}{2} \cos 2\pi (f_c - f_{m2}) t}$$



$$\text{B.W.} = 2 \times \text{Maximum Frequency.}$$

$$P_t = P_c + P_{USB} + P_{LSB}.$$

$$\therefore P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \mu_1^2}{8R} + \frac{A_c^2 \mu_2^2}{8R} + \frac{A_c^2 \mu_1^2}{8R} + \frac{A_c^2 \mu_2^2}{8R}$$

$$\therefore P_t = \frac{A_c^2}{2R} \left[ 1 + \frac{\mu_1^2 + \mu_2^2}{2} \right].$$

$$\therefore P_t = P_c \left[ 1 + \frac{\mu_1^2 + \mu_2^2}{2} \right] \quad \checkmark$$

$$\text{let, } \mu_t^2 = \mu_1^2 + \mu_2^2 + \dots$$

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \dots} = \text{Total modulation index.}$$

$$\therefore P_t = P_c \left[ 1 + \frac{\mu_t^2}{2} \right] \quad \leftarrow \underline{\text{H.B.}}$$

$$\therefore P_t = P_c + \frac{P_c \mu_t^2}{2}$$

$$\therefore \eta = \frac{\mu_t^2}{2 + \mu_t^2} \quad \leftarrow \underline{\text{H.B.}}$$

∴ Note:

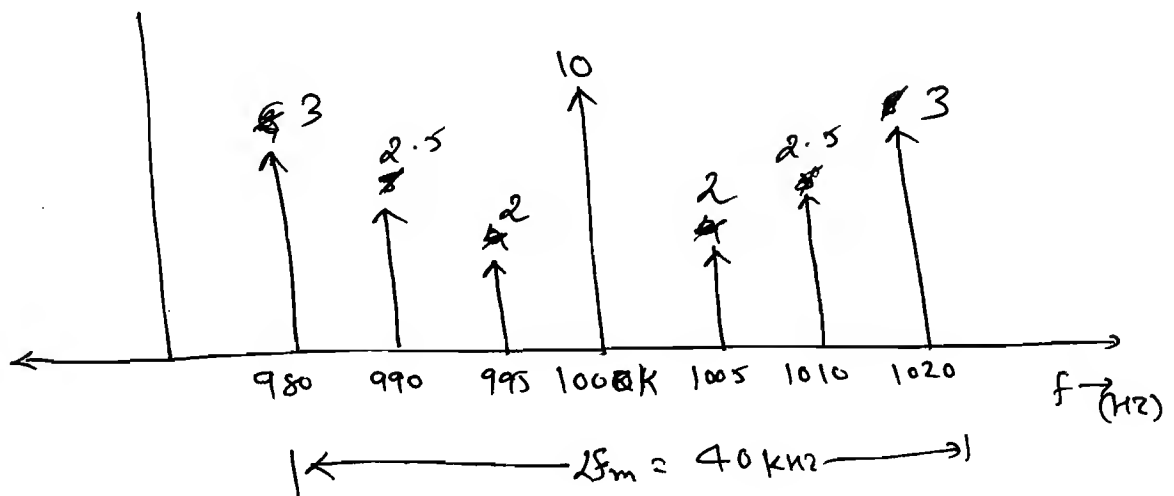
⇒ The B.W. & Power is depends only on the magnitude Spectrum. (not on phase Spectrum).

Ex-1 = A Carrier signal  $c(t) = 20 \cos 2\pi 10^6 t$  is modulated by a message signal having 3 frequencies 5 KHz, 10 KHz & 20 KHz. The Corresponding Modulation indices are 0.4, 0.5, 0.6. Sketch the and calculate the B.W., Power & Modulation efficiency.

Sol<sup>n</sup>:

$f_c = 1000 \text{ KHz}$	$\mu_1 = 0.4$	$A_c = 20$
$f_{m1} = 5 \text{ KHz}$	$\mu_2 = 0.5$	
$f_{m2} = 10 \text{ KHz}$	$\mu_3 = 0.6$	
$f_{m3} = 20 \text{ KHz}$		

A ⇒



→  $B_w = 2 f_{m3}$   
 $= 2 \times 20$   
 $B_w = 40 \text{ KHz}$

$$\therefore P_c = \frac{A_c^2}{2R}$$

$$\therefore P_c = \frac{400}{2(1)}$$

$$\therefore \boxed{P_c = 200 \text{ Watts}}$$

$$\therefore P_t = P_c \left[ 1 + \frac{\mu_t^2}{2} \right]$$

$$\therefore \mu_t^2 = \mu_1^2 + \mu_2^2 + \mu_3^2$$

$$\mu_t^2 = 0.16 + 0.25 + 0.36$$

$$\mu_t^2 = 0.77$$

$$\therefore P_t = 200 \left[ 1 + \frac{0.77}{2} \right]$$

$$\therefore \boxed{P_t = 277 \text{ W}}$$

$$\therefore \eta = \frac{\mu_t^2}{2 + \mu_t^2} = \frac{0.77}{2.77}$$

$$\therefore \boxed{\eta = 27.8}$$

Ex. 2 consider an Am signal  
 $s(t) = [10 + 4 \sin 2\pi 10^3 t - 8 \cos 2\pi 10^4 t] \times \cos 2\pi f_c t$   
 Determine the B.W. & power.

Sol<sup>n</sup>:  
 $s(t) = 10 \left[ 1 + 0.4 \sin 2\pi 10^3 t - 0.8 \cos 2\pi 10^4 t \right] \times \cos 2\pi f_c t$

$$\therefore A_c = 10, \quad \mu_1 = 0.4, \quad \mu_2 = -0.8$$

$$f_{m1} = 1 \text{ kHz}, \quad f_{m2} = 10 \text{ kHz}$$

$$\therefore BW = 2 f_m$$

$$BW = 20 \text{ kHz}$$

$$\mu_t^2 = 0.16 + 0.64$$

$$\mu_t^2 = 0.80$$

$$\rightarrow P_c = \frac{A_c^2}{2R}$$

$$R_c = \frac{100}{2}$$

$$\therefore P_c = 50 \text{ W}$$

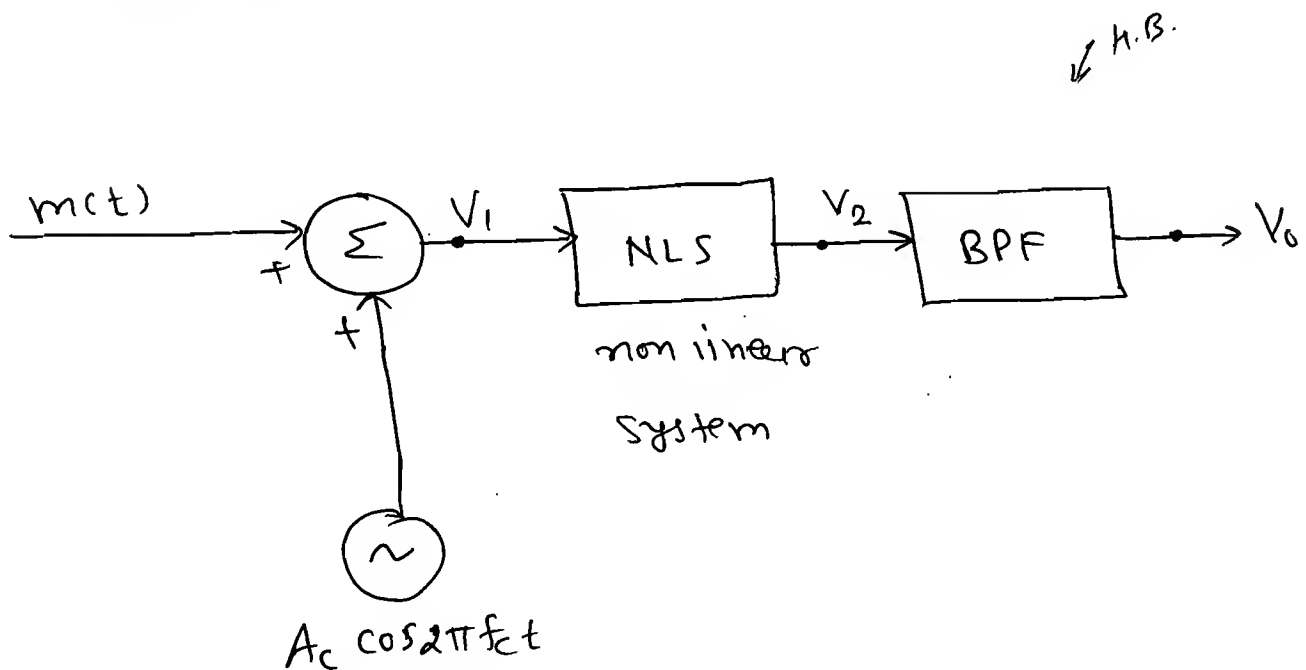
$$\mu = |K_a A_m|$$

$$\therefore P_t = 50 \left[ 1 + \frac{0.80}{2} \right]$$

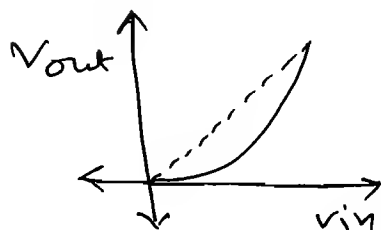
$$\therefore P_t = 70 \text{ W}$$

\* Generation of AM signal :-

⊙ Square Law Modulator:



→ The generalized characteristic of NLS:



$$V_{out} = a V_{in} + b V_{in}^2$$

↙ H.B.

$$\Rightarrow V_1 = [m(t) + A_c \cos 2\pi f_c t]$$

$$\therefore V_2 = aV_1 + bV_1^2$$

$$\begin{aligned} \therefore V_2 &= am(t) + aA_c \cos 2\pi f_c t \\ &\quad + bm^2(t) + 2bmA_c m(t) \cdot \cos 2\pi f_c t \\ &\quad + bA_c^2 \cos^2 2\pi f_c t \end{aligned}$$

→ The bandpass filter is used to select the ~~freq.~~ Component of Am signal.

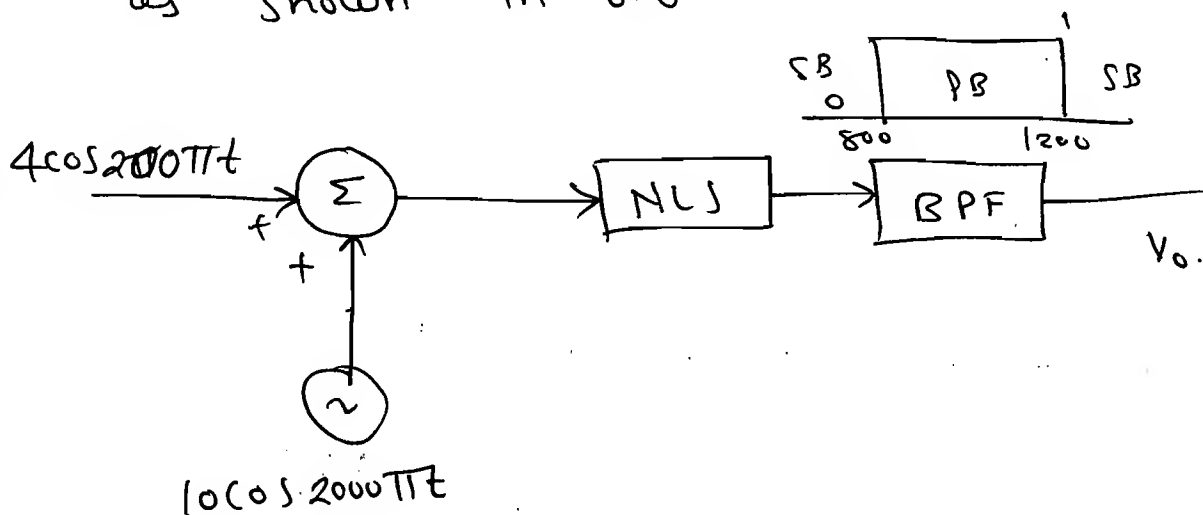
⇒ Output of bandpass filter.

$$\therefore V_0 = aA_c \cos 2\pi f_c t + 2bA_c m(t) \cdot \cos 2\pi f_c t$$

$$\therefore V_0 = aA_c \left[ 1 + \left( \frac{2b}{a} \right) m(t) \right] \cos 2\pi f_c t$$

$$K_a = \frac{2b}{a}$$

Ex-1 Consider a square law modulator as shown in fig.





The BPF is ideal filter having passband from 800 Hz to 1200 Hz. Det. modulation index, B.W. & power of AM signal.

$$V_2 = V_1 + 0.1 V_1^2$$

Ans:

$$V_1 = 4 \cos 2000\pi t + 10 \cos 2000\pi t.$$

$$\therefore V_2 = V_1 + 0.1 V_1^2.$$

$$\therefore V_2 = [4 \cos 2000\pi t + 0.1 [4 \cos 2000\pi t + 10 \cos 2000\pi t]^2]$$

$$\therefore V_2 = 4 \cos 2000\pi t + 10 \cos 2000\pi t + 0.16 \cos^2 2000\pi t + 0.8 \cos 2000\pi t \cdot \cos 2000\pi t - 1 \cdot \cos^2 2000\pi t.$$

$$\therefore V_2 = 10 \cos 2000\pi t + 8 \cos 2000\pi t \cdot \cos 2000\pi t.$$

$$V_2 = 10 [1 + 0.8 \cos 2000\pi t] \cos 2000\pi t.$$

$$\therefore V_2 = A_c [1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t.$$

$$\therefore \boxed{A_c = 10V}$$

$$f_m = 100 \text{ Hz}$$

$$\therefore \mu = 0.8$$

$$f_c = 1000 \text{ Hz}.$$

$$\therefore B_W = 2 \times 100$$

$$\therefore \boxed{B_W = 200 \text{ Hz.}}$$

$$P_t = P_c \left[ 1 + \frac{\mu^2}{2} \right].$$

$$\therefore P_t = 50 \left[ 1 + \frac{0.64}{2} \right].$$

$$P_c = \frac{A_c^2}{2R}$$

$$\boxed{P_t = 66 \text{ W}}$$

$$P_c = \frac{100}{2} = 50 \text{ watts.}$$

\* Crate point of view!  $\downarrow$  H.B.

$$\mu = K_a A_m.$$

$$\therefore \boxed{\mu = (2b/a) A_m} = 0.2 \times 4 = 0.8.$$

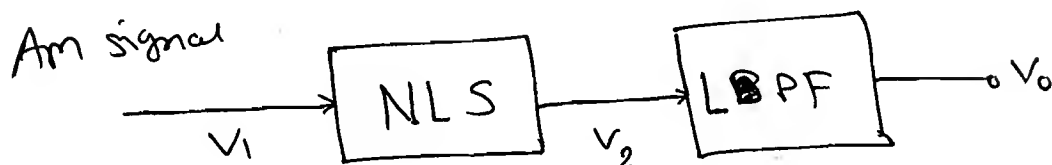
$$\therefore \boxed{K_a = 2b/a}$$

$$\therefore K_a = 2 \times 0.1 = 0.2$$

\* Demodulation of AM signal:

- ① Square law demodulator
- ② Envelope detector
- ③ Synchronous detector.

① Square Law demodulator:



$$V_1 = A_c \cos 2\pi f_c t + K_a m(t) A_c \cos 2\pi f_c t.$$

$$\therefore V_2 = aV_1 + bV_1^2$$

$$\therefore V_2 = aA_c \cancel{\cos 2\pi f_c t} + aK_a m(t) A_c \cancel{\cos 2\pi f_c t}$$

$$+ bA_c^2 \cancel{\cos^2 2\pi f_c t} + 2bA_c^2 K_a m(t) \cos^2 2\pi f_c t$$

$$+ bK_a^2 m^2(t) A_c^2 \cos^2 2\pi f_c t.$$

$$\downarrow$$

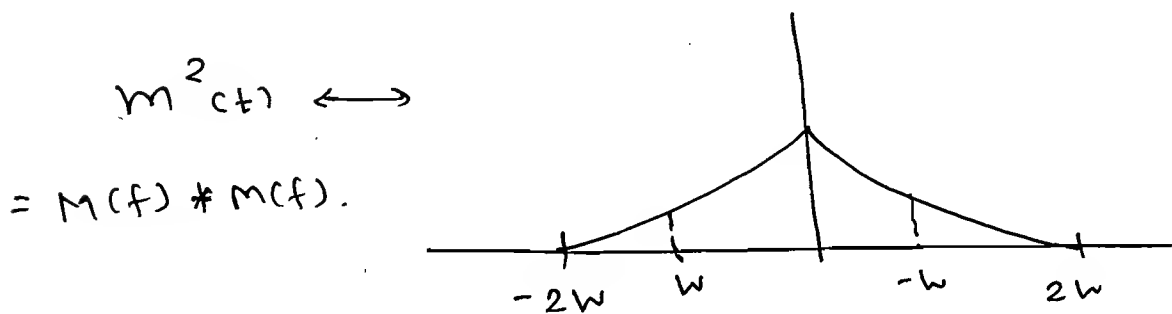
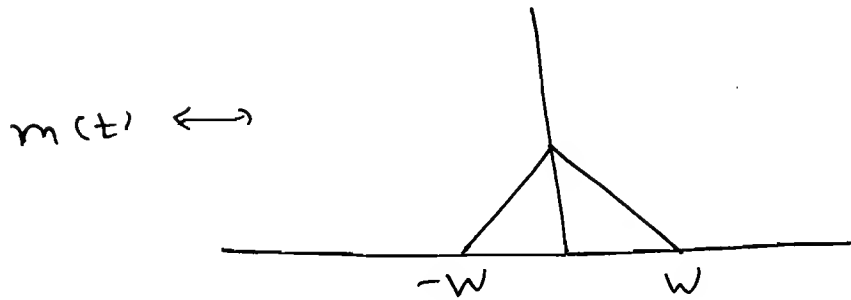
$$\frac{m^2(t)}{2} + \frac{m^2(t) \cdot \cancel{\cos^2 2\pi f_c t}}{2}$$

$$\downarrow$$

$$\frac{m(t)}{2} + \frac{m(t) \cos 2\pi f_c t}{2}$$

$$\therefore V_o = bA_c^2 m(t) K_a \cos. + \frac{bK_a^2 m^2(t) A_c^2}{2}$$

$$V_o = \underbrace{bA_c^2 K_a [m(t)]}_{\text{signal}} + \underbrace{\frac{bK_a^2 A_c^2}{2} [m^2(t)]}_{\text{noise.}}$$



$\Rightarrow$  The output of demodulator consist of  $m(t)$  and  $m^2(t)$ .

$\Rightarrow m^2(t)$  is also having significant low frequencies so it is not eliminated.

$$\rightarrow \frac{\text{Signal Component}}{\text{Noise Component}} = \frac{2}{K_a m(t)}$$

$$\therefore \boxed{\frac{S}{N} = \frac{2}{K_a A_m \cos 2\pi f_m t}} \leftarrow \text{H.B.} =$$

$$\therefore \left(\frac{S}{N}\right)_{\min} = \frac{2}{K_a A_m} = 2/\mu.$$

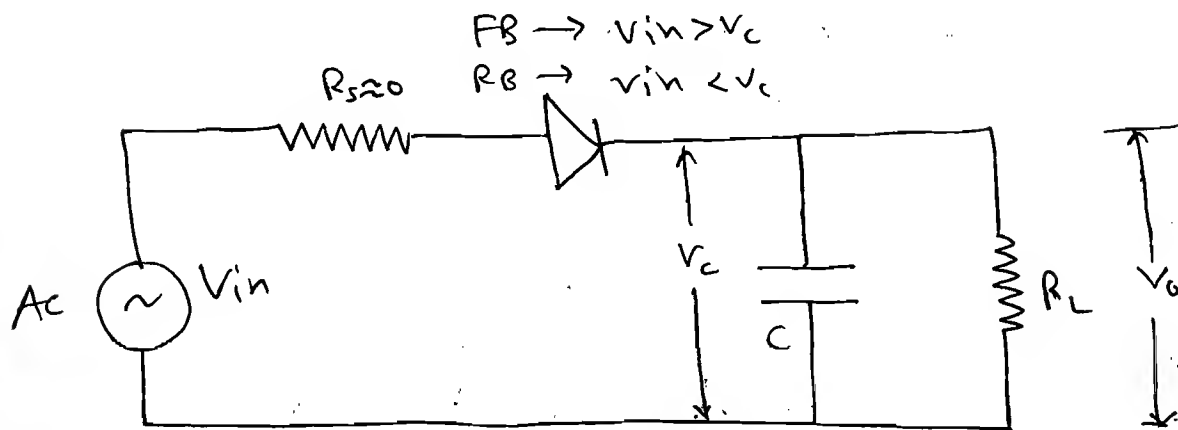
$$\left(\frac{S}{N}\right)_{\min} = \frac{2}{\mu} = 0.2 \quad ; \quad \mu = 10$$

$$= 20 \quad ; \quad \mu = 0.1$$

$$= 1 \quad ; \quad \mu = 2.$$

( $\mu$  should be  $< 2$  in this case).

## ② Envelope Detector:

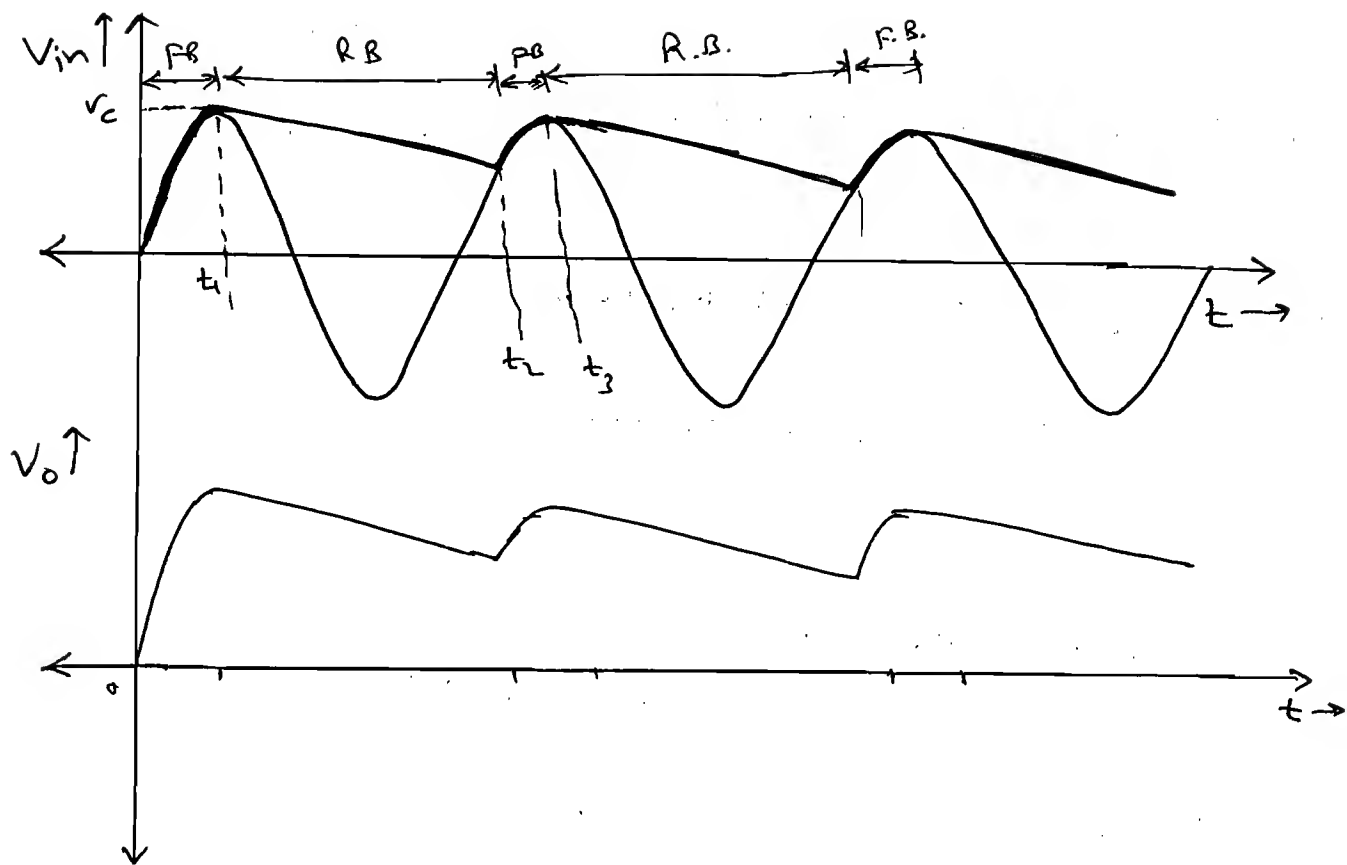


$\Rightarrow$  Hardware implementation is very simple. ( $\mu \leq 1$ ).

$\rightarrow$  In AM signal the peak amplitude of the carrier which is also called as envelope is varied according to the message signal. So, the envelope of the AM signal represent the message signal. Envelope detector is used to track the peak amplitude of an AM signal.

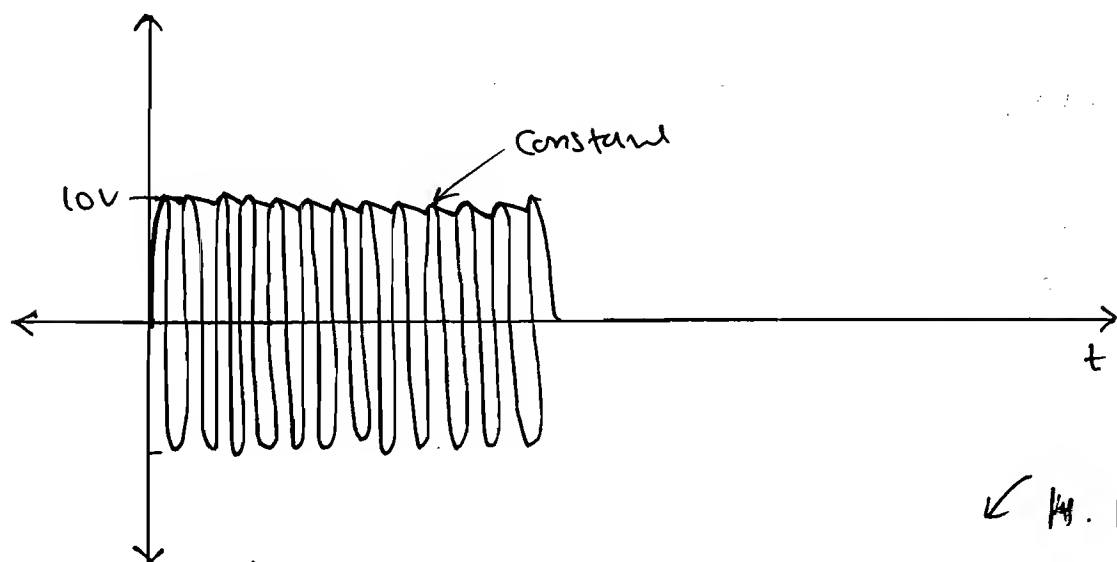
$\Rightarrow$   $R_s C \rightarrow$  very low (charging time constant)

$R_L C \rightarrow$  very high (discharging time constant)



$\Rightarrow$   $A_m$  is very high freq. signal means time period is very very less. means  $V_o$  seen like as follow:

eg. if  $A_m$  is  $10 \cos 2\pi f_c t$  then  $V_o$  is ~~10V~~ 10V const.



IMP

Imp

W.B.

$$\rightarrow A_c [1 + K_m(t)] \cos 2\pi f_c t$$

$$\rightarrow A_c \cos 2\pi f_c t$$

$$\rightarrow e^t \cos 2\pi f_c t$$

$$V_o = A_c [1 + K_m(t)]$$

$$V_o = A_c$$

$$V_o = e^t$$

$$\rightarrow A_c \cos 2\pi f_c t + B \sin 2\pi f_c t$$

$$V_o = \sqrt{A_c^2 + B^2} \cdot \angle$$

$$\Rightarrow V_o = A_c [1 + K_a m(t)]$$

$$V_o = \underbrace{A_c}_{\text{DC}} + A_c K_a m(t)$$

DC can be eliminated by filter.

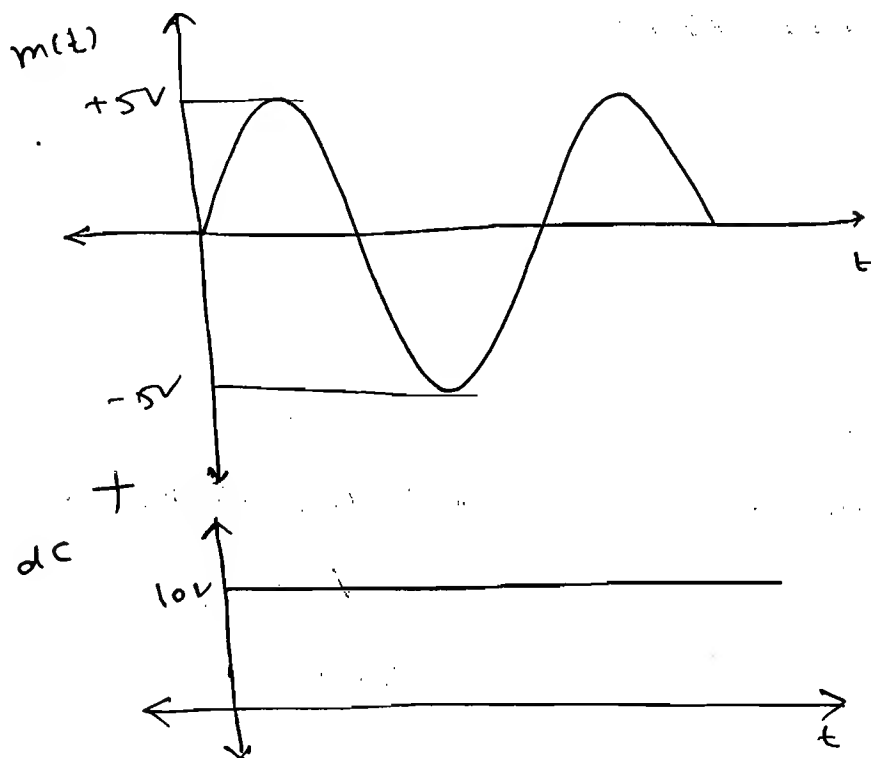
$$V_o = A_c K_a m(t)$$

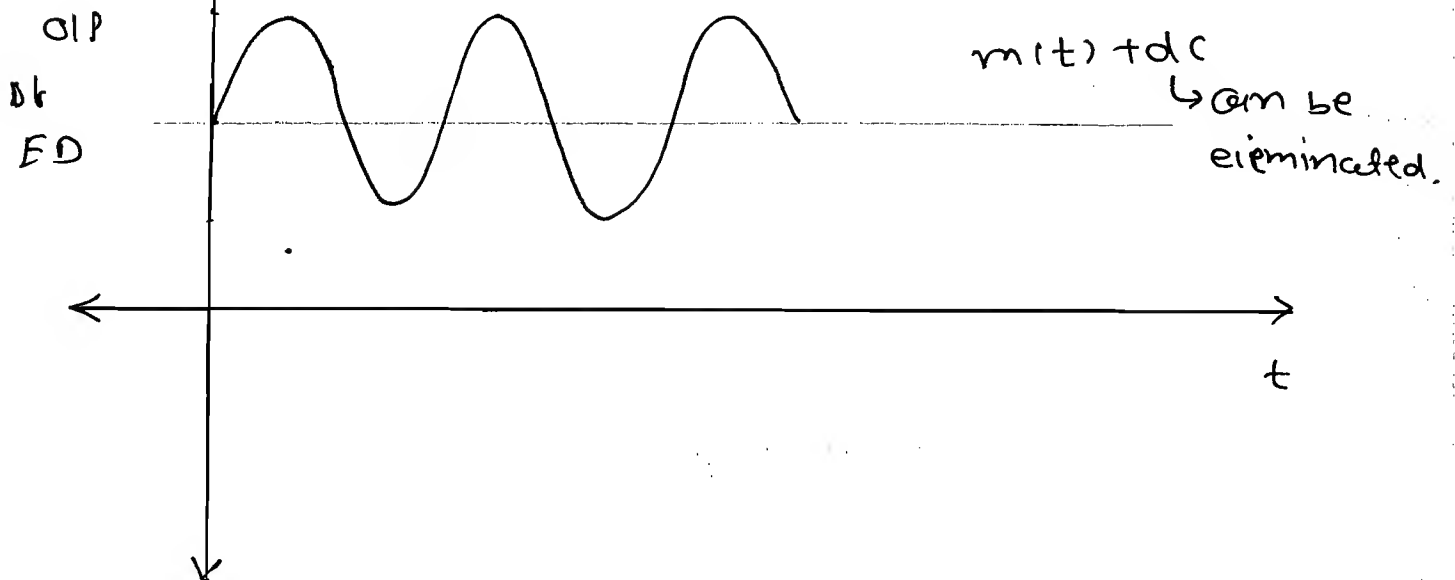
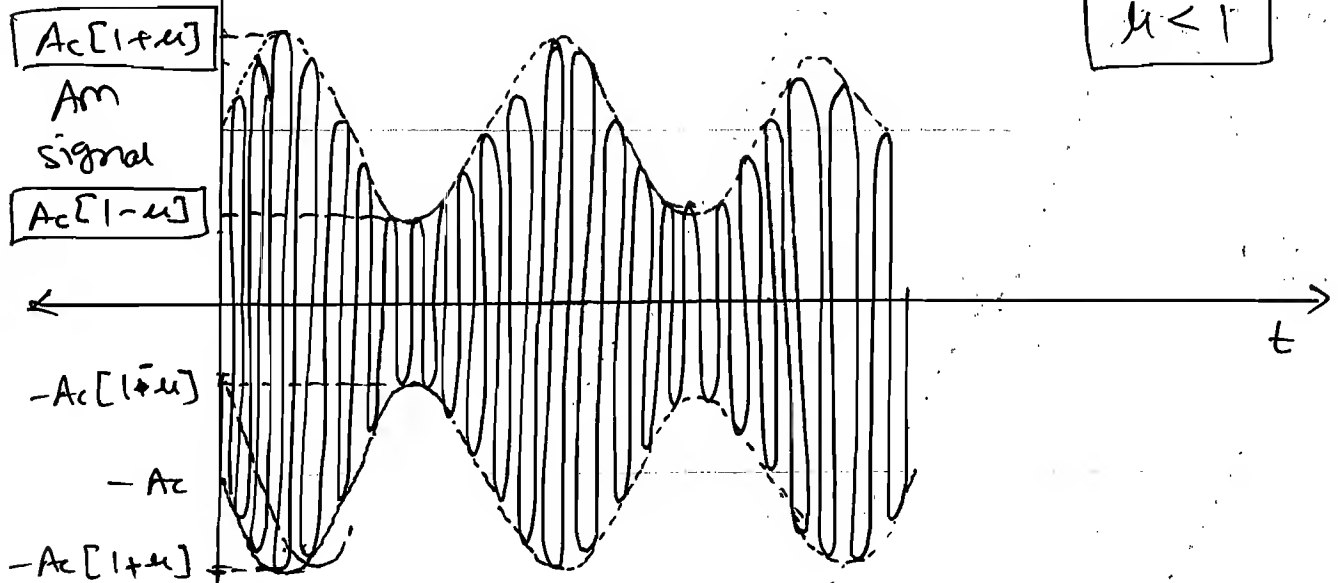
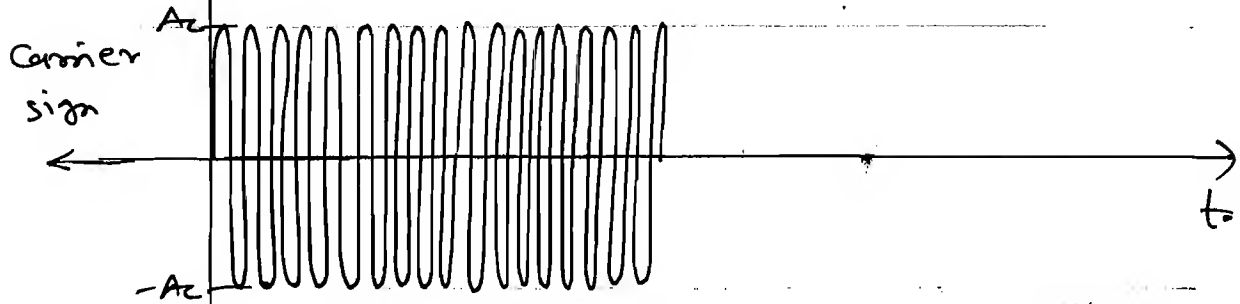
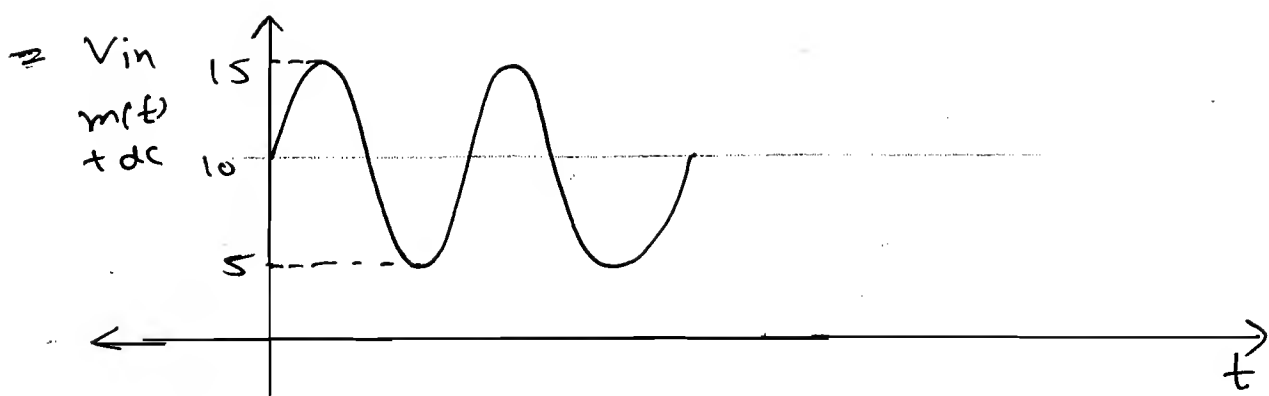
$$\text{if } K_a = 1/A_c$$

$$\therefore V_o = m(t)$$

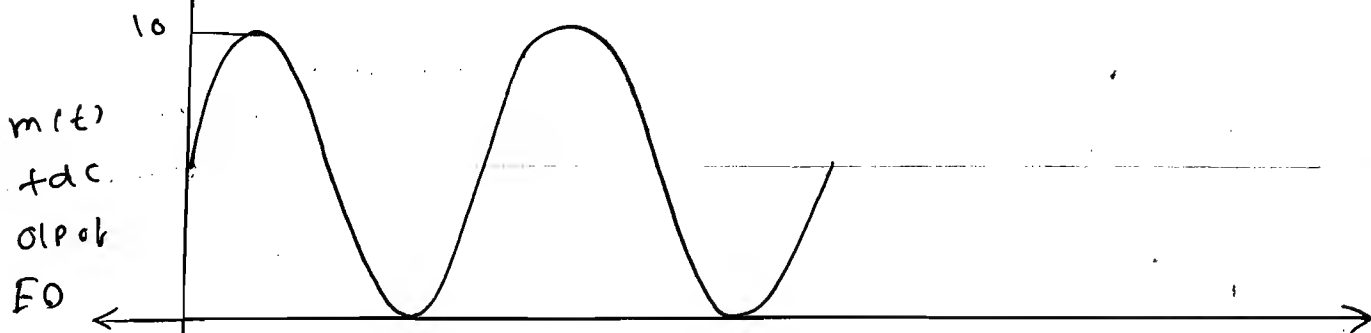
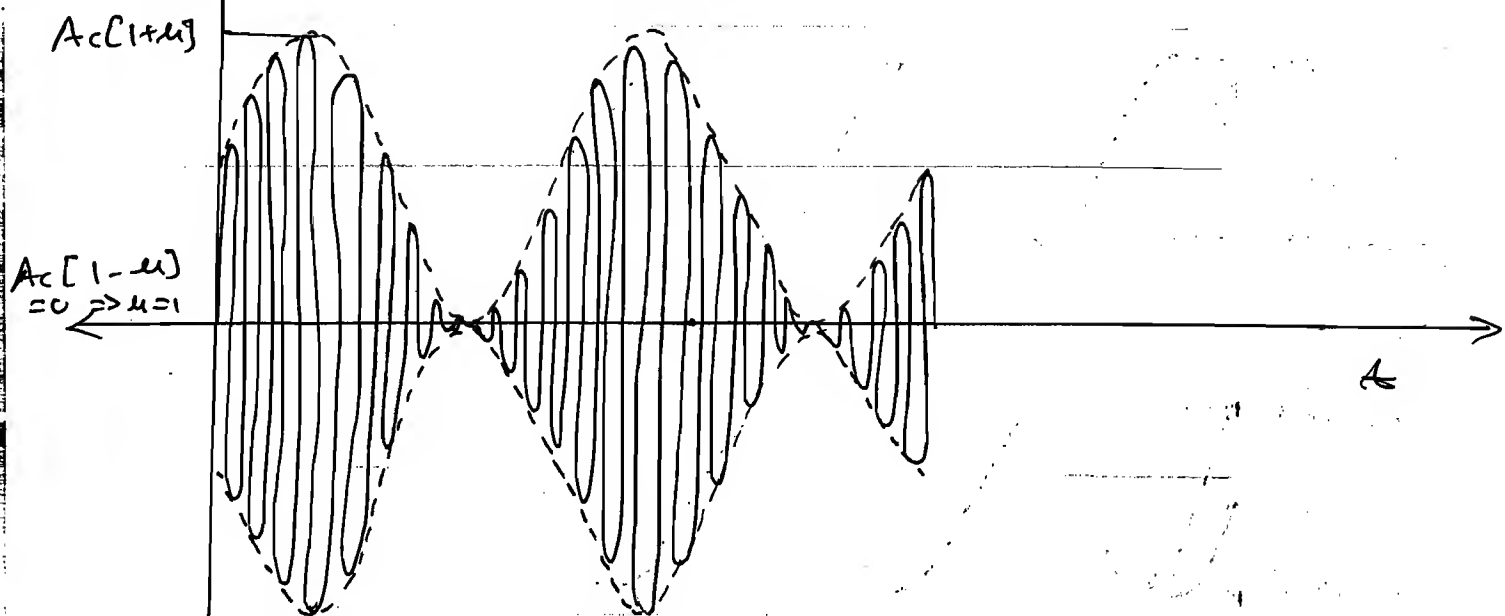
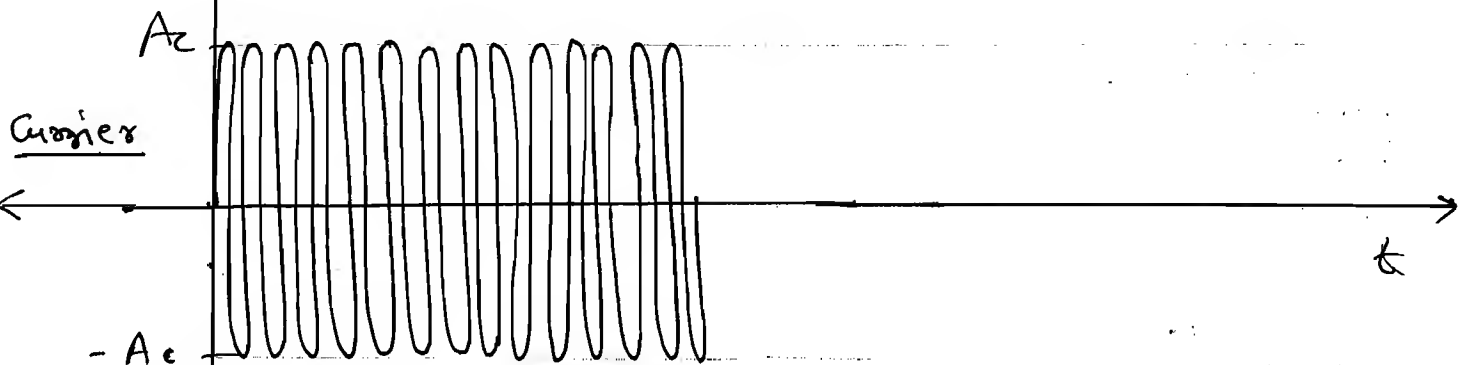
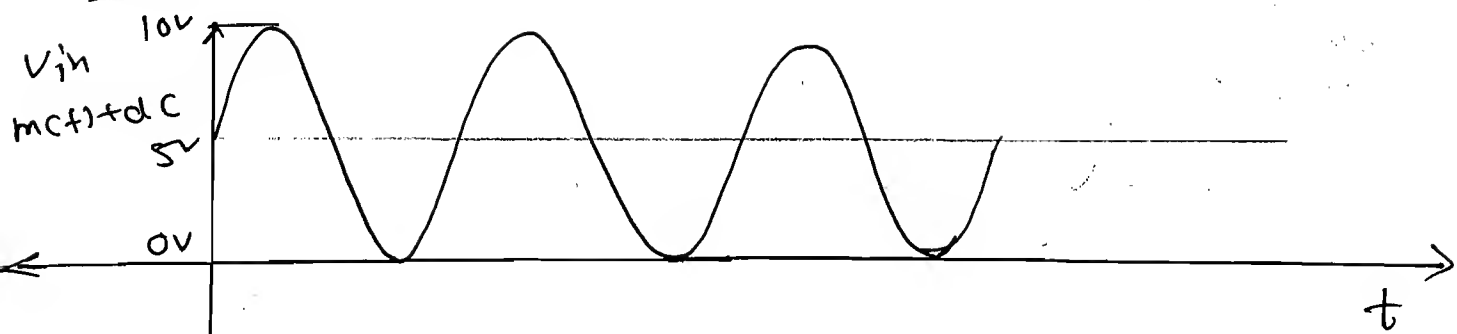
### \* Limitation of envelope Detector:

→ Consider a <sup>sin</sup> signal whose amplitude varies from +5 to -5. A dc component of 10V is added to the message signal before multiplying with carrier.





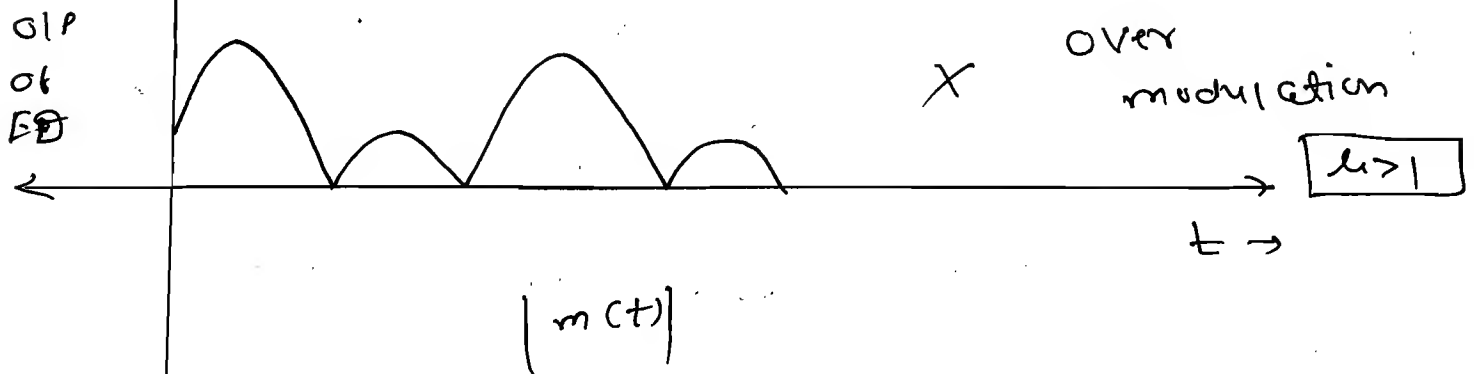
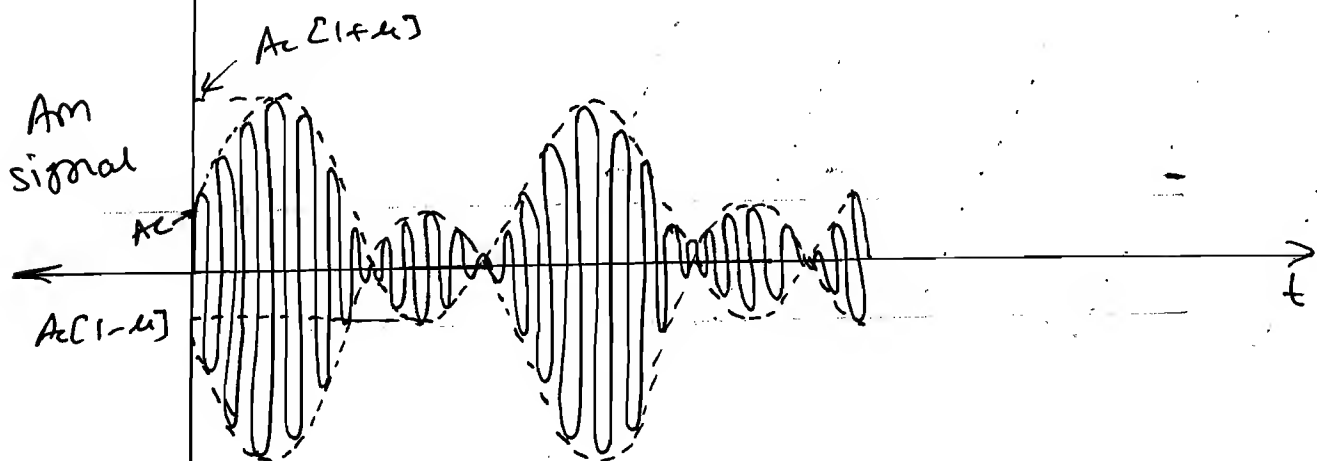
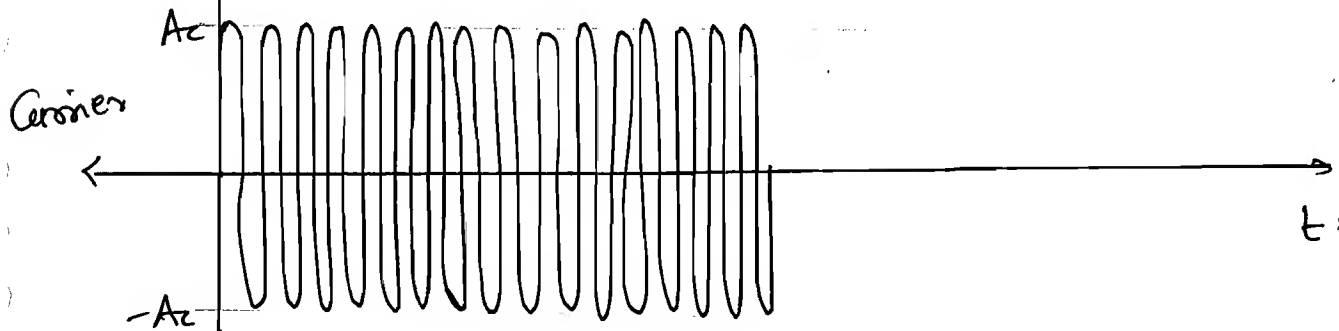
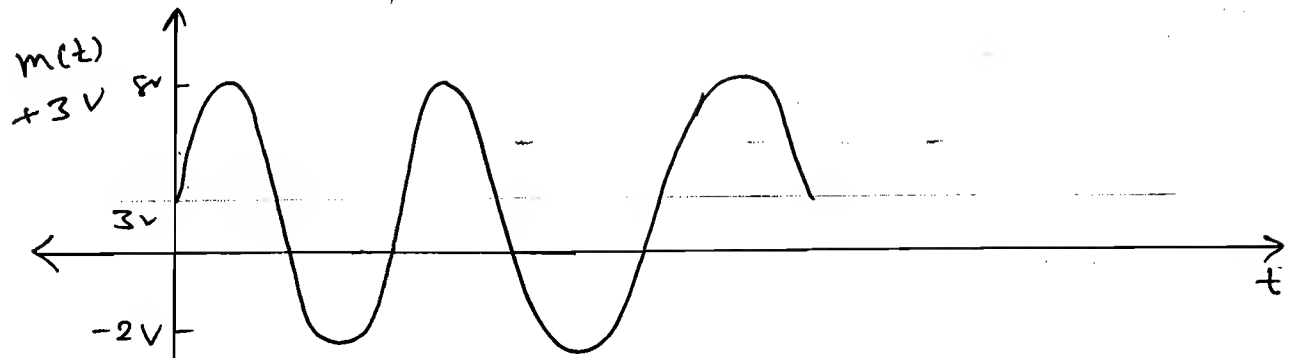
\*  $\mu = 1$

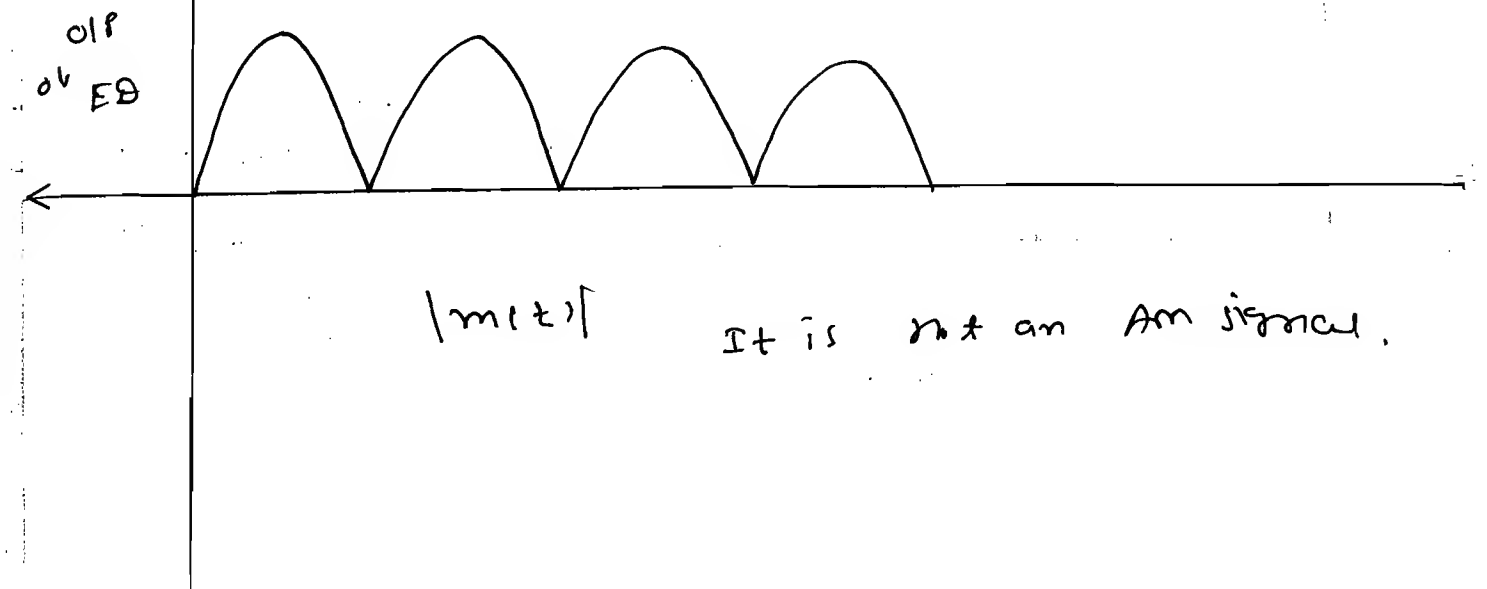
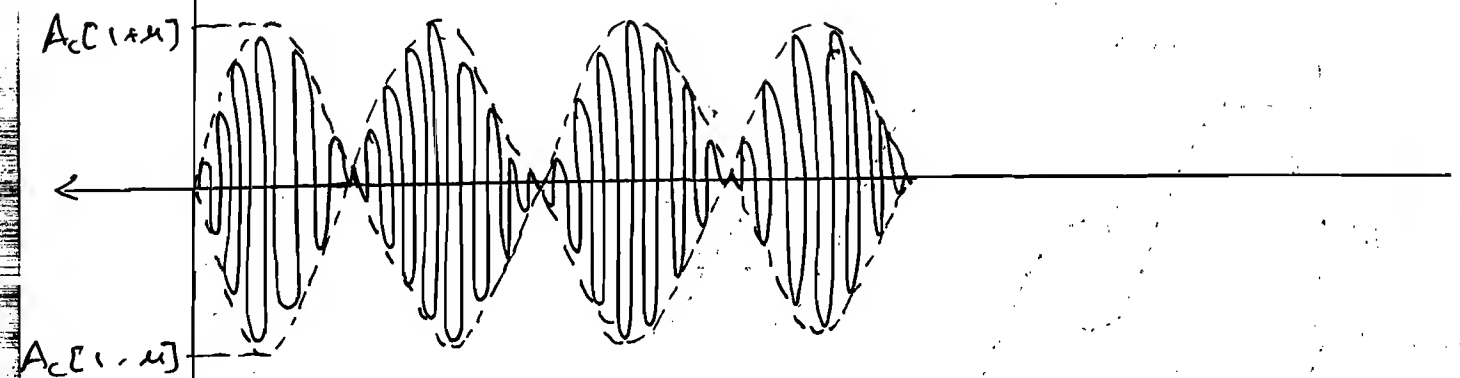
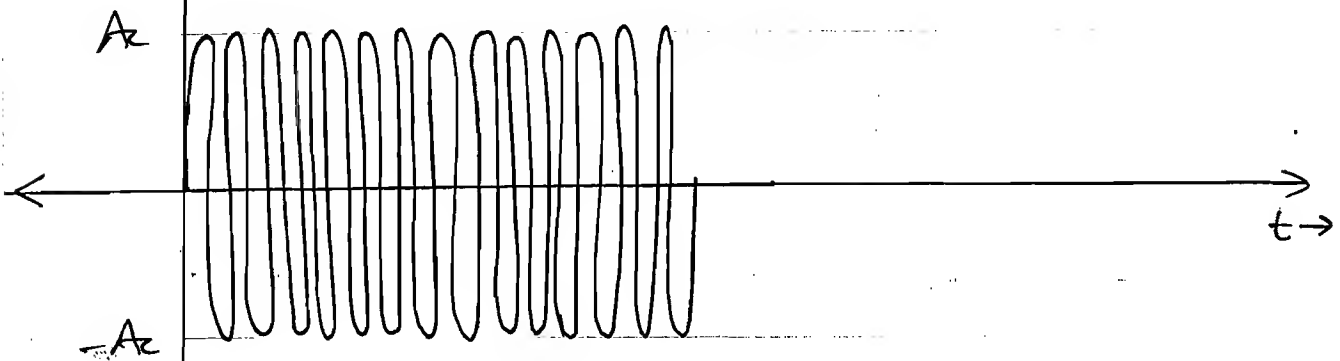
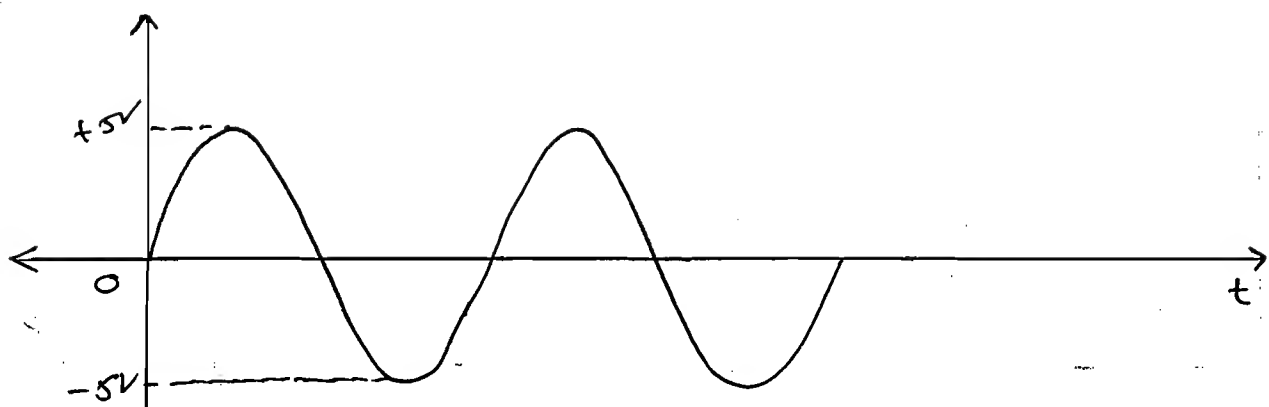


$m(t) + \cancel{dc}$



\*  $\boxed{\mu > 1}$





It is not an Am signal.

$$\textcircled{1} \quad \mu \leq 1 \longrightarrow V_o = \frac{O/P}{A_c} [1 + K_a m(t)] \quad \swarrow \underline{H.B.}$$

$$\textcircled{2} \quad \mu > 1 \longrightarrow V_o = |A_c [1 + K_a m(t)]|$$

$$\textcircled{3} \rightarrow [dc + m(t)] \cos 2\pi f_c t$$

$$\rightarrow dc \cos 2\pi f_c t + m(t) \cos 2\pi f_c t$$

$\textcircled{3}$  A dc component is added to the message signal & multiplied with carrier to generate the Am signal but the demodulation is possible only when  $m(t) + dc$  is positive.

$\textcircled{4}$   $K_a$  is used in analysis to normalized the peak amplitude of the message signal so that the modulation index is  $\leq 1$ .

$$\mu = K_a A_m$$

When,  $K_a = 1$  there is possibility of over modulation.

$$\therefore \mu = A_m \quad m(t) = 2 \cos 2\pi \times 10^3 t$$

$$\therefore \mu = 2$$

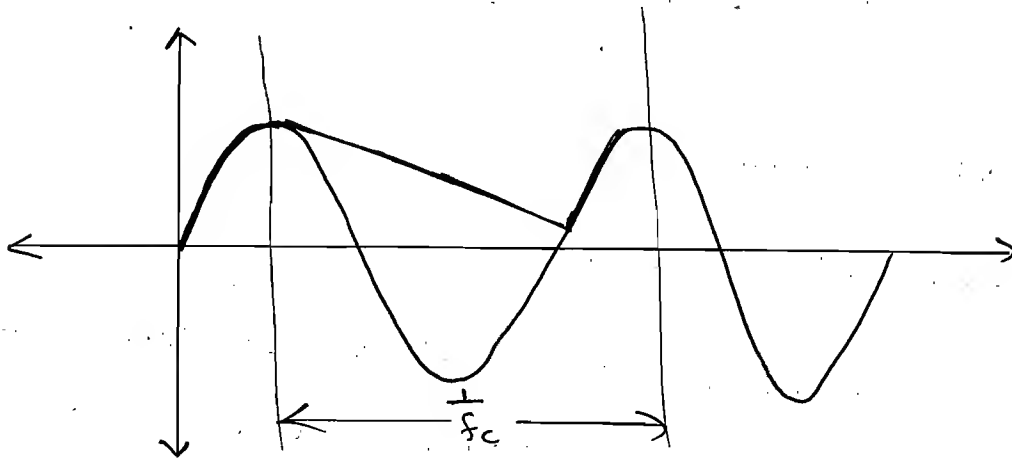
$\textcircled{5}$  The time constant of envelope detector should satisfy following conditions.

$$\textcircled{1} \quad R_L C \gg \frac{1}{f_c}$$

$$\textcircled{2} \quad R_L C \ll \frac{1}{f_m}$$

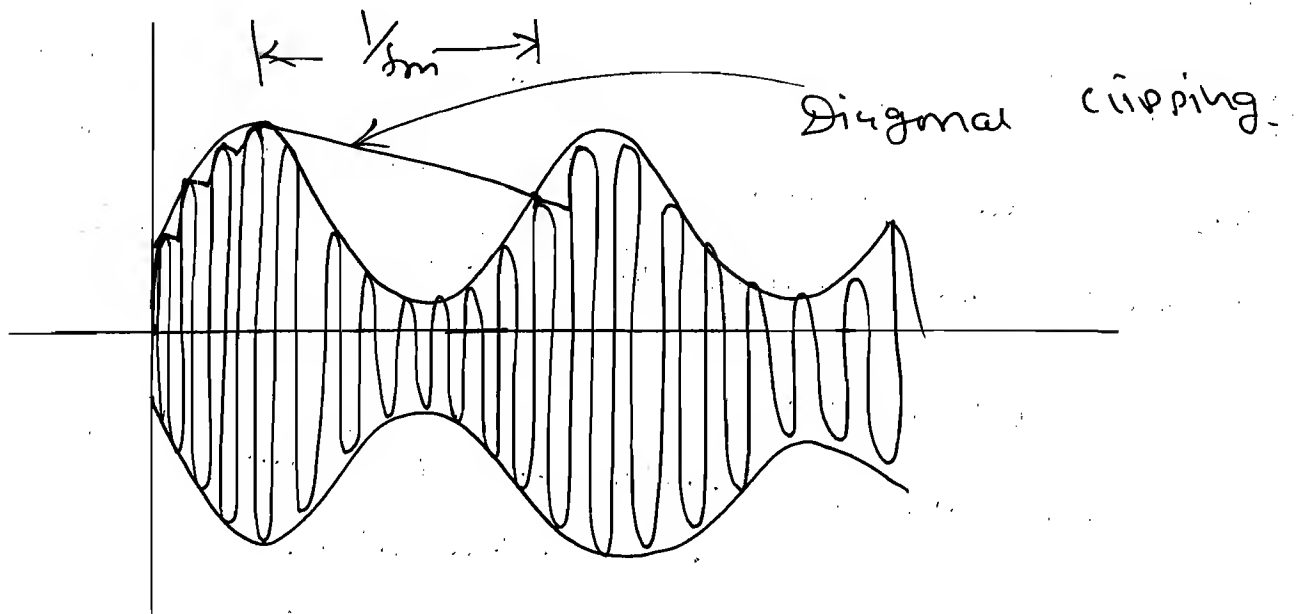
$$\frac{1}{f_c} \ll R_L C \ll \frac{1}{f_m} \quad \leftarrow \underline{H.B.}$$

①



In this case we can not able to track the output (or) envelope.

② \*



Gate-2004

Ex-1 An Am signal is demodulated using an envelop detector the carrier freq. is 1 MHz and the msg. freq. is 2 kHz. A suitable value for the time constant of the envelop detector is,

- (A) 0.2  $\mu$ s (C) 20  $\mu$ s  
(B) 1  $\mu$ s (D) 500  $\mu$ s.

Ans:

$$\frac{1}{f_c} \ll R_L C \ll \frac{1}{f_m}$$

$$\therefore \frac{1}{1 \text{ MHz}} \ll R_L C \ll \frac{1}{2 \text{ kHz}}$$

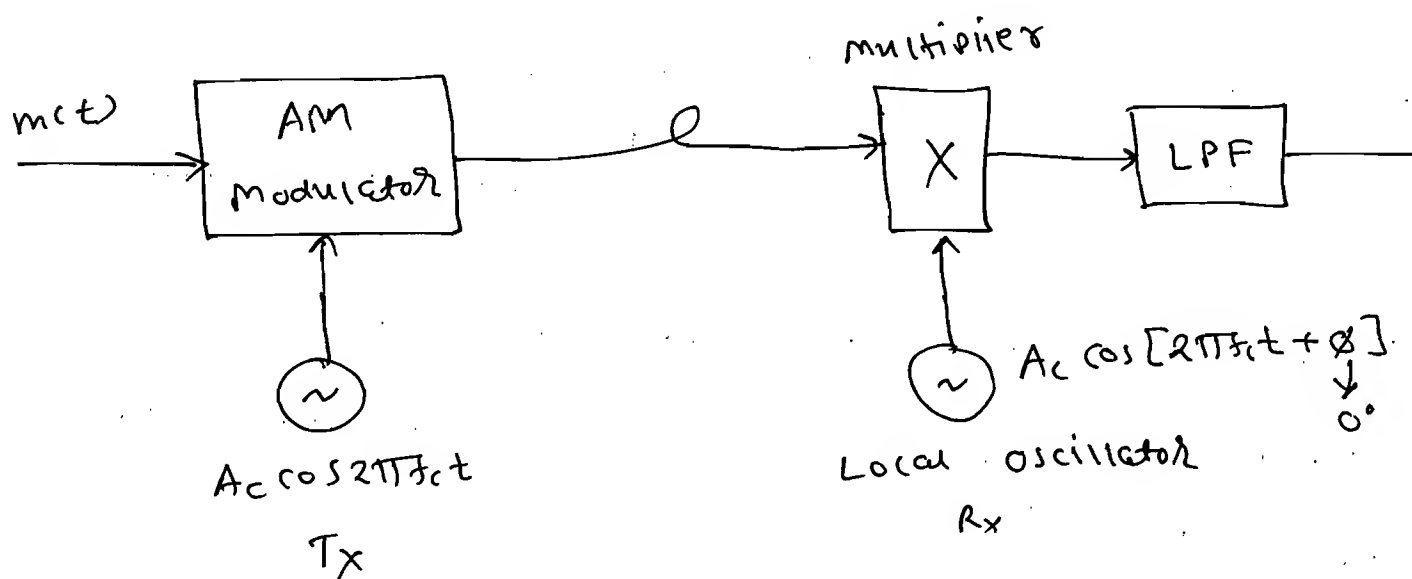
$$\therefore \frac{1}{10^6} \ll R_L C \ll \frac{1}{2 \times 10^3}$$

$$\therefore \boxed{1 \mu\text{s} \ll R_L C \ll 500 \mu\text{s}}$$

So, valid ans is (c) 20  $\mu\text{s}$ .

### (3) Synchronous Detector:-

$\Rightarrow$  Synchronous detector is capable of demodulating the overmodulating <sup>AM</sup> signal also



$\Rightarrow$  Output of multiplier,

$$\rightarrow (Am)(Lo)_o$$

$$= [A_c \cos 2\pi f_c t + A_c m(t) \cos 2\pi f_c t] A_c \cos 2\pi f_c t$$

$$= A_c^2 \cos^2 2\pi f_c t + A_c^2 m(t) \cos^2 2\pi f_c t$$

$$= \frac{A_c^2}{2} + \frac{A_c^2 \cos 2\pi (2f_c) t}{2} + \frac{A_c^2 m(t)}{2} + \frac{A_c^2 m(t) \cos 2\pi (2f_c) t}{2}$$

$$\therefore \text{O/P of LPF} = \frac{A_c^2}{2} m(t).$$

$\Rightarrow$  Assume that the phase shift is existing b/w carrier used at transmitter and carrier generated at receiver.

$\Rightarrow$  Then O/P of multiplier,

$$[A_c \cos 2\pi f_c t + A_c m(t) \cos 2\pi f_c t] \times A_c \cos[2\pi f_c t + \phi]$$

$\Rightarrow$  finally O/P of LPF is

$$\left[ \frac{A_c^2}{2} m(t) \right] \cos \phi.$$

if  $\phi = 0^\circ \rightarrow \frac{A_c^2}{2} m(t).$

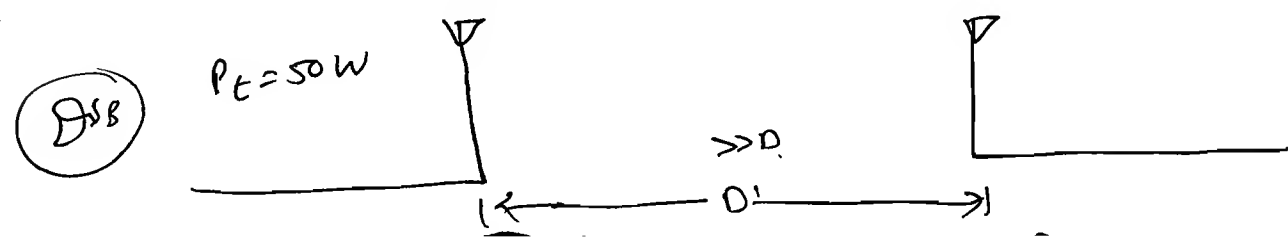
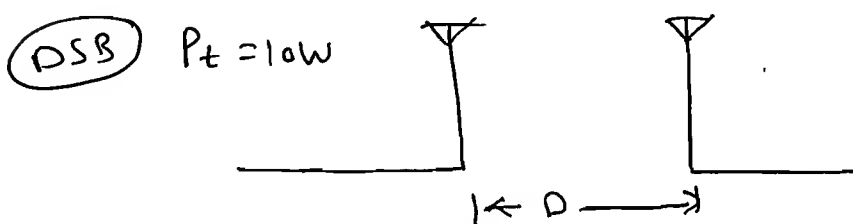
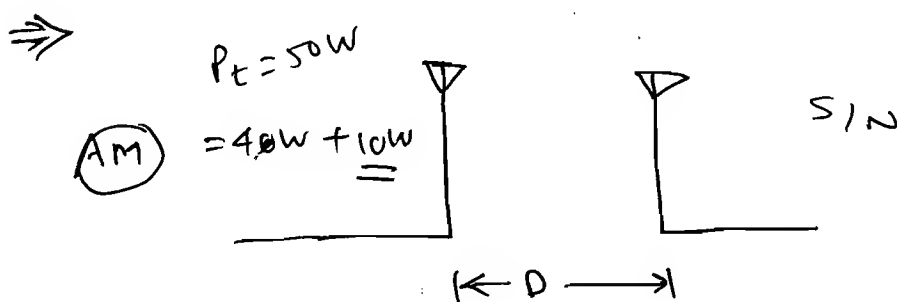
$\phi = 90^\circ \rightarrow 0 \rightarrow \boxed{\text{Quadrature null effect}}$

$\rightarrow$  To overcome the Quadrature null effect Synchronization should be maintained b/w  $T_x$  &  $R_x$ . So, Additional Hardware is required to maintain Synchronization. The Hardware complexity of the synchronization detector is very high when compared with envelope detector. So envelope detector is used as Demodulator for AM.

## ② DSBSC Modulation:- (Double side Band Suppressed Carrier)

⇒ In AM the maximum efficiency possible is 33.3%; in the case of single tone modulation. So, the maximum sideband power is only 33.3% of the total power. When the carrier is suppressed the total power & sideband power are equal. So, modulation efficiency is 100%.

→ To cover the same distance in a wireless communication system DSB modulation require less power with compared with AM. If the same power is used in DSB the distance b/w Tx & Rx is increased.



$$\Rightarrow S(t) = A_c \cancel{\cos 2\pi f_c t} + A_c k_a m(t) \cos 2\pi f_c t.$$

$\downarrow$   
 $k_a = 1$

$\Rightarrow$  Time domain eq<sup>n</sup> of DSB...

$$S(t) = A_c m(t) \cos 2\pi f_c t.$$

H.B.

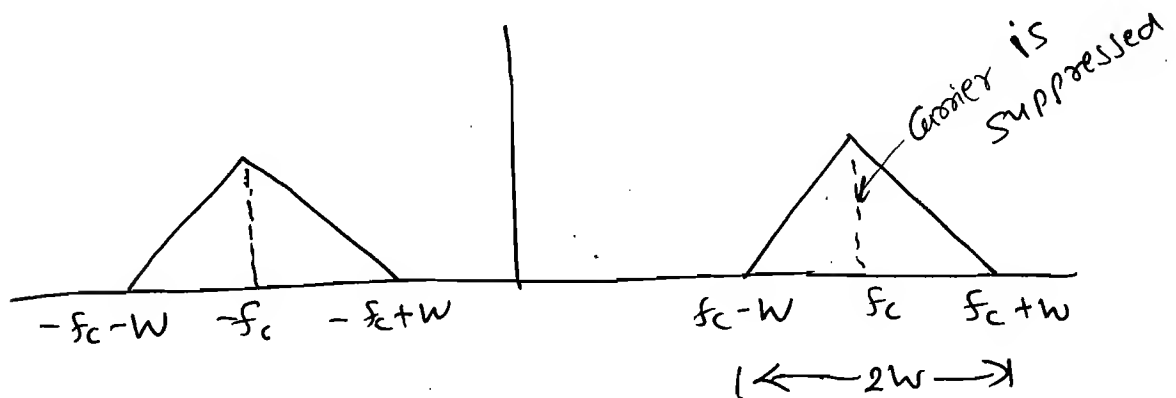
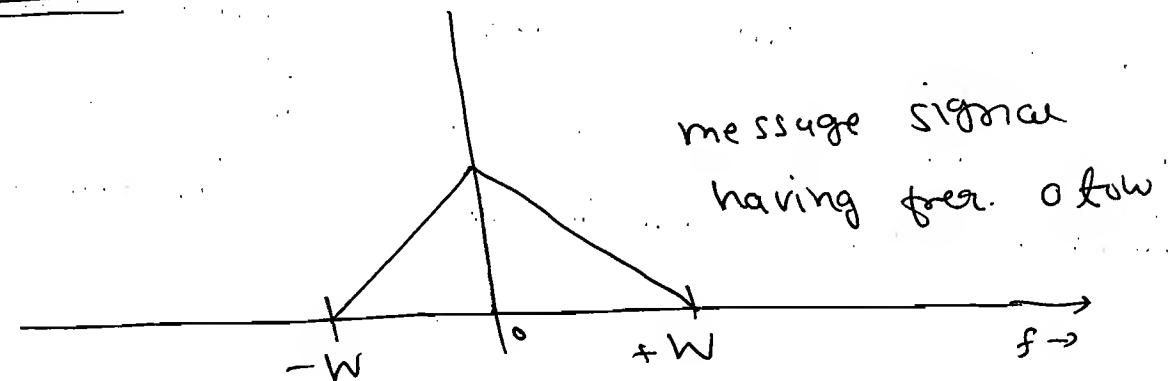
$$S(t) = m(t) \cdot c(t).$$

$\Rightarrow$  Freq. domain eq<sup>n</sup> of DSB,

$$S(f) = \frac{A_c}{2} [M(f - f_c) + M(f + f_c)]$$

H.B.

$\Rightarrow$  Spectrum:





# \* Single tone Modulation of DSB.

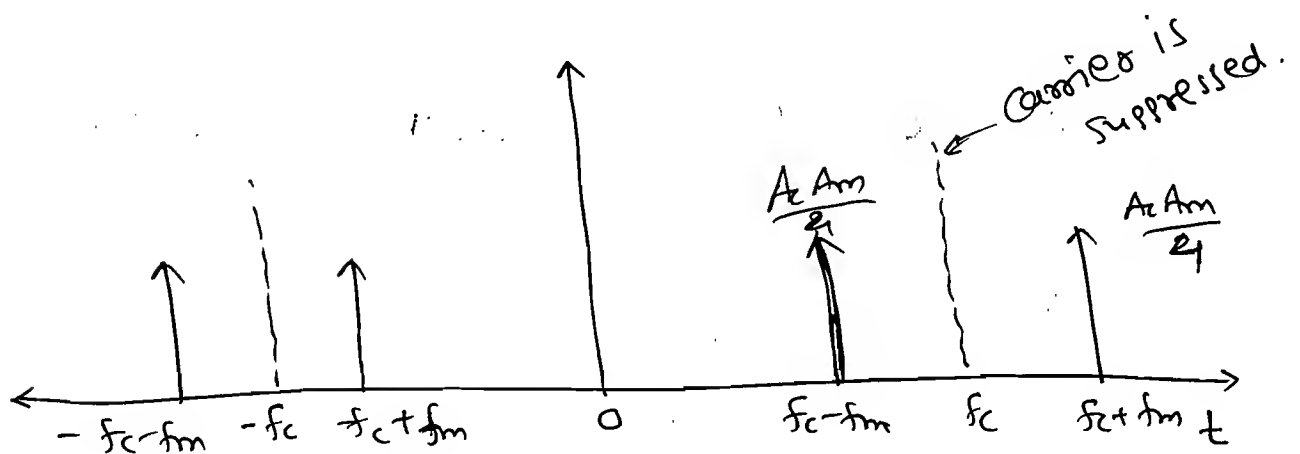
$$\Rightarrow S(t) = A_c m(t) \cdot \cos 2\pi f_c t$$

$$= A_c \cdot A_m \cdot \cos 2\pi f_m t \cdot \cos 2\pi f_c t$$

$$= \frac{A_c \cdot A_m}{2} [\cos 2\pi (f_c + f_m) t + \cos 2\pi (f_c - f_m) t]$$

$$\therefore S(t) = \underbrace{\frac{A_c A_m}{2} \cos 2\pi (f_c + f_m) t}_{\text{USB}} + \underbrace{\frac{A_c A_m}{2} \cos 2\pi (f_c - f_m) t}_{\text{LSB}}$$

H.B



## \* Power Calculation:

$$\Rightarrow P_t = P_{\text{USB}} + P_{\text{LSB}}$$

$$\therefore = \frac{\left( \frac{A_c A_m}{2\sqrt{2}} \right)^2}{R} + \frac{\left( \frac{A_c A_m}{2\sqrt{2}} \right)^2}{R}$$

$$P_t = \frac{A_c^2 A_m^2}{8R} + \frac{A_c^2 A_m^2}{8R}$$

$$\therefore \boxed{P_t = \frac{A_c^2 A_m^2}{4R}} \quad \text{H.B.}$$

$$P_t = \frac{A_c^2 A_m^2}{4R} \quad (W) \quad H.B.$$

$$\therefore \eta = 100\%$$

$$\Rightarrow P_t = \cancel{P_c} + \frac{P_c \mu^2}{2}$$

$$\therefore P_t = \frac{P_c \mu^2}{2}$$

$$\therefore P_c = \frac{A_c^2}{2R}, \quad K_a = 1 \Rightarrow A_m = \mu.$$

$$\therefore P_t = \frac{A_c^2 A_m^2}{4R} \quad H.B.$$

Ex-1 A carrier signal  $c(t) = 20 \cos 2\pi 10^6 t$  is modulated by a message signal  $m(t) = 5 \cos 2\pi 10^4 t$  to generate a DSB signal. Sketch the spectrum & calculate the B.W., power & modulation efficiency.

Ans:  $c(t) = 20 \cos 2\pi 10^6 t.$

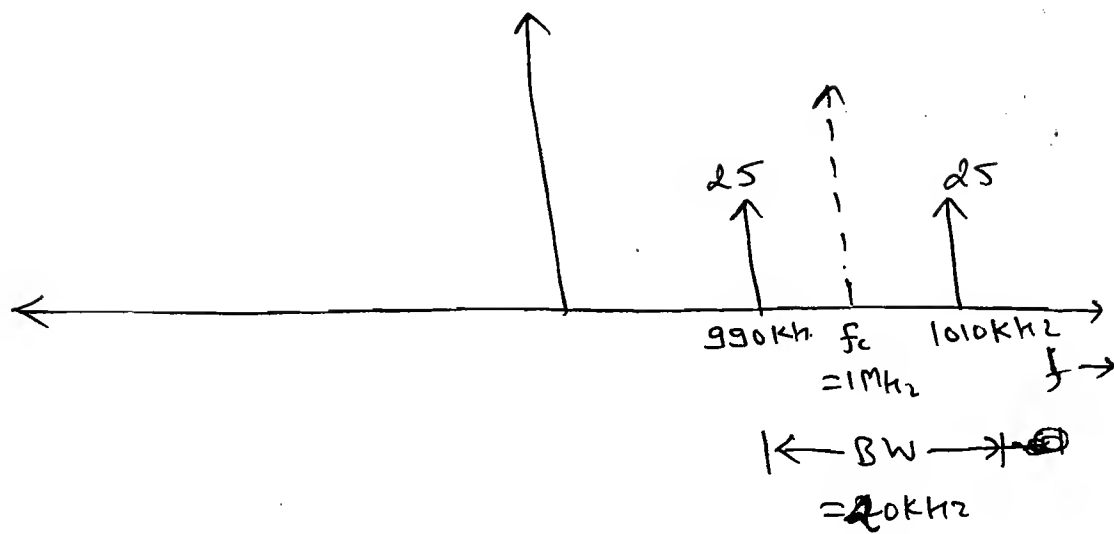
$$\Rightarrow A_c = 20.$$

$$f_c = 10^6 \text{ Hz} = 1 \text{ MHz.}$$

$$\Rightarrow m(t) = 5 \cos 2\pi 10^4 t$$

$$\Rightarrow f_m = 10 \text{ KHz}$$

$$A_m = 5$$



$$\Rightarrow BW = 2f_m = 2 \times 10 \text{ KHz}$$

$$BW = 20 \text{ KHz}$$

$$\Rightarrow P_t = \frac{A_c^2 A_m^2}{2R}$$

$$n = 100\%$$

$$\therefore P_t = \frac{400 \times 25}{2 \times R}$$

$$\therefore P_t = 2.5 \text{ kW}$$

Ex-2 Repeat the above problem when the msg signal  $m(t) = 5 \cos 2\pi 10^4 t + 2 \cos 8\pi 10^3 t$ .

Ans:

$$f_{m1} = 10 \text{ KHz}, \quad f_{m2} = 4 \text{ KHz}$$

$$A_{m1} = 5 \text{ V}, \quad A_{m2} = 2 \text{ V}$$

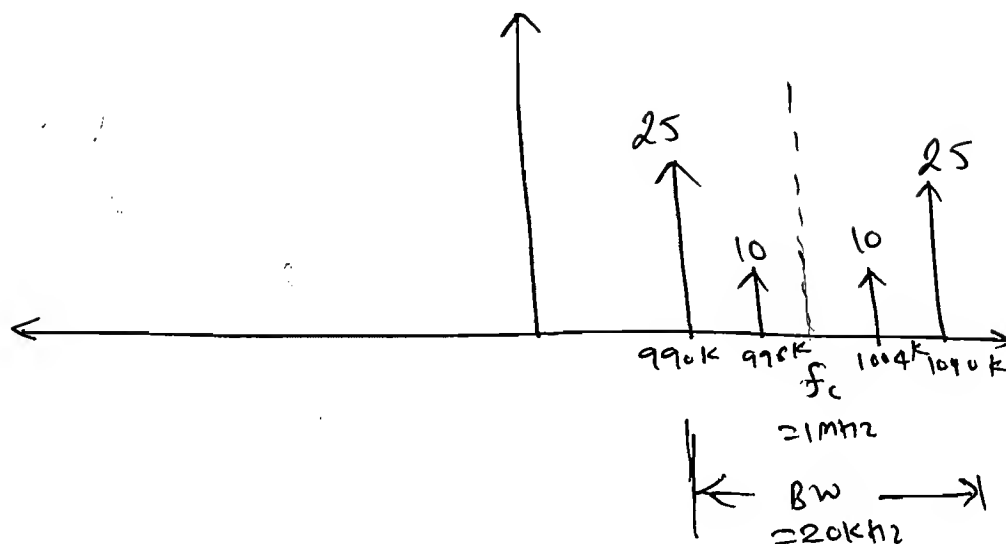
$$\therefore P_t = \frac{A_c^2}{2R} [A_1^2 + A_2^2]$$

$$\therefore P_t = \frac{400}{2} [A_{m1}^2 + A_{m2}^2]$$

$$\therefore P_t = 100 [25 + 4].$$

$$\therefore P_t = 2.9 \text{ kW}$$

$$\eta = 100\%$$

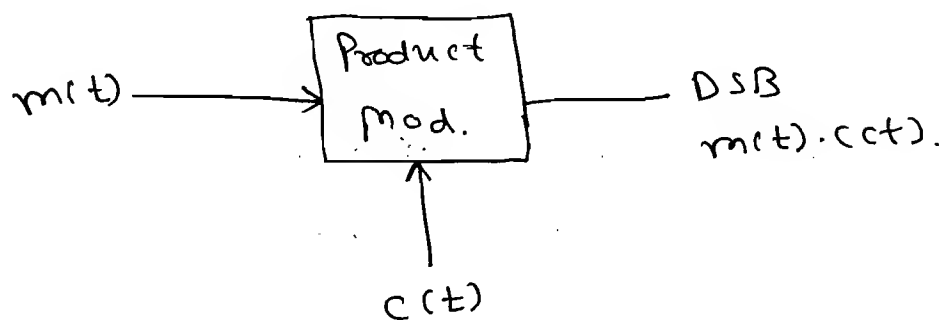


$$\Rightarrow Bw = 2 f_m.$$

$$Bw = 20 \text{ kHz}$$

\* Generation of DSB signal:

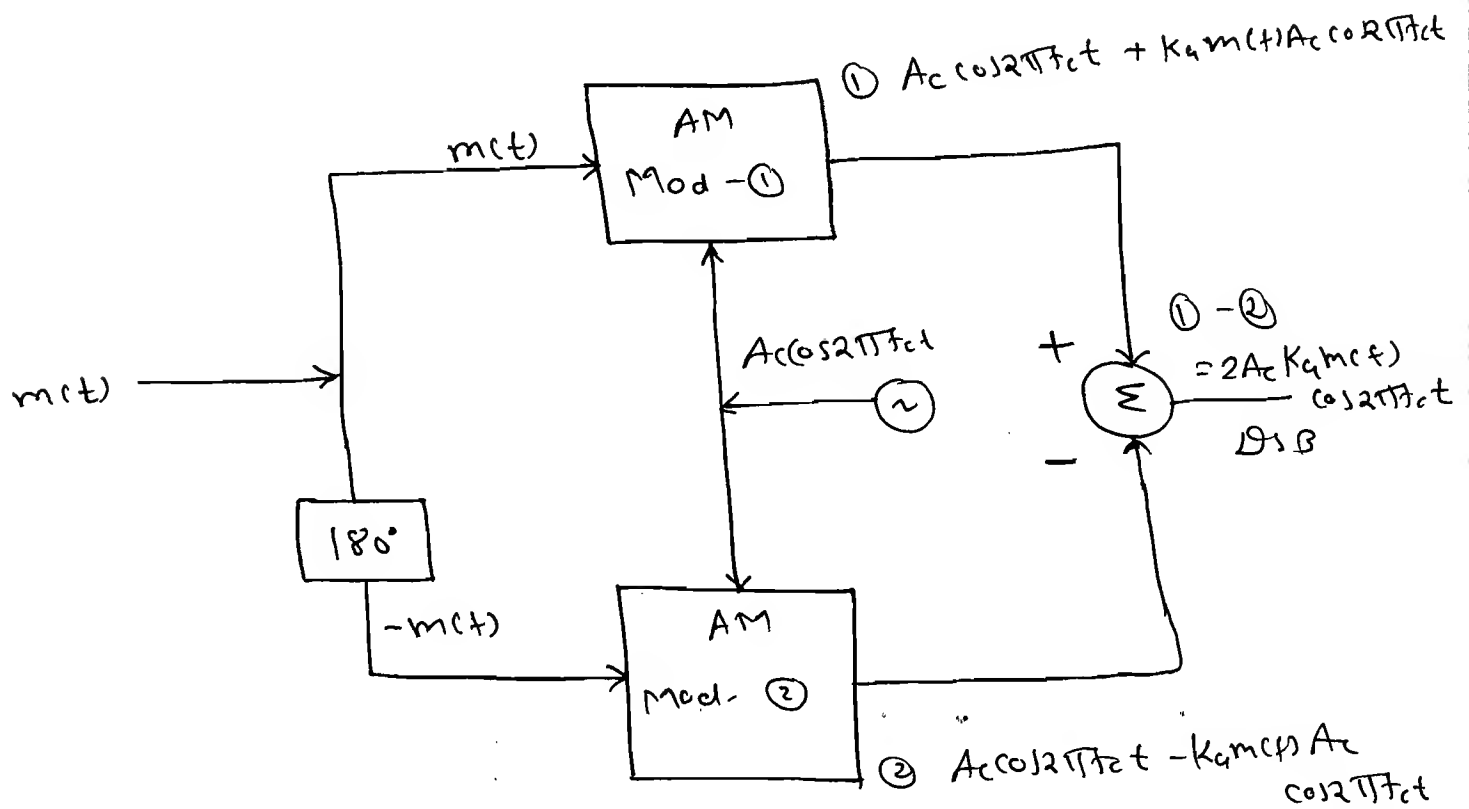
→ Any modulator which generate DSB signal it is also called a product modulator.



① Balanced Modulator.

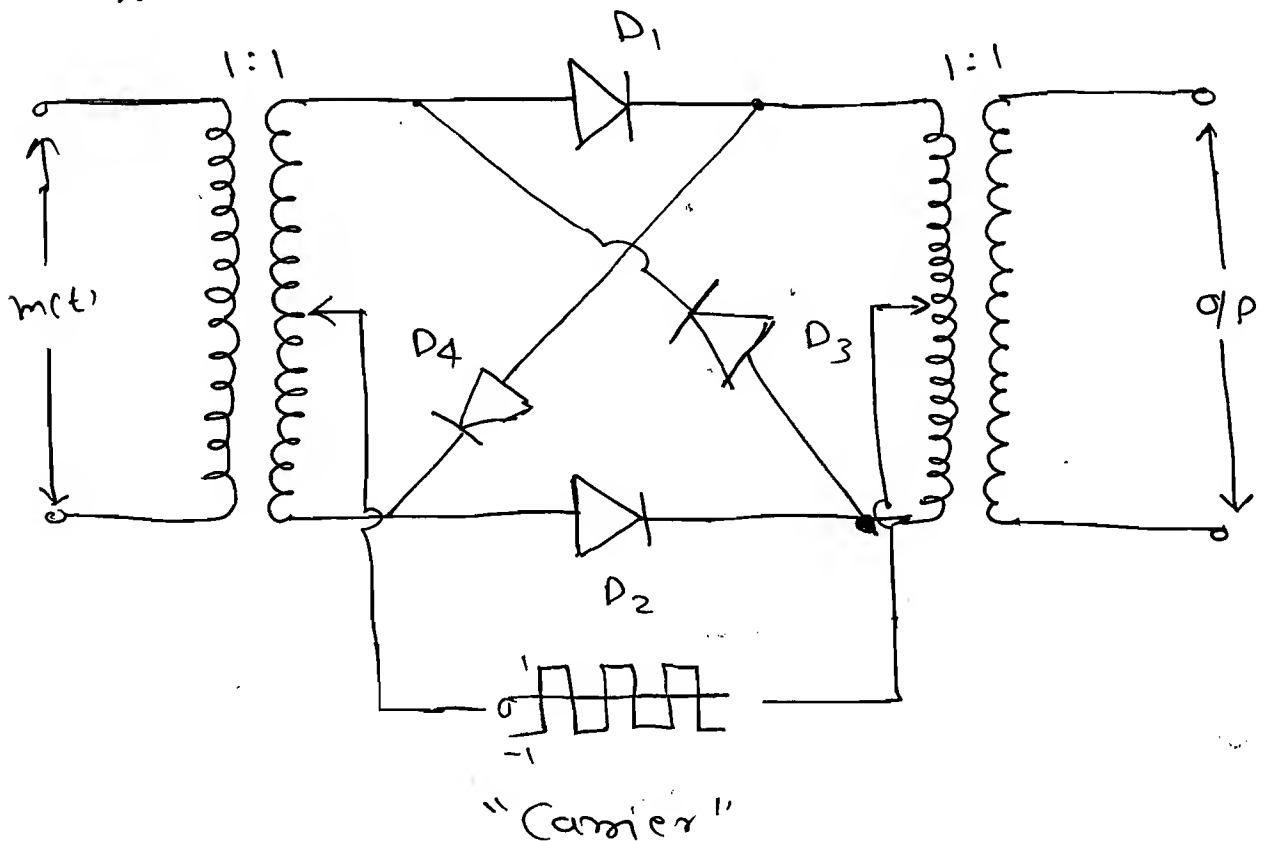
② Ring modulator.

## ① Balanced Modulator:



$\Rightarrow$  "Balanced modulator can be used as "Multiplier"."

## ② Ring Modulator:

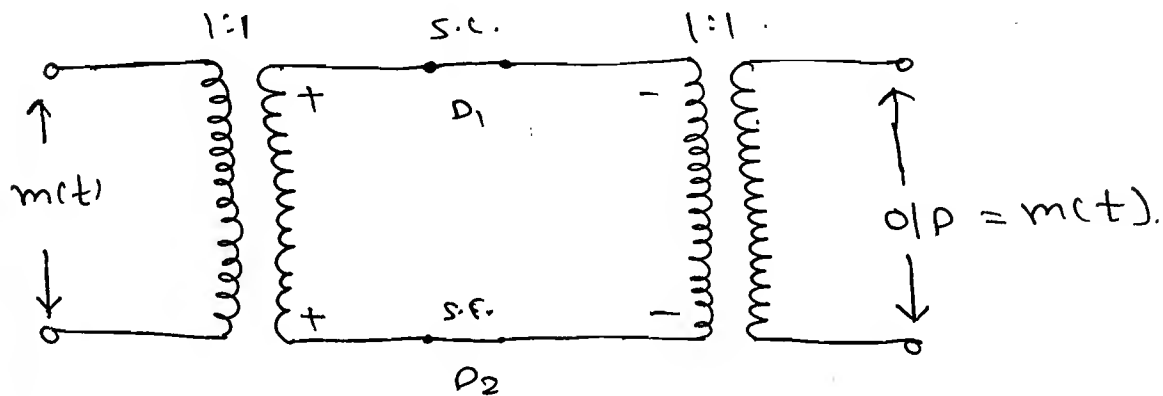


→ Carrier is taken as square wave for convenience.

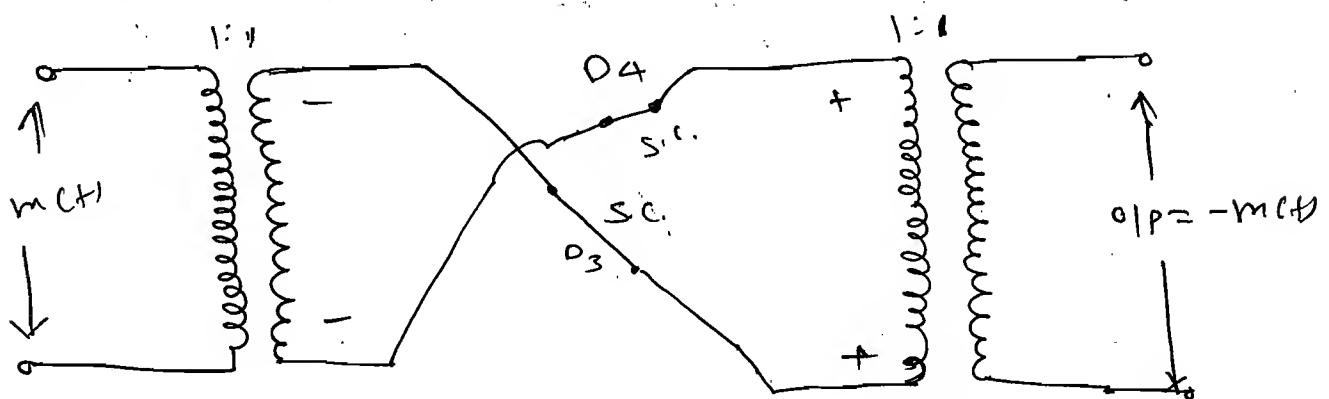
NOTE: ⇒ In all practical application we use sinusoidal signal because it contains only one freq  $f_c$ .

⇒ Other signal, according to the fourier series there will be infinite freq.

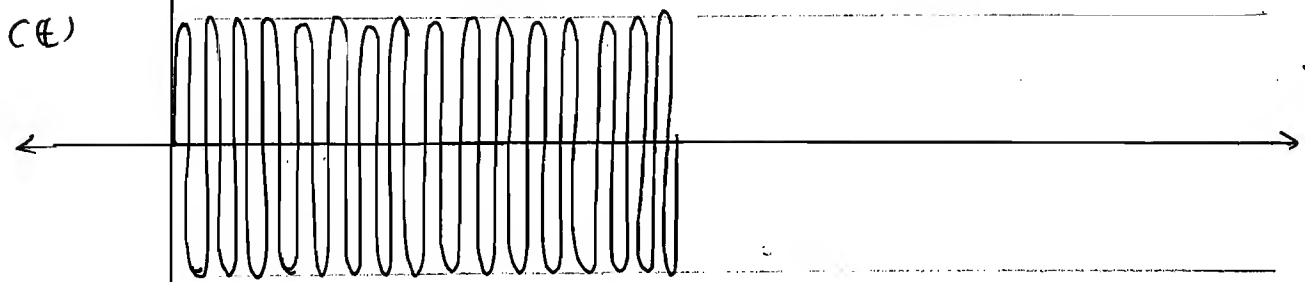
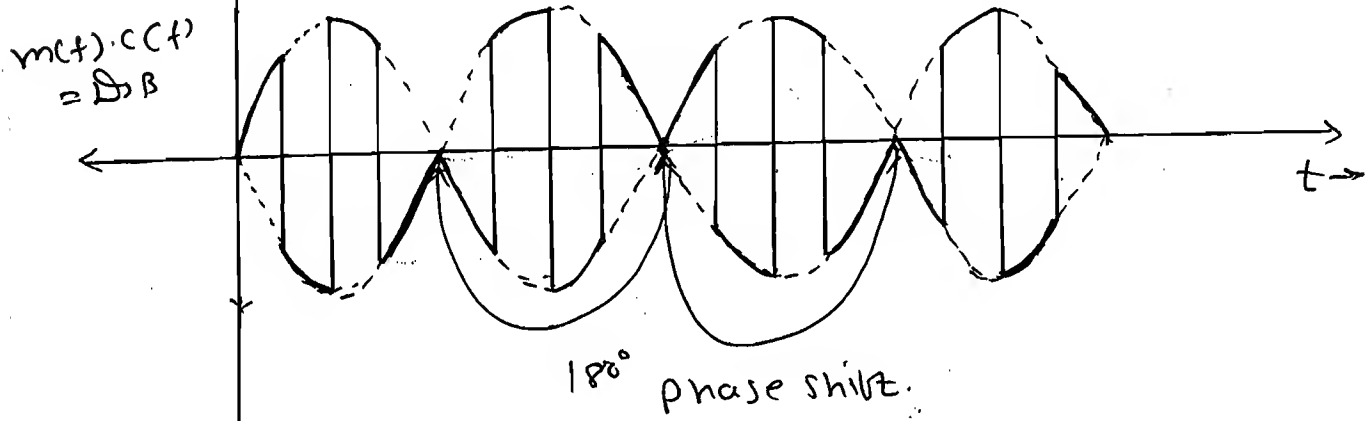
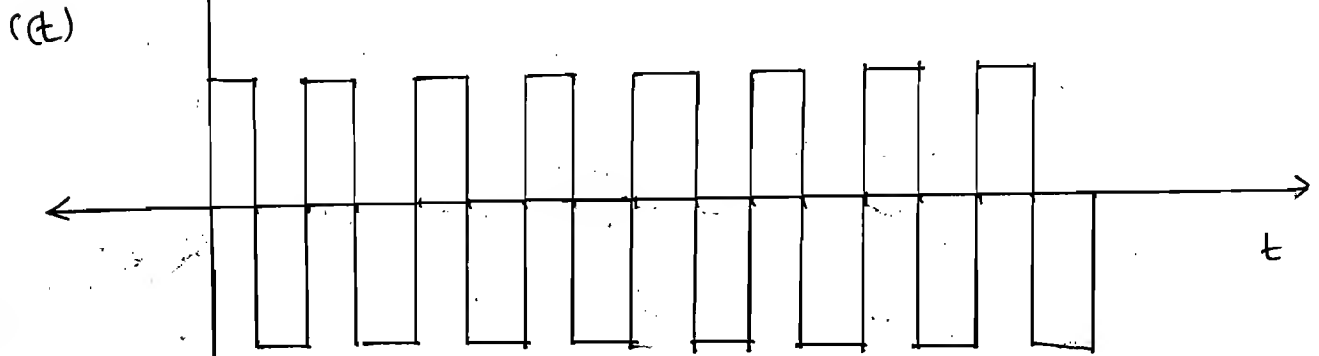
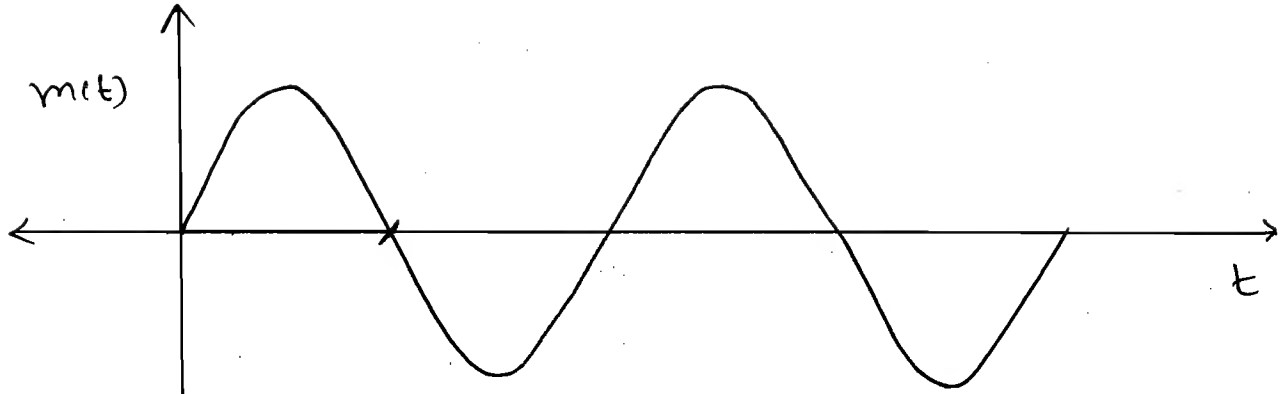
⇒ Carrier Polarity is '+ve':



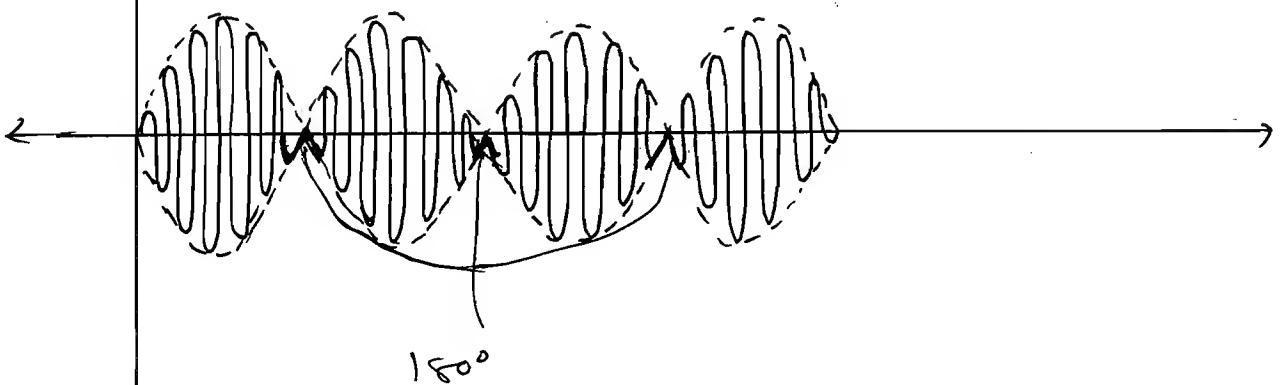
⇒ Carrier Polarity is '-ve':



⇒ In DSB when ~~carrier~~ signal changes its polarity (or) crosses 'zero' line then there will be  $180^\circ$  phase shift in DSB signal.



$c(t) * m(t)$



## \* Demodulation of DSB signal:

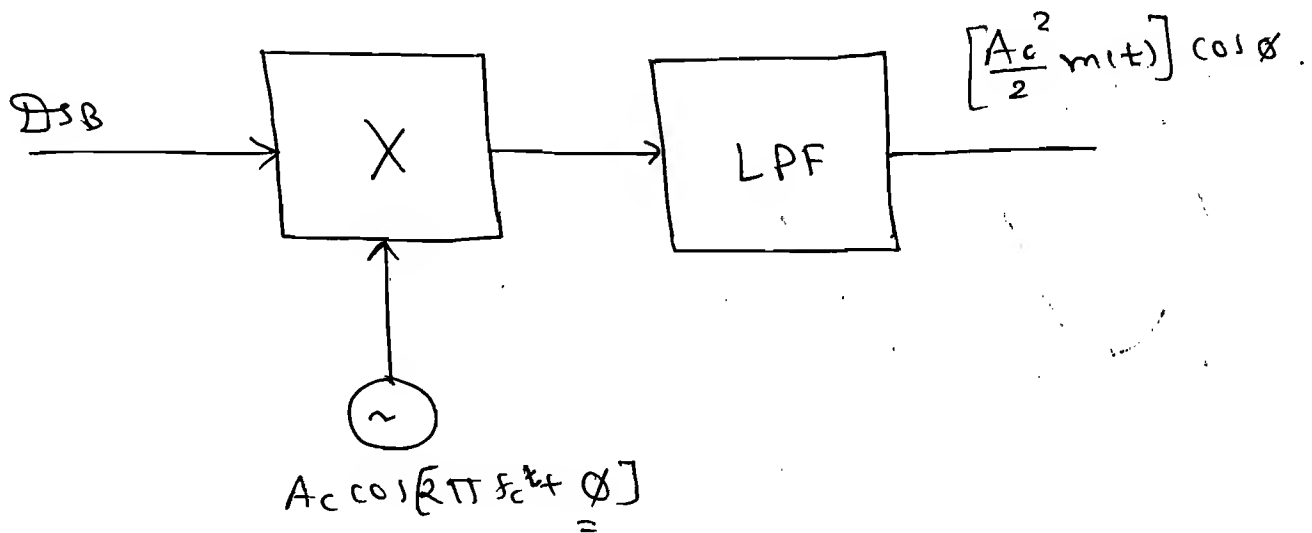
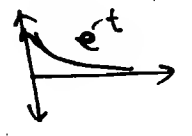
→ When a DSB signal is passed through an Envelope detector, the o/p is " $|m(t)|$ ".

So, Envelope detector is not used as

demodulator. Therefore, Synchronous Detector

is used as demodulator.

But, the signal which has <sup>amp. of</sup> +ve  $m(t)$  then it can be also demodulated by E.D for e.g.  $e^{-t}$



⇒ o/p of Multiplier = (DSB)(L.O).

$$\begin{aligned}
 &= [A_c m(t) \cos 2\pi f_c t] A_c \cos 2\pi f_c t \\
 &= A_c^2 m(t) \cos^2 2\pi f_c t \\
 &= \frac{A_c^2}{2} m(t) + \frac{A_c^2}{2} m(t) \cos 2\pi (2f_c) t.
 \end{aligned}$$

⇒ o/p of LPF =  $\frac{A_c^2}{2} m(t)$ .

⇒ Modulation concept used only in demodulation of AM by Envelope detector. It is not for Synchronous demodulator for DSB.



$\Rightarrow$  If there are some phase shift  $\phi$ .

then,  $(D_{SB})(L_0)_0$

$$= A_c m(t) \cdot \cos(2\pi f_c t) \cdot A_c \cos(2\pi f_c t + \phi).$$

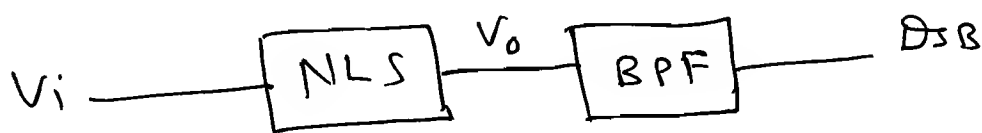
$$= \frac{A_c^2}{2} m(t) \cancel{\cos(4\pi f_c t + \phi)} + \frac{A_c^2}{2} m(t) \cdot \cos \phi.$$

$$\Rightarrow \text{O/P of LPF} = \frac{A_c^2}{2} m(t) \cdot \cos \phi.$$

$\rightarrow$  The Hardware Complexity of DSB receiver is very high when compared with AM.

Ex-1 A DSB signal is generated using non-linear system having characteristic  $V_0 = aV_i + bV_i^3$ ,  $V_i = [m(t) + \cos 2\pi f_c t]$ . The O/P of the non-linear system is passed through a bandpass filter to select the DSB signal. Determine the value of  $f_c$  so that carrier freq. of DSB signal is 1 MHz.

- (A) 1 MHz (B) 3 MHz (C) 0.33 MHz (D) 0.5 MHz.

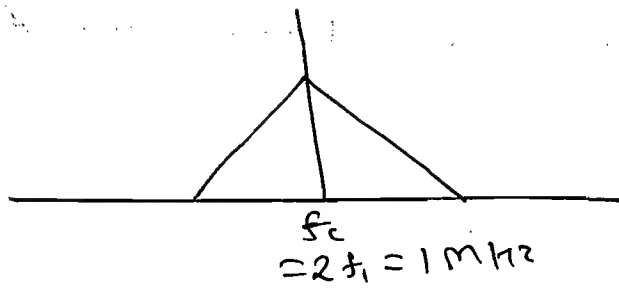


Ans:  $V_0 = aV_i + bV_i^3$ .

$$\therefore V_0 = a \cancel{m(t)} + a \cancel{\cos 2\pi f_c t} + b \cancel{m^3(t)} + b \cancel{\cos^3 2\pi f_c t}$$

$$+ b \cancel{3m^2(t) \cdot \cos 2\pi f_c t} + 3bm(t) \cdot \cos^2 2\pi f_c t.$$

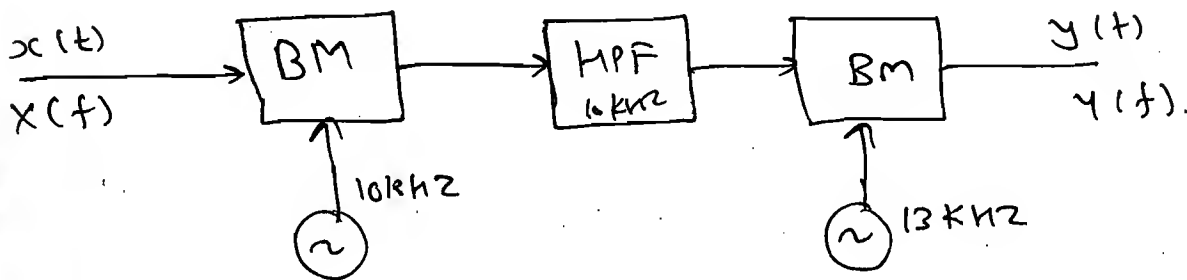
$$V_0 = 3b \cancel{m^2(t)} + \frac{3bm(t)}{2} \cos 2\pi (2f_c) t$$



$$\boxed{f_1 = 0.5 \text{ MHz}}$$

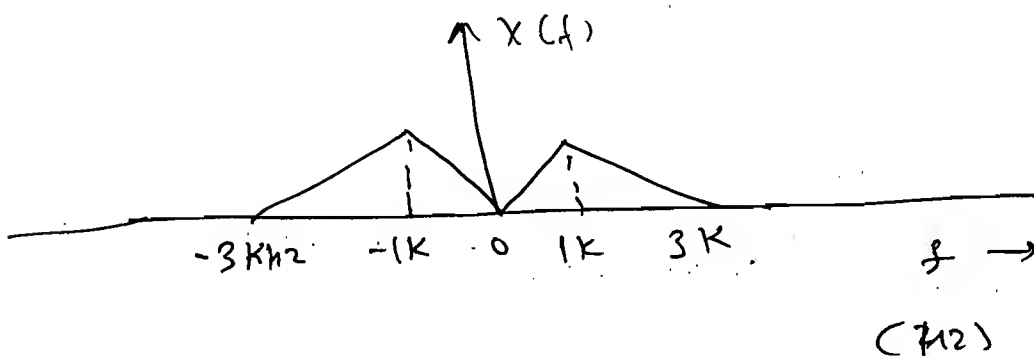
GATE: 2004

Ex-2 Consider the system shown in fig. Determine the +ve freq. at which  $y(t)$  is having spectral peaks.

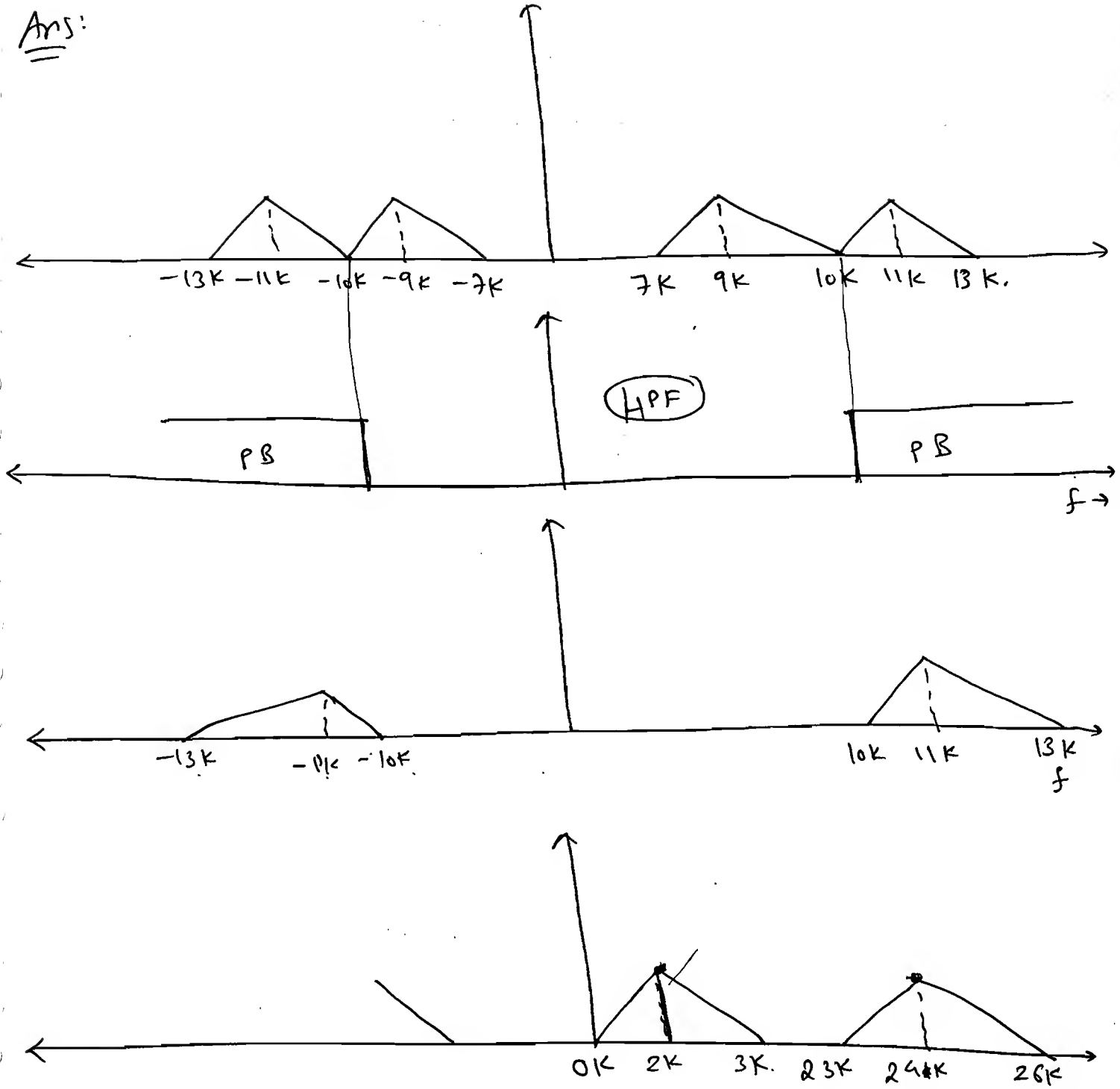


- (A) 2 kHz & 14 kHz  
 (B) 2 kHz & 24 kHz  
 (C) 2 kHz & 4 kHz  
 (D) 1 kHz & 24 kHz.

→ Spectral of i/p signal,

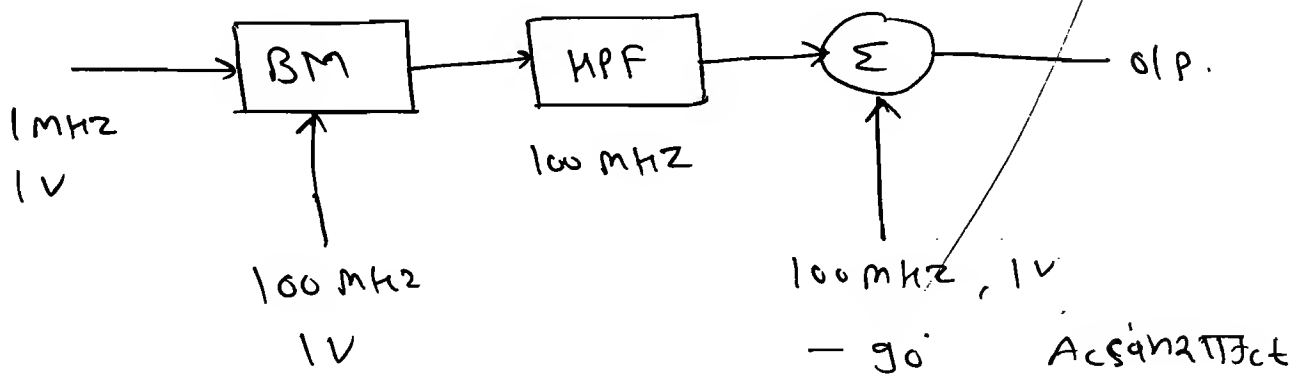


Ans:



So, Ans: (B) 2 kHz & 24 kHz.

Ex-3 Consider a system as shown in fig.



determine the envelope of the o/p signal.

Ans:

$$\rightarrow A_c \cos 2\pi f_c t \cdot A_m \cos 2\pi f_m t$$

$$= \frac{A_c A_m}{2} \cos 2\pi (f_c + f_m)t + \frac{A_c A_m}{2} \cos 2\pi (f_c - f_m)t$$

$$= \frac{1}{2} \cos 2\pi 101M t + \frac{1}{2} \cos 2\pi 99M t$$

→ The o/p of HPF.

$$\Rightarrow \frac{1}{2} \cos 2\pi (f_c + f_m)t$$

Now.

$$\Rightarrow \text{o/p} = \frac{A_c A_m}{2} \cos 2\pi (f_c + f_m)t + A_c \sin 2\pi f_c t$$

$$= \frac{A_c A_m}{2} [\cos 2\pi f_c t \cdot \cos 2\pi f_m t - \sin 2\pi f_c t \cdot \sin 2\pi f_m t] + A_c \sin 2\pi f_c t$$

$$= \underbrace{\left[ \frac{A_c A_m}{2} \cos 2\pi f_m t \right]}_A \cos 2\pi f_c t + \underbrace{\left[ A_c - \frac{A_c A_m}{2} \sin 2\pi f_m t \right]}_B \sin 2\pi f_c t$$

$$\therefore \text{o/p} = \sqrt{A^2 + B^2}$$

$$\text{o/p} = \sqrt{\frac{A_c^2 A_m^2}{4} \cos^2 2\pi f_m t + A_c^2 - 2 \frac{A_c A_m}{2} \sin 2\pi f_m t + \frac{A_c^2 A_m^2}{4} \sin^2 2\pi f_m t}$$

$$= \sqrt{\frac{1}{4} + 1 - \sin 2\pi f_m t}$$

$$\boxed{\text{o/p} = \sqrt{5/4 - \sin 2\pi f_m t}}$$

### ③ SSB Modulation: (Single Side Band Mod.)

→ The BW of the AM and DSB signal is same. In order to multiplex more number of signal the BW of the signal should be as low as possible. So, in this method only one side band is transmitted through the channel.

$$\rightarrow A_c m(t) \cdot \cos 2\pi f_c t$$

$$= A_c A_m \cos 2\pi f_c t \cdot \cos 2\pi f_m t$$

$$= \underbrace{\frac{A_c A_m}{2} \cos 2\pi (f_c + f_m) t}_{\text{USB}} + \underbrace{\frac{A_c A_m}{2} \cos 2\pi (f_c - f_m) t}_{\text{LSB}}$$

⇒ Time domain signal of SSB is,

$$\boxed{\frac{A_c A_m}{2} \cos 2\pi t (f_c \pm f_m)} \leftarrow \text{H.B.}$$

+ → USB

- → LSB

$$\Rightarrow \frac{A_c A_m}{2} \cos 2\pi f_m t \cdot \cos 2\pi f_c t \mp \frac{A_c A_m}{2} \sin 2\pi f_m t \cdot \sin 2\pi f_c t$$

$$\text{Now, } m(t) = A_m \cos 2\pi f_m t$$

$$\hat{m}(t) = A_m \sin 2\pi f_m t$$

= Hilbert transform.

$$\rightarrow \frac{A_c m(t)}{2} \cos 2\pi f_c t \quad \mp \quad \frac{A_c \hat{m}(t)}{2} \sin 2\pi f_c t.$$

H.B.

$- \rightarrow$  USB

$+ \rightarrow$  LSB.

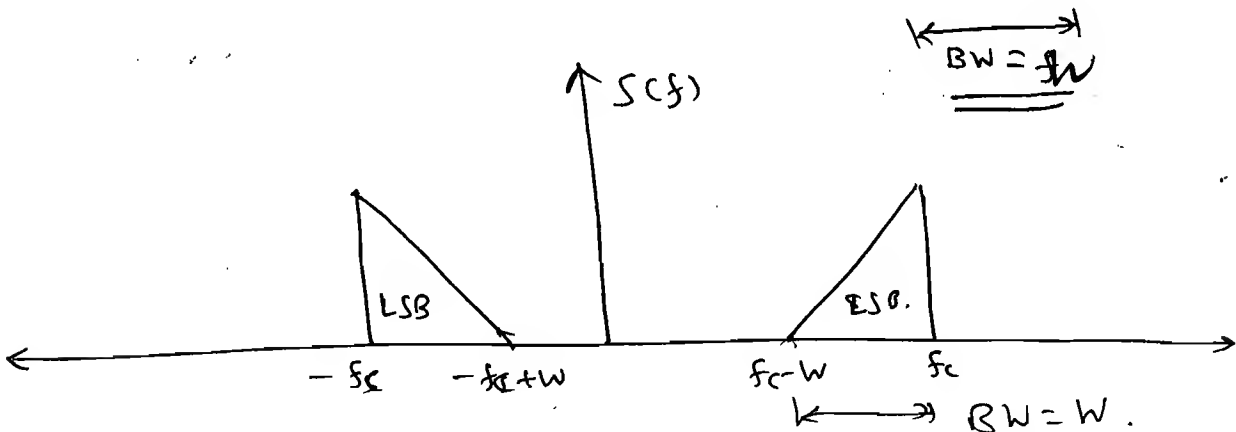
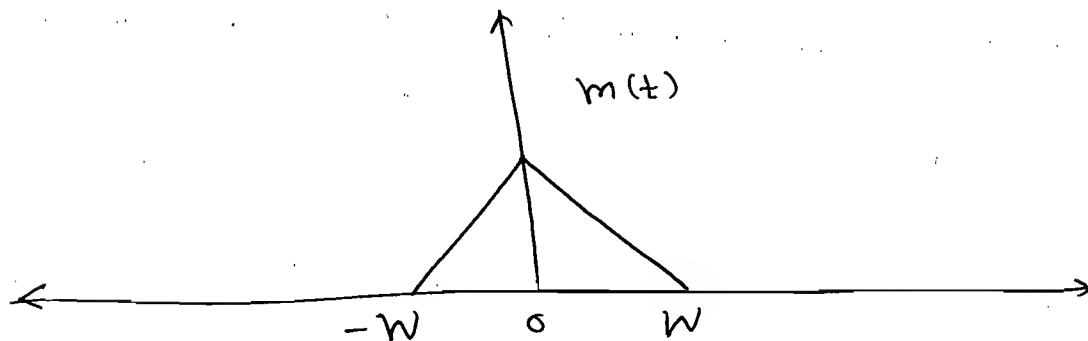
\* Power Calculation:

$$\Rightarrow P_t = \frac{A_c^2 A_m^2}{4R}$$

$$P_t = \frac{A_c^2 A_m^2}{8R} + \frac{A_c^2 A_m^2}{8R}$$

USB                      ~~LSB~~

$$\therefore P_t = \frac{A_c^2 A_m^2}{8R} \quad \leftarrow \text{H.B.}$$



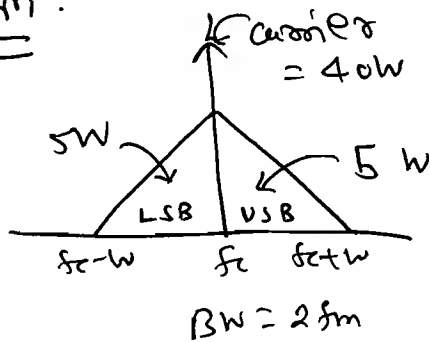
→ The Bw & Power are decreasing by 50% when compared with the DSB signal.

⇒ When DSB modulation is replaced with SSB modulation then the power saving is 50%.

(\*)



① AM:



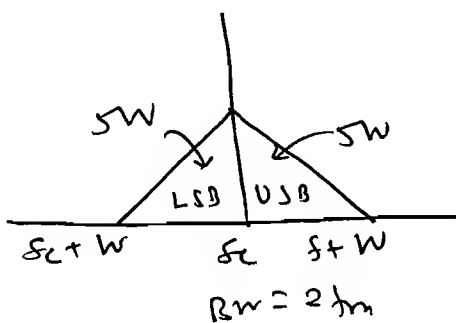
$$P_t = 50W = 40W + 5W + 5W$$

80%      10%    10%

$$\mu = 0.707$$

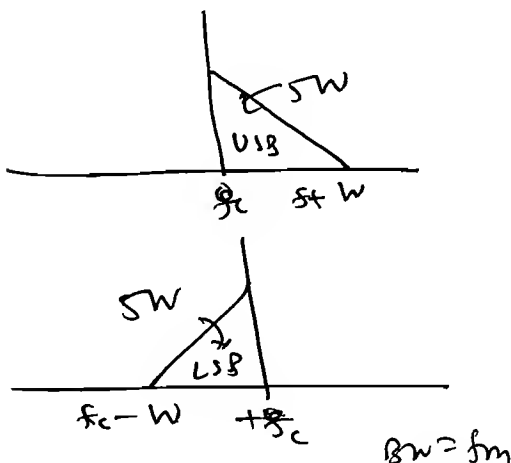
$$\alpha = 20\%$$

② DSB:



$$P_t = 10W = 5W + 5W$$

③ SSB:



$$P_t = 5W$$

\* Power Saving:  $\leftarrow h_B$

Case-(i): AM to DSB:

$$\therefore \text{Power Saving (\%)} = \frac{\text{Power saved}}{\text{Total power}} \times 100 \%$$

$$= \frac{40}{50} \times 100 \%$$

$$= \frac{P_c}{P_t} \times 100 \%$$

$$= \frac{P_c}{P_c \left[ 1 + \frac{\mu^2}{2} \right]} \times 100 \%$$

$$\boxed{\text{Power Saving (\%)} = \frac{2}{\mu^2 + 2} \times 100 \%}$$

Case-(ii): AM to SSB.

$$\therefore \text{Power Saving (\%)} = \frac{\text{Power saved}}{\text{total power}} \times 100 \%$$

$$= \frac{40 + 5}{50} \times 100 \%$$

$$= \frac{P_c + \frac{P_c \mu^2}{4}}{P_c \left[ 1 + \frac{\mu^2}{2} \right]} \times 100 \%$$

$$\boxed{\text{Power Saving (\%)} = \left[ \frac{1 + \frac{\mu^2}{4}}{1 + \frac{\mu^2}{2}} \right] \times 100 \%}$$

Case-(iii): DSB to SSB.

$$\Rightarrow \boxed{\text{Power saving} = \frac{5}{10} = 50 \%}$$



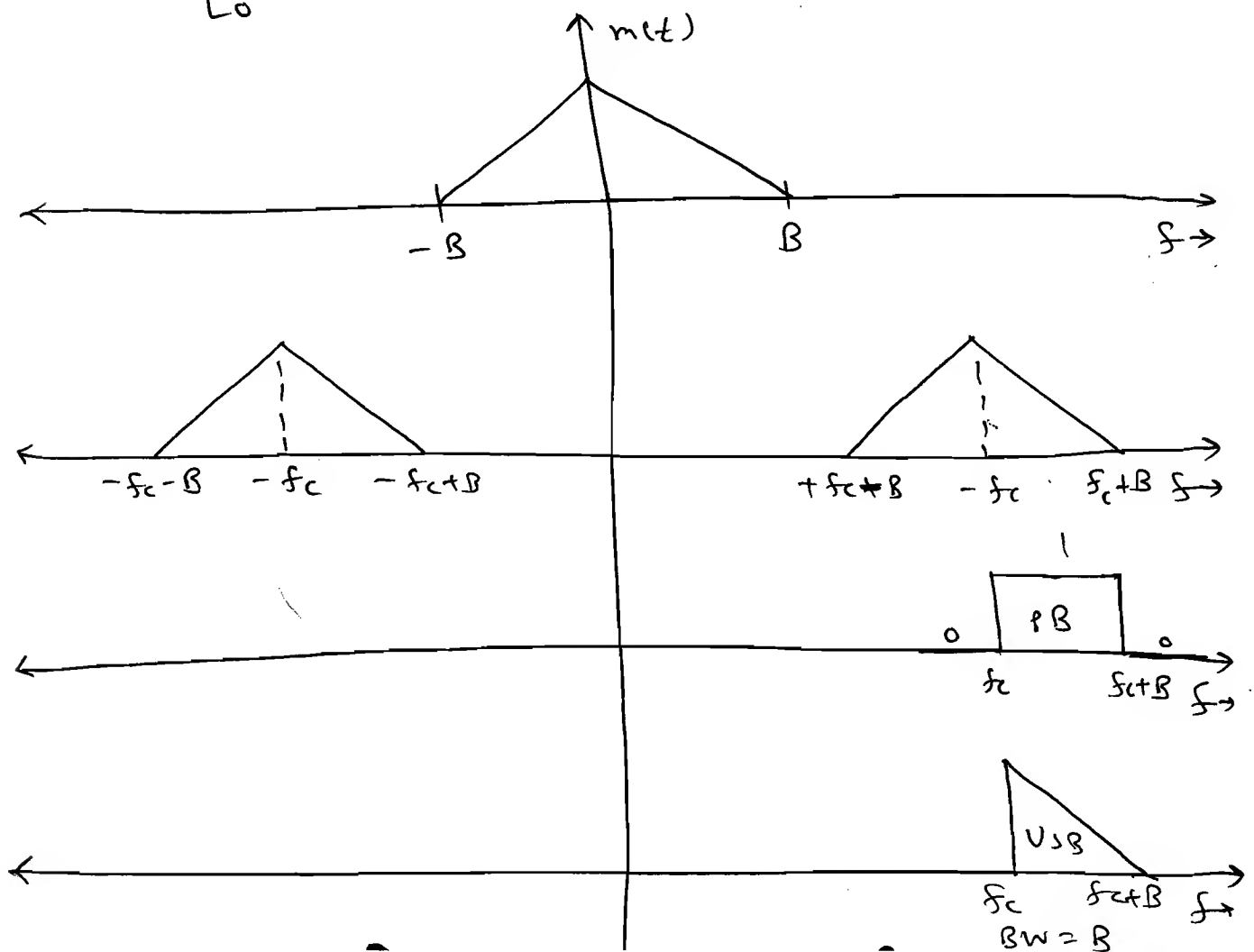
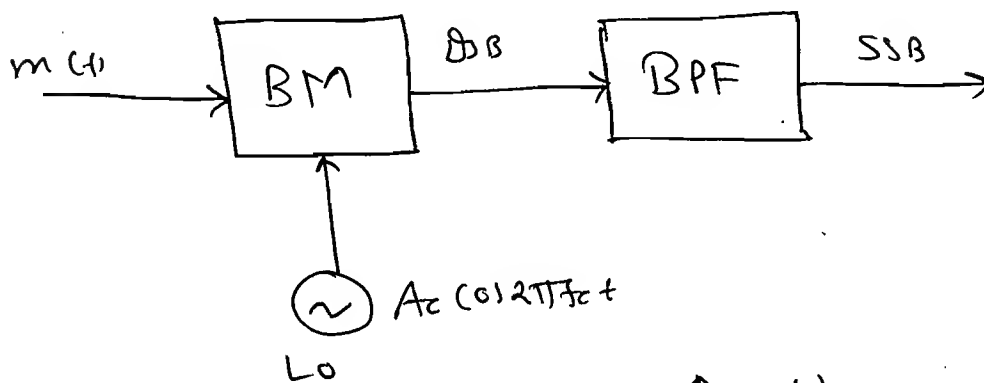
# \* Generation of SSB signal:-

① Frequency discrimination method (or)  
(Filtered method)

② Phase discrimination method:

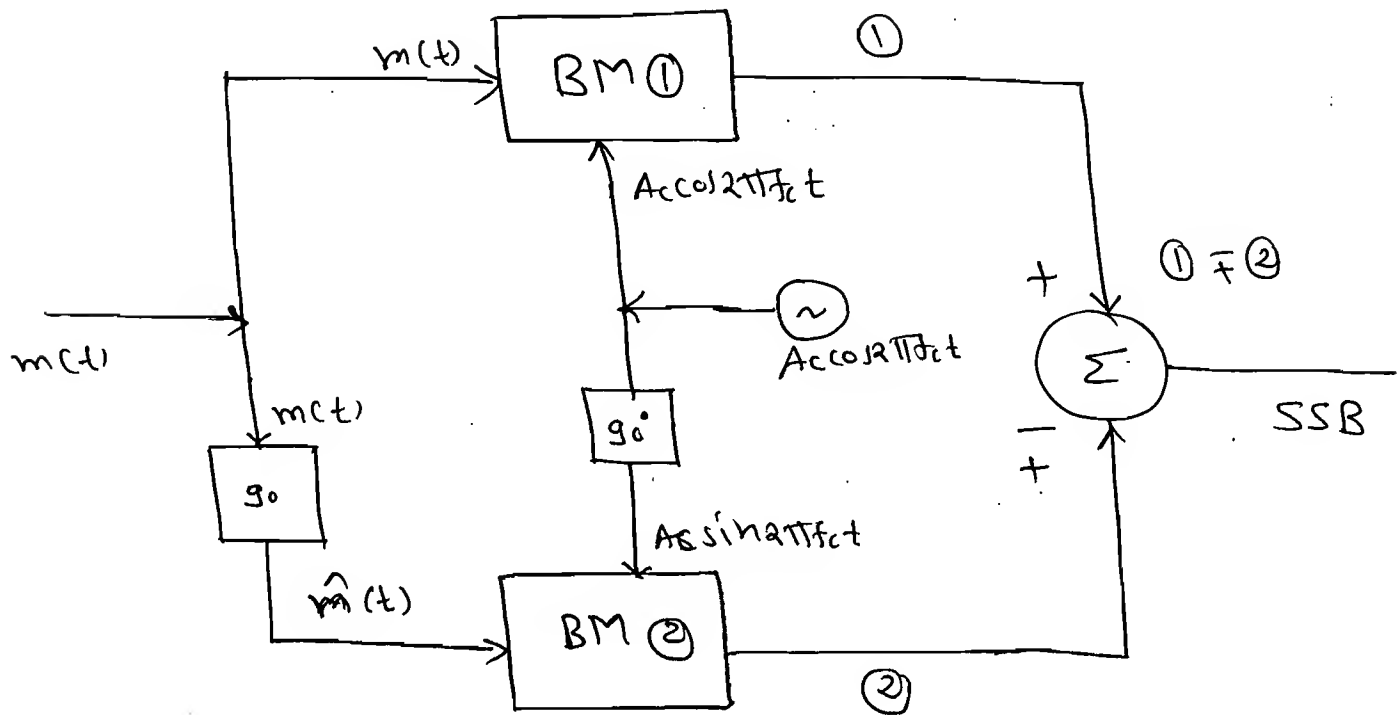
H.B.  
=

① Frequency discrimination method:



## ② Phase Discrimination Method :-

$$\Rightarrow S(t) = \frac{A_c}{2} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t$$



### disadvantage

$\Rightarrow$  Hardware Complexity is very high.

### \* Demodulation of SSB signals :-

$$\Rightarrow \text{SSB} \rightarrow \text{EP} \rightarrow \text{O/P} = \sqrt{\left(\frac{A_c}{2} m(t)\right)^2 + \left(\frac{A_c \hat{m}(t)}{2}\right)^2}$$

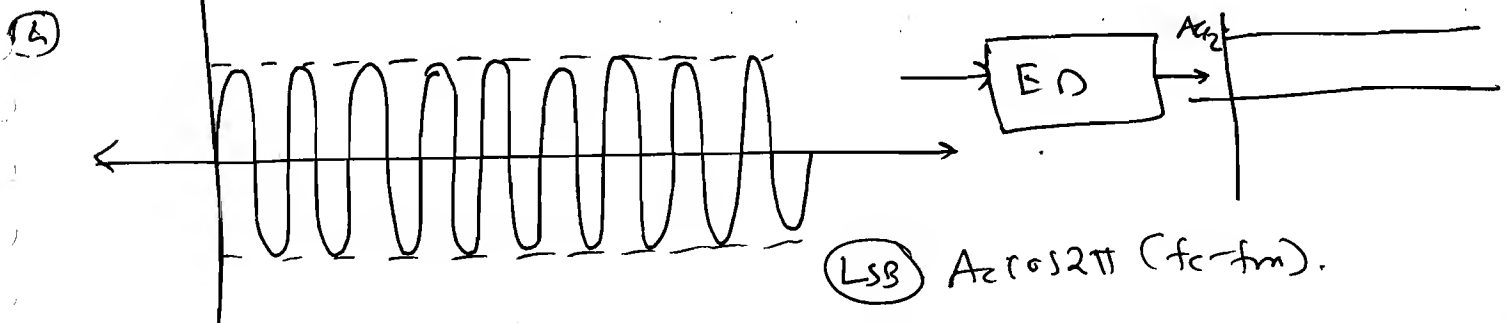
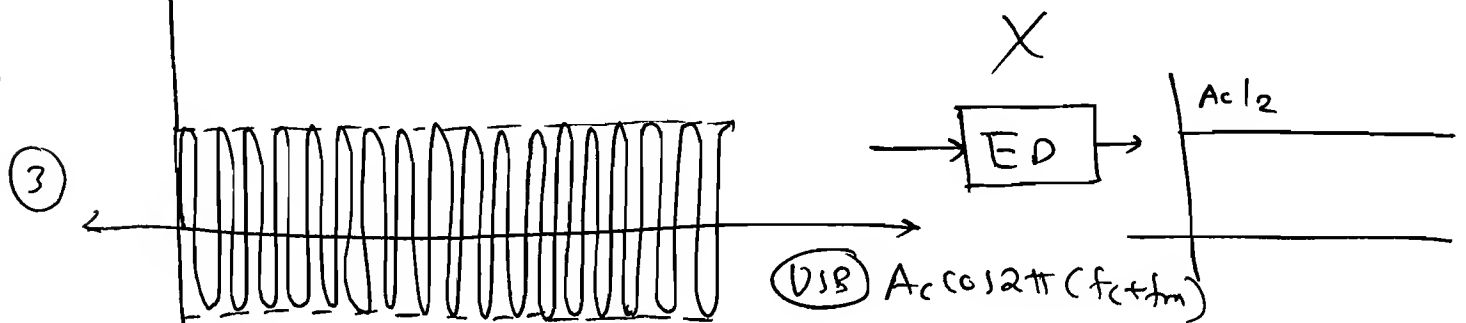
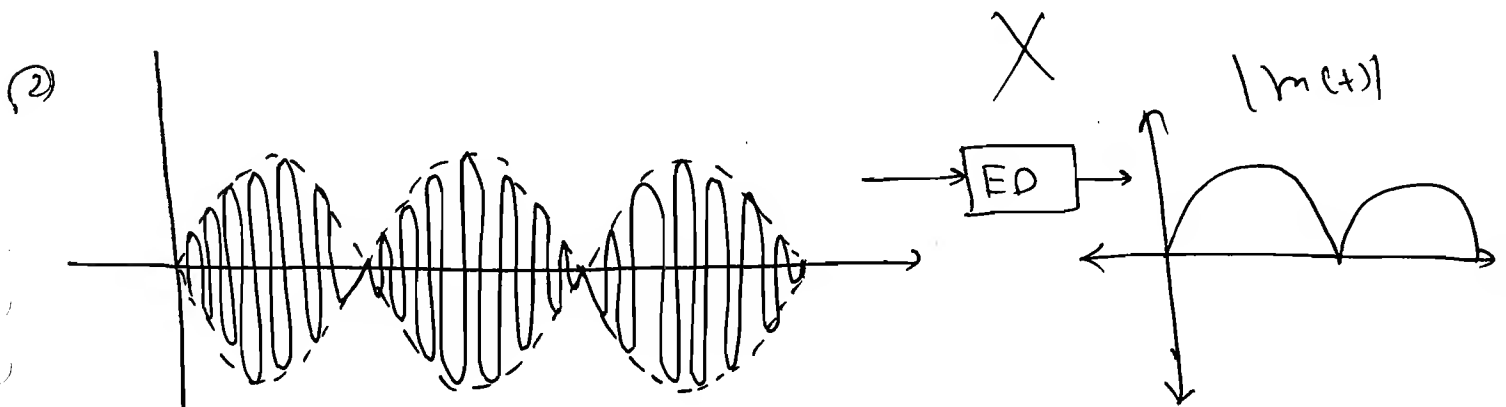
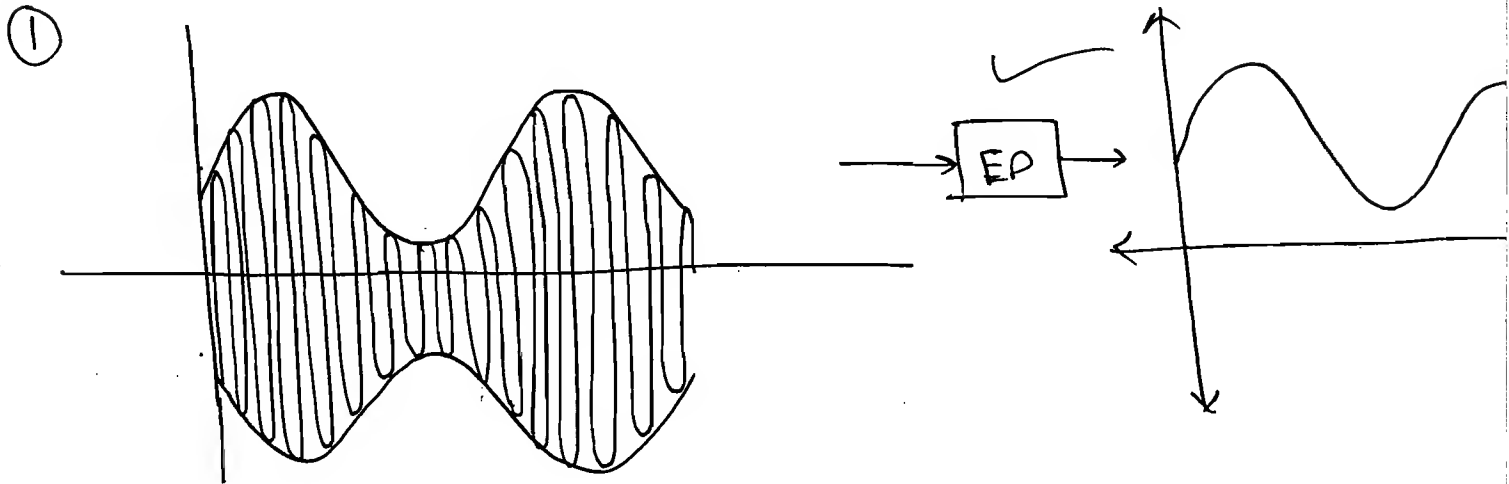
$$\text{let, } m(t) = \cos 2\pi f_m t$$

$$\therefore \text{O/P} = \left( \frac{A_c^2}{4} \cos^2 2\pi f_m t + \frac{A_c^2}{4} \sin^2 2\pi f_m t \right)^{1/2}$$

$$\text{O/P} = \frac{A_c}{2}$$

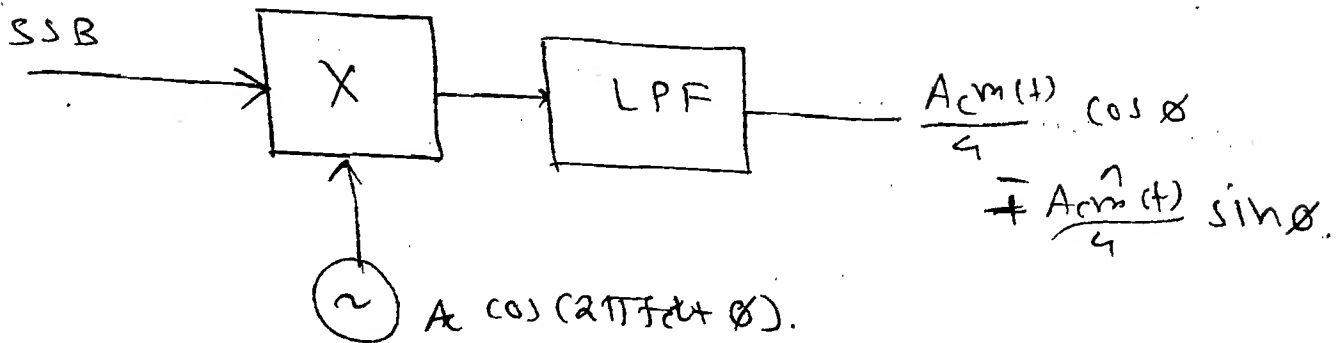


$\Rightarrow$  When the  $m(t) = \cos 2\pi f_m t$ . The o/p of an envelope detector is constant. So, ED is not used as demodulation for SSB.



# \* Synchronous Demodulator:

✓ H-B.



$\Rightarrow$  O/P of multiplier:

$$\begin{aligned} & \frac{A_c^2}{2} m(t) \cdot \cos^2 2\pi f_c t + \frac{A_c^2}{2} \hat{m}(t) \cdot \cos 2\pi f_c t \cdot \sin 2\pi f_c t \\ &= \frac{A_c^2}{4} m(t) + \frac{A_c^2 m(t)}{4} \cancel{\cos 2\pi(4f_c)t} + \frac{A_c^2 \hat{m}(t)}{4} \cancel{\sin 4\pi f_c t} \\ & \quad + \frac{A_c^2 \hat{m}(t)}{4} \cdot (0) \end{aligned}$$

$\Rightarrow$  O/P of LPF

$$\text{O/P} = \frac{A_c^2}{4} m(t)$$

if there is phase shift of  $\phi$  then

$$\text{O/P} = \frac{A_c^2}{4} m(t) \cos \phi + \frac{A_c^2}{4} \hat{m}(t) \sin \phi$$

$$\Rightarrow \text{When } \phi = 0^\circ \Rightarrow \frac{A_c^2}{4} m(t)$$

$$\phi = 90^\circ \Rightarrow \frac{A_c^2}{4} \hat{m}(t)$$

here, No Quadrature null effect.

$\Rightarrow$  When the Phase Shift is bet<sup>n</sup>  $0^\circ$  and  $90^\circ$  the o/p of the demodulator ~~are~~  $m(t)$  and  $\hat{m}(t)$  and demodulation is not possible. So, Synchronization required bet<sup>n</sup> Tx and Rx.

\* Advantages of SSB over DSB.  $\checkmark$  H.B.

- ① BW is reduced by 50%.
- ② Required power for same distance is reduced by 50%.
- ③ No Quadrature null effect.  $\checkmark$

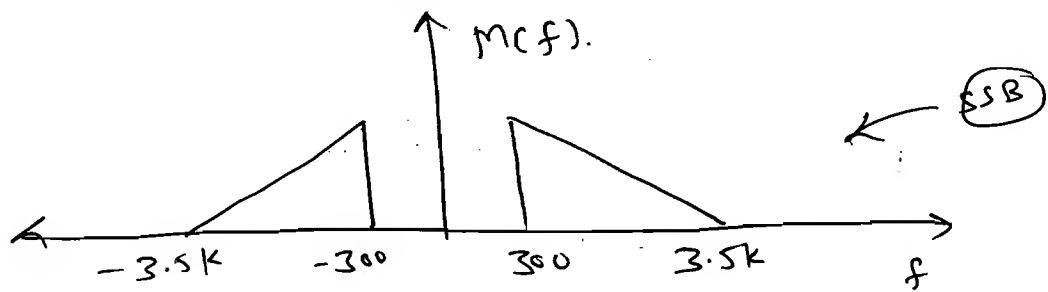
\* Disadvantages of SSB:

$\rightarrow$  As the practical filters do not have sharp cut-off freq., it is not possible to suppress one side-band completely.

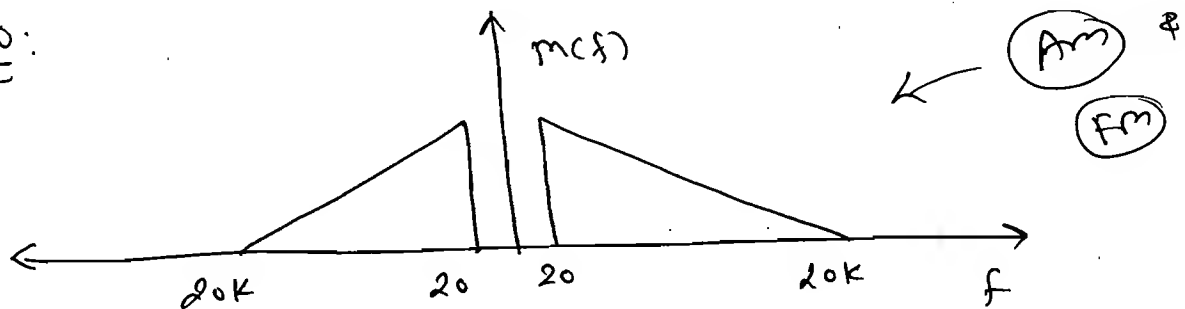
\* Application: (Most imp concept):

$\Rightarrow$  SSB modulation is suitable only for the transmission of Voice signals only.  
e.g. Telephone System.

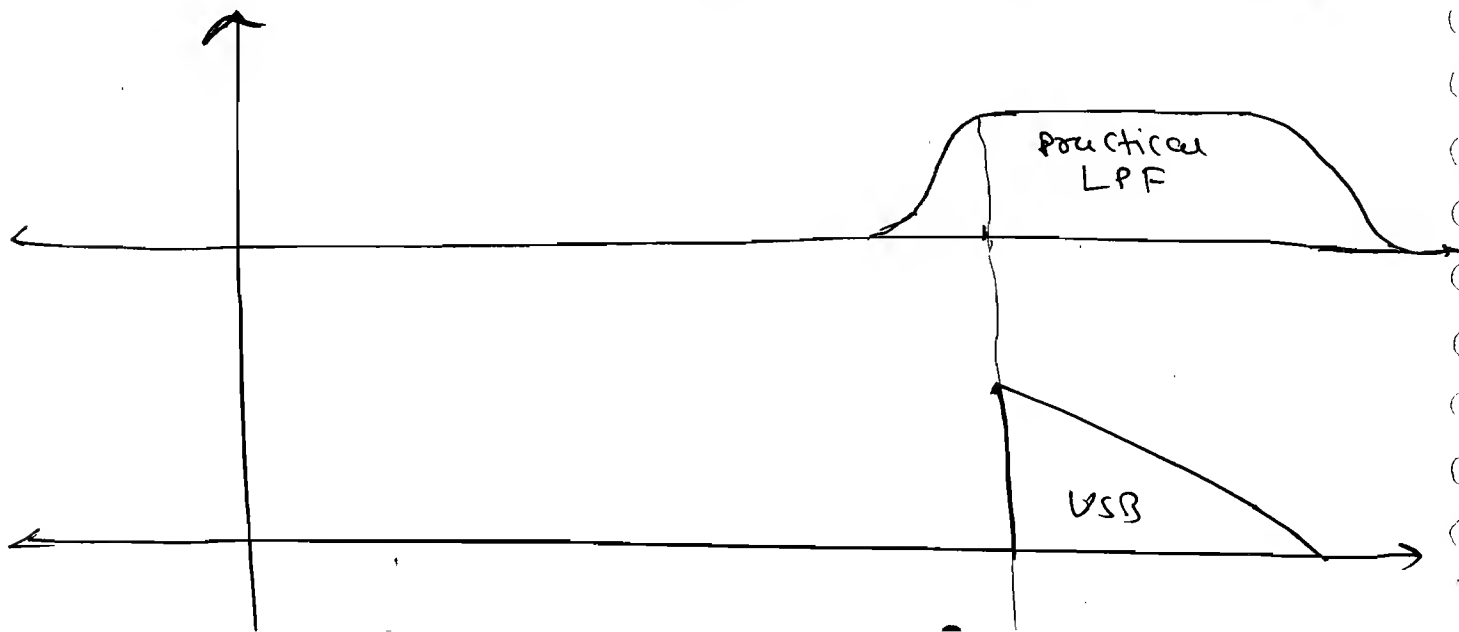
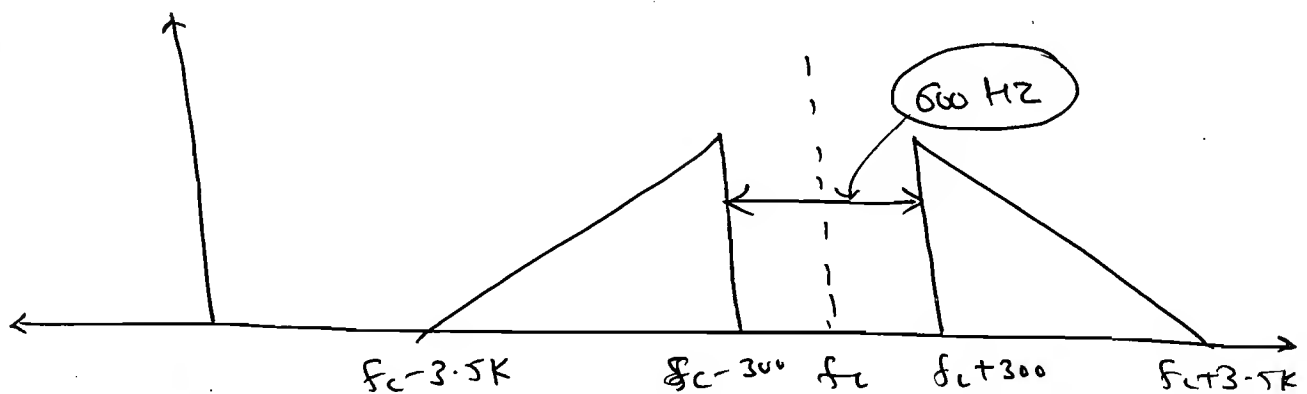
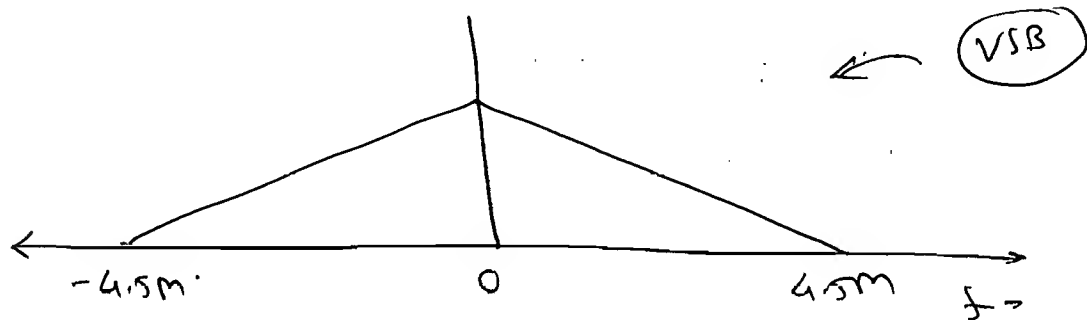
① Voice:



② Audio:



③ Vedio:

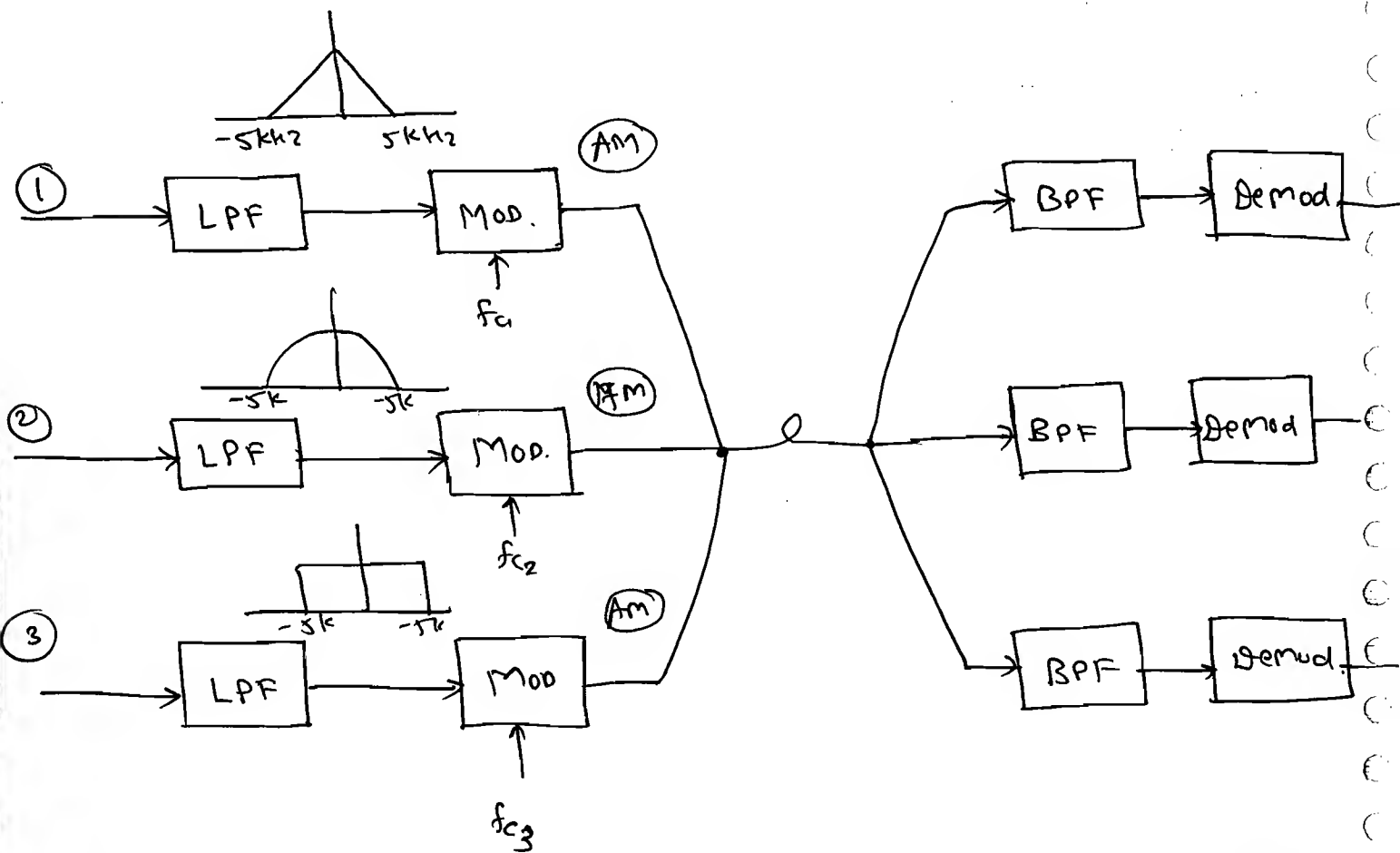


⇒ In the case of Voice transmission a Spectral gap of 600 Hz is existing bet<sup>n</sup> USB and LSB. So, it is possible to eliminate one side band completely. even though filter is not having sharp cut-off frequency.



# \* Frequency Division Multiplexing (FDM):

⇒ Transmission of more than one signal through some ~~transmission~~ ~~line~~ communication channel is called as Multiplexing.



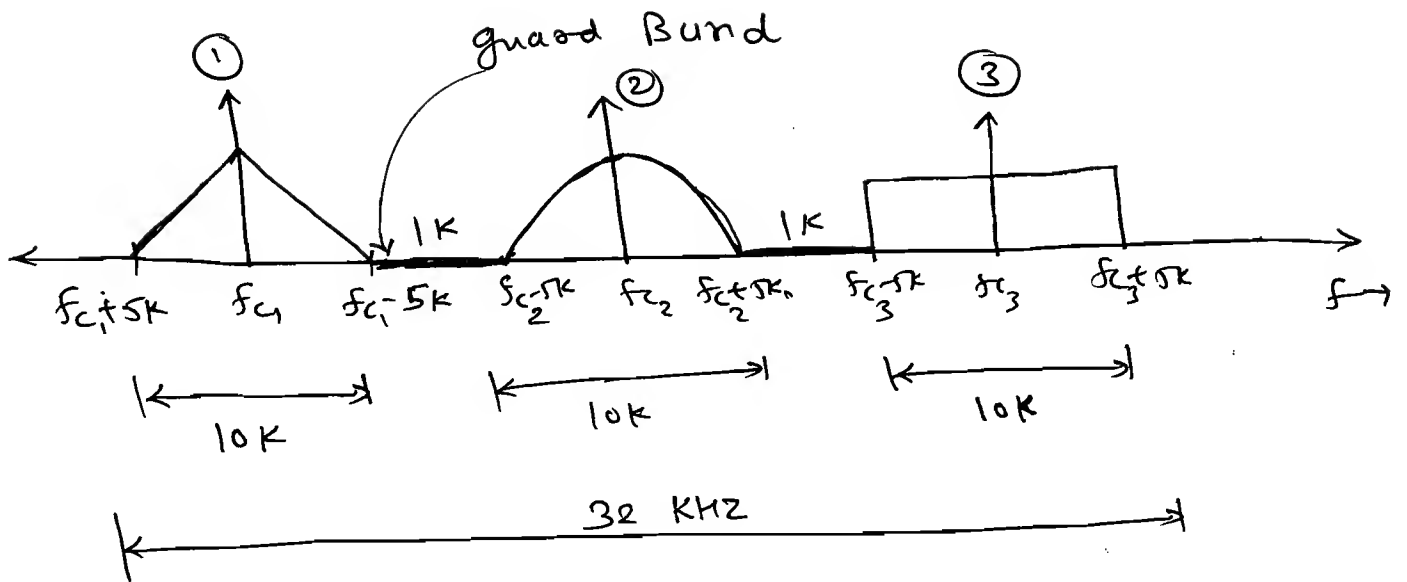
⇒ The LPFs are used to eliminate the insignificant high frequencies and cut-off freq. depends on the application.

→ Assume that all the signals are band-limited to 5 kHz.

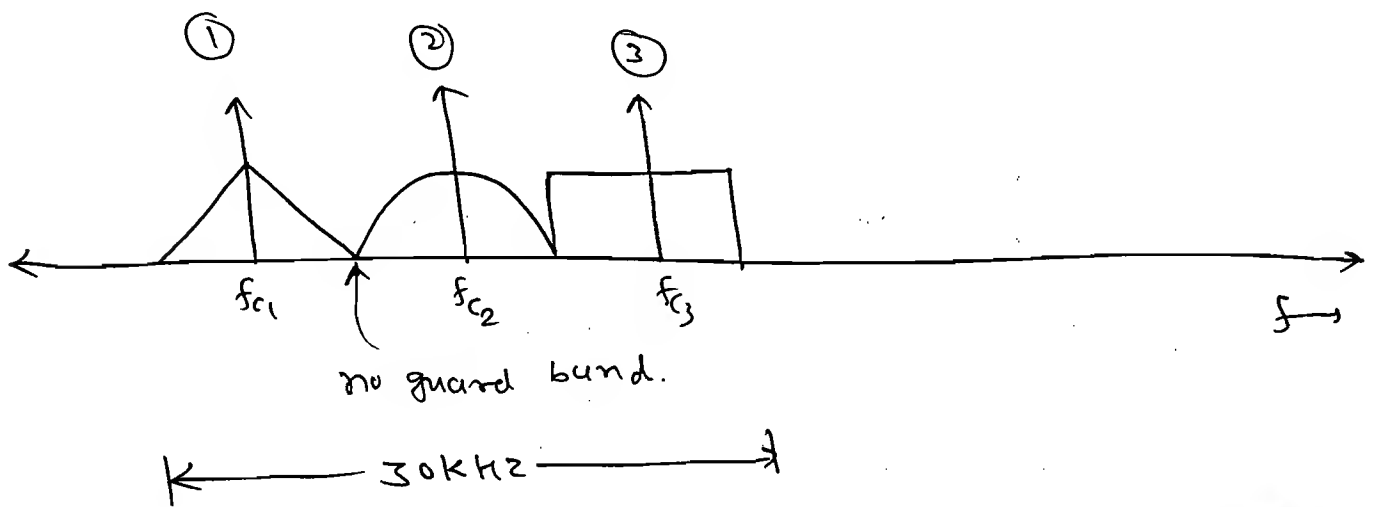
⇒ Assuming all modulators as AM, the spectrum of the multiplexed signal is as shown in fig.



⇒



⇒



⇒ Practically guard band is required. Because BPF will not sharp cut-off frequencies.

→ Bandpass filters are used at the receiver to select the required signal.

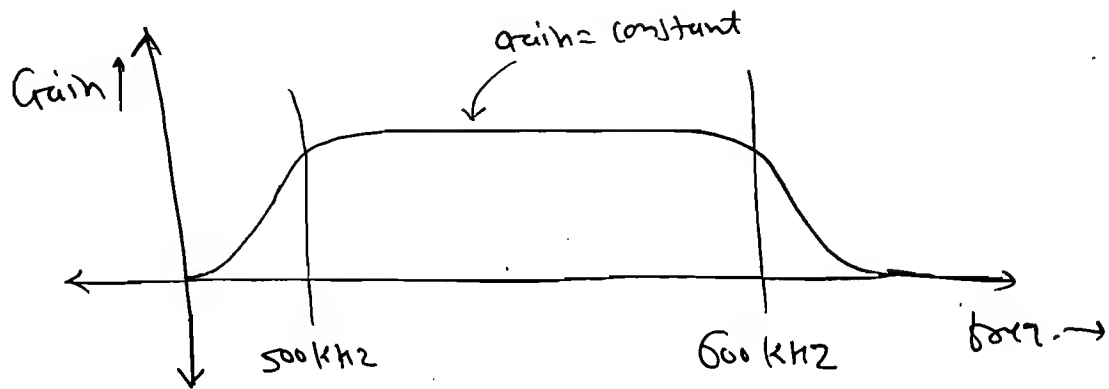
→ The no. of signals which can be multiplexed through a channel is depends on the channel B.W. & signal B.W.

⇒ Channel BW is defined as the range of freqs. that the channel is capable of

transmitting without distortion.

⇒ In order to multiplex more no. of signals; Channel BW should be as high as possible and signal BW should be as low as possible.

⇒ For Channel BW.



freq. response of Channel.

⇒ To transmit a signal without any distortion the gain of the channel should be constant. For an ideal channel the gain is constant from 0 to  $\infty$ .

⇒ In order to transmit a signal without any distortion BW of channel should be greater than the BW of the signal.

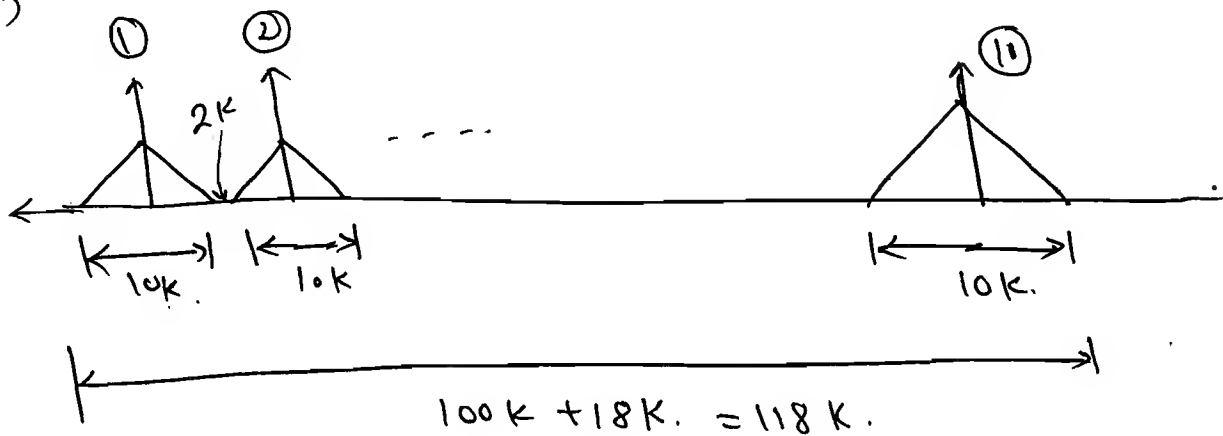
⇒ Twisted Pair → 500 kHz ← means the gain of channel is remain constant through range of 500 kHz frequencies.  
Coaxial Cable → 500 MHz.  
FOC → GHz.

Ex-1 10 signals are band limited to 5 kHz are transmitted through a channel after modulation using FDM, the guard band is 2 kHz. Determine the B.W. of the multiplexed signal.

- i) If all Modulators are AM.
- ii) If all modulators are DSB.
- iii) If all Modulators are SSB.

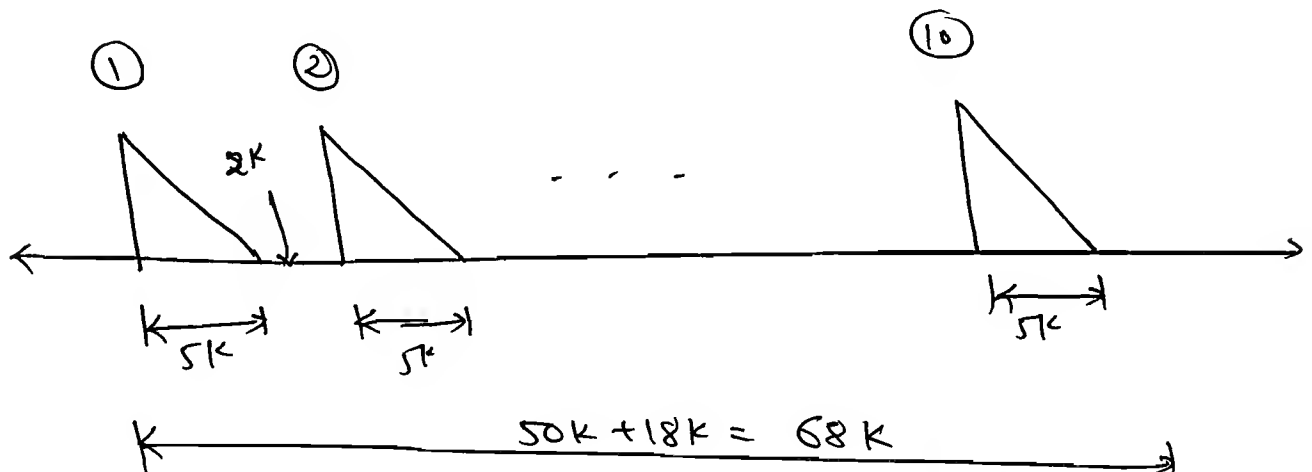
Ans:

(i)



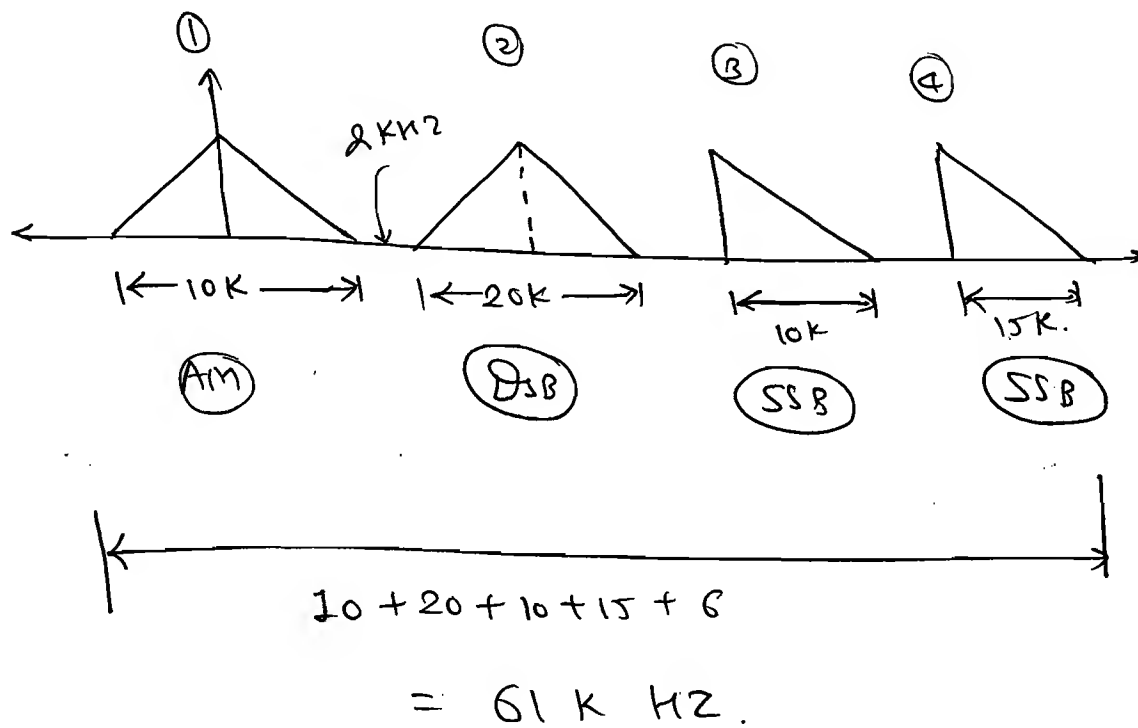
(ii) Same as above but there are no carriers.

(iii)



Q-2 4 signals each bandlimited to 5K, 10K, 10K, 15K, are transmitted through a channel after modulation using FDM. In modulators used are AM, DSB, SSB & SSB respectively. Assuming guard band of 2 kHz. Determine the minimum BW of channel required.

Ans:



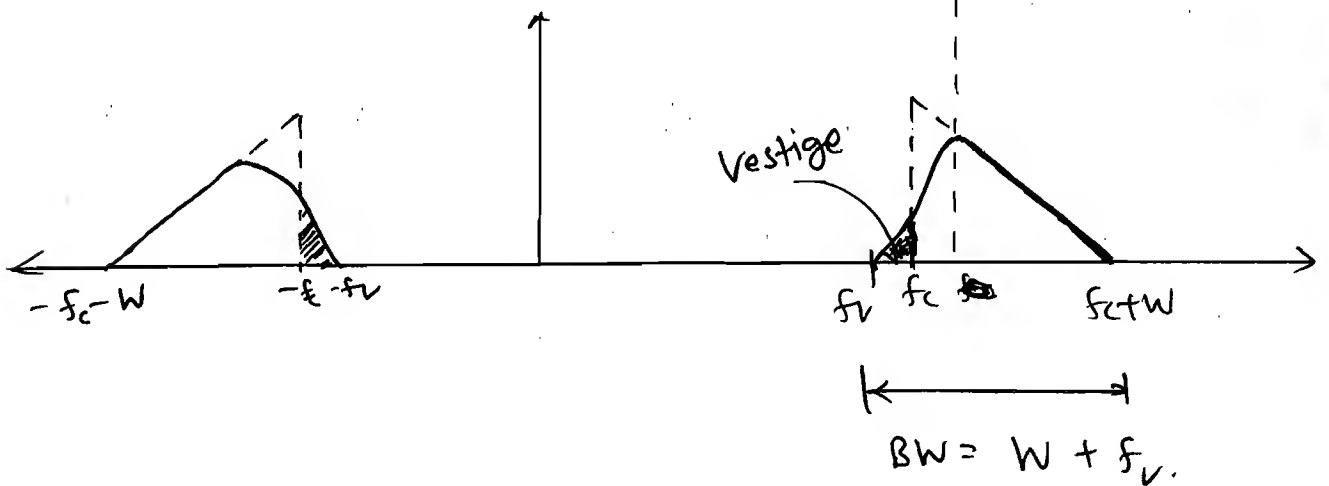
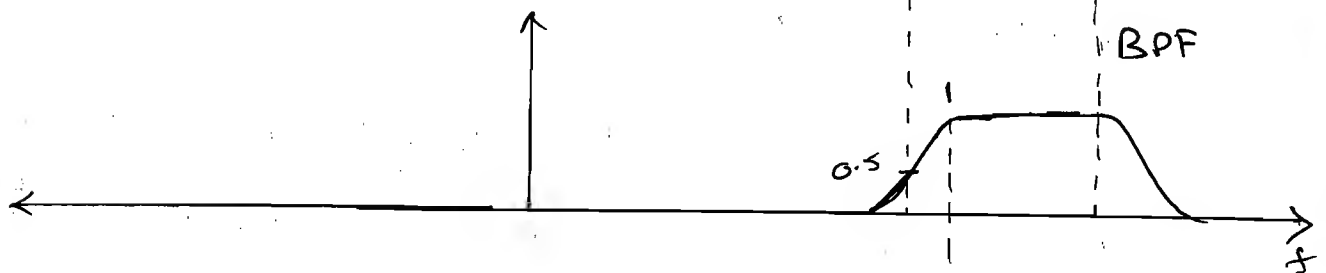
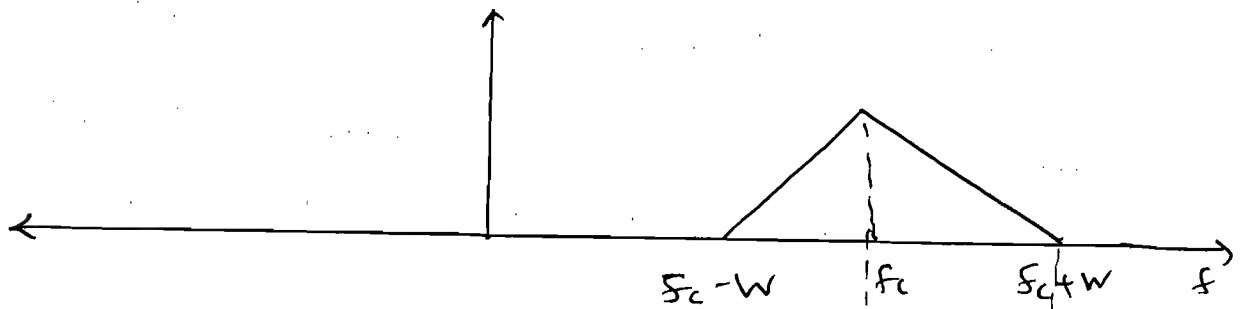
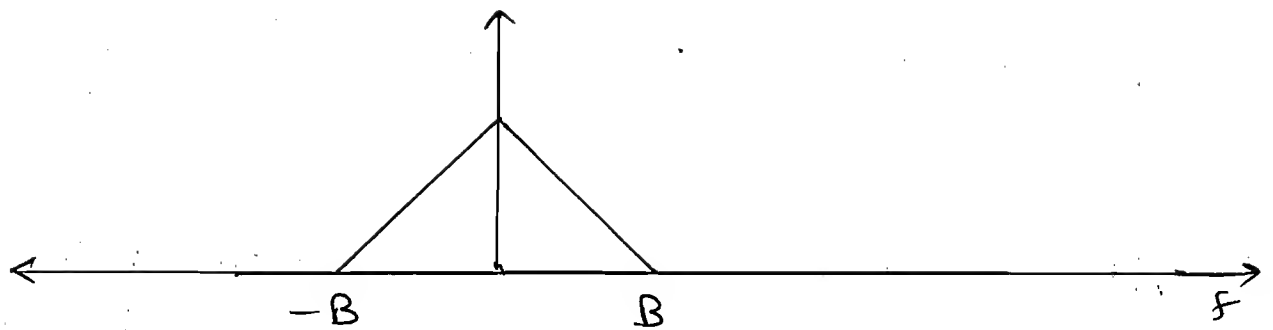
#### ④ Vestigial Side Band Modulation: (VSB).

→ Video signal consist of significant freq. upto 4.5 MHz. If AM (or) DSB is used the BW is 9 MHz and it is not possible to transmit more no. of signals. So AM and DSB are not used for video signal transm.

→ SSB modulation is suitable for voice transmission only.

→ The Generation and Demodulation technique of VSB are same as SSB except few modification at the Band pass filter.

→ The Bandpass filter should be designed so that the gain is not constant in the entire upper side band.

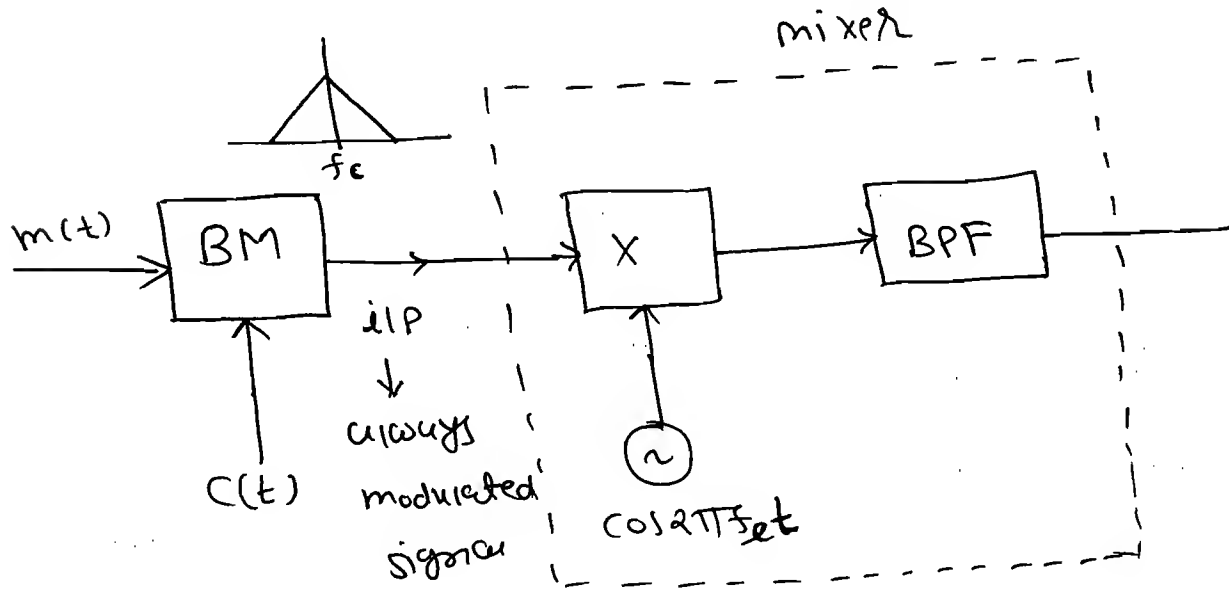


$\Rightarrow$  BW of Vestige depends on design of BPF.

# \* Mixer: H.B.

→ Mixing (or) Heterodyning.

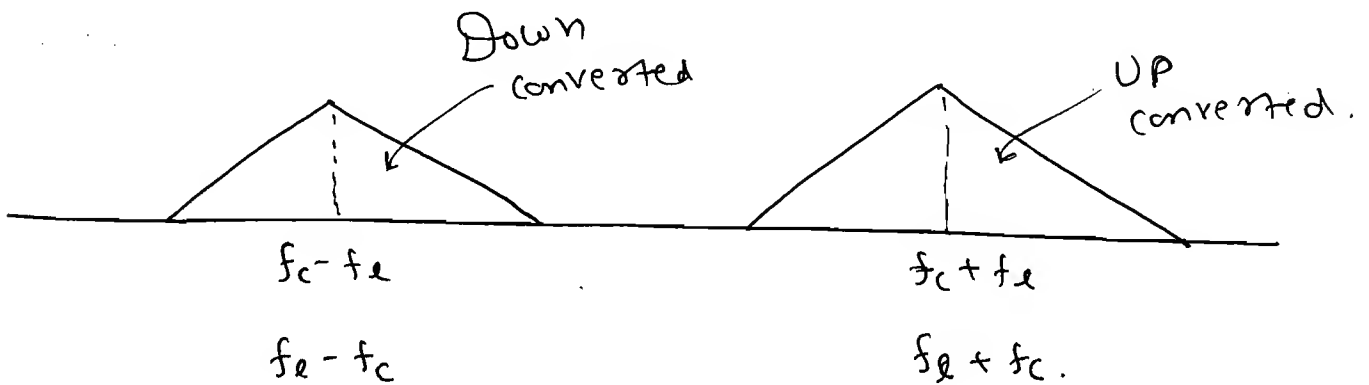
→ Mixer is used to change the carrier frequency of a modulated signal.



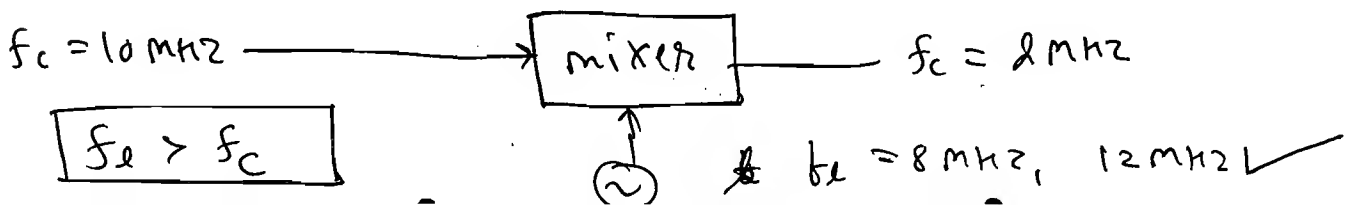
⇒ O/P of multiplier =  $m(t) \cdot \cos(2\pi f_c t) \cdot \cos(2\pi f_e t)$ .

① Condition - ① :-  $f_c > f_e$       ② Condition:  $f_e > f_c$

$$\Rightarrow \frac{1}{2} [m(t) \cos(2\pi(f_c + f_e)t) + m(t) \cos(2\pi(f_c - f_e)t)]$$



(\*)



# ☆ Angle Modulation

⇒ The generalized eq<sup>n</sup> of the carrier is

$$\begin{array}{c} \downarrow A_m \quad \downarrow F_m \quad \downarrow P_m \\ A_c \cos(2\pi f_c t + \phi) \\ \hline \downarrow \\ \theta(t) \end{array} \quad \leftarrow \underline{H.B.}$$

$$\Rightarrow A_c \cos(\theta(t)).$$

→ " $2\pi f_c t + \phi$ " is called as the angle of the carrier.

⇒ Whenever the frequency ( $\omega$ ) the phase changes angle also changes. So FM & PM are also called as Angle Modulation Technique.

## \* Frequency Modulation:-

Definition:

⇒ FM is defined as the process in which the frequency of the carrier is varied according to the message signal.

⇒ Freq. of carrier before modulation is  $f_c$  and after modulation  $f_i$  where

$$\Rightarrow \boxed{f_i = f_c + k_f m(t)} \quad \leftarrow \underline{H.B.}$$

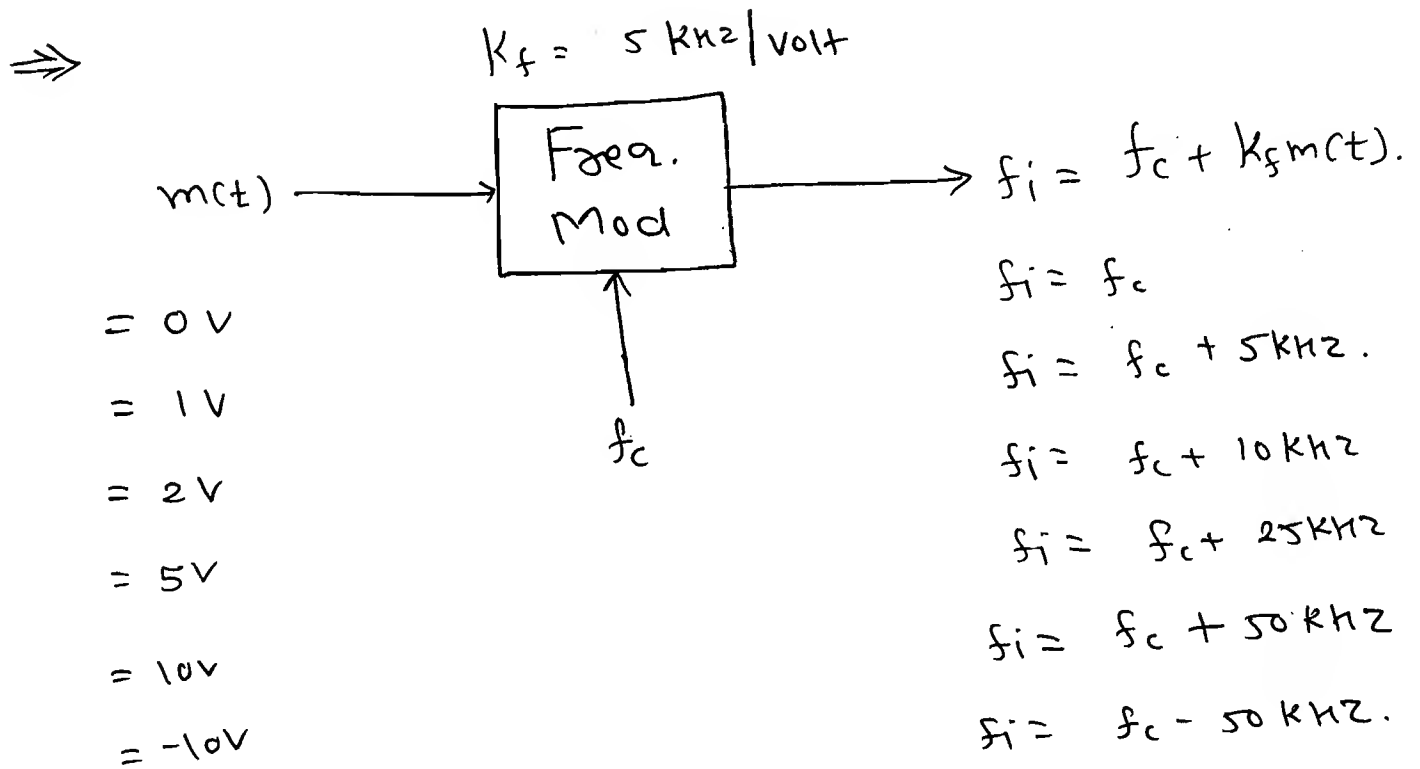


$K_f$  = Frequency Sensitivity of modulator  
 $(\frac{\text{KHz}}{\text{Volt}})$

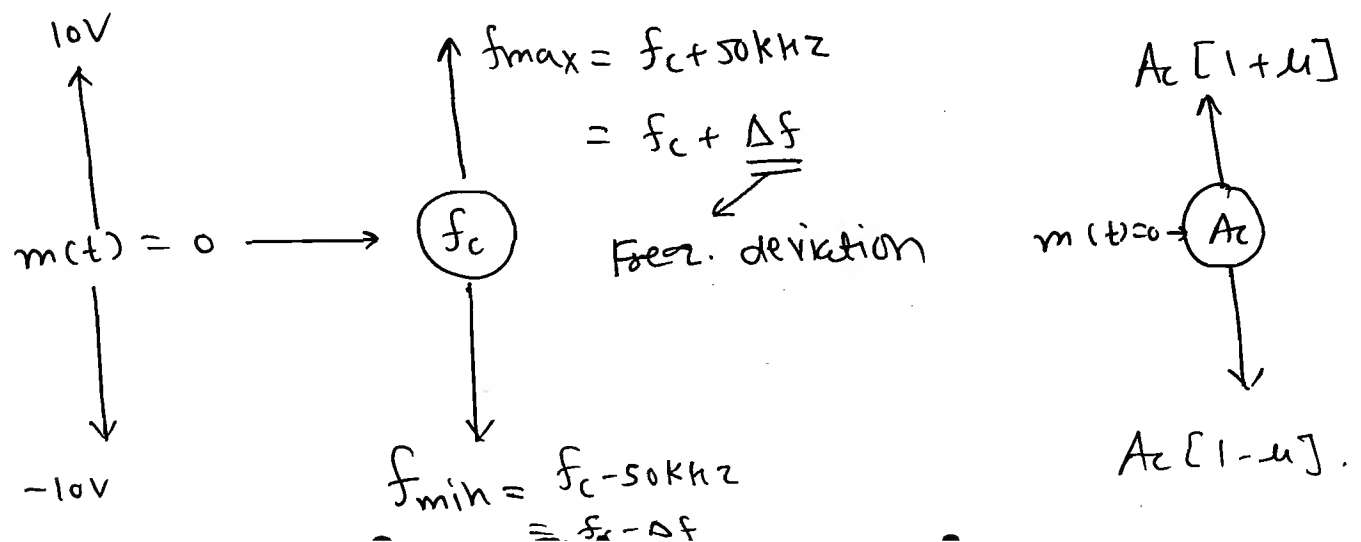
$\Rightarrow$  If the Carrier is not modulated  $m(t)=0$

and  $\boxed{f_i = f_c}$

$\rightarrow$  ' $K_f$ ' indicate the change in the freq. per 1V change of the message signal.



$\rightarrow$  FM modulator is a Voltage to Frequency converter.



⇒ In the case of single tone modulation,

$$m(t) = A_m \cos 2\pi f_m t.$$

$$\rightarrow f_i = f_c + K_f m(t).$$

$$f_i = f_c + K_f A_m \cos 2\pi f_m t.$$

$$\rightarrow f_{\max} = f_c + K_f A_m = f_c + \Delta f.$$

$$\rightarrow f_{\min} = f_c - K_f A_m = f_c - \Delta f.$$

$$\Rightarrow \Delta f = K_f A_m \checkmark$$

Generalized formula.

← H.B.

$$\Delta f = K_f [m(t)]_{\max}$$

$$\Rightarrow A_c \cos(\theta(t)).$$

$$\rightarrow \theta(t) = \omega_c t$$

$$\therefore \theta(t) = 2\pi f_i t.$$

$$\therefore f_i \frac{d}{dt} \theta(t) = 2\pi f_i$$

← H.B.

$$\therefore \boxed{f_i = \frac{1}{2\pi} \frac{d}{dt} (\theta(t))}$$

$$\Rightarrow \theta(t) = 2\pi \int f_i dt$$

$$= 2\pi \int [f_c + K_f m(t)] dt.$$

$$\boxed{\theta(t) = 2\pi f_c t + 2\pi k_f \int m(t) dt.} \leftarrow \underline{H.B.}$$

→ The above eq<sup>n</sup> represent the angle in the case of FM.

→ So, Time domain eq<sup>n</sup> of FM for multitone Modulation is,

$$\therefore \boxed{S(t) = A_c \cos \left[ 2\pi f_c t + 2\pi k_f \int m(t) dt \right].} \quad \swarrow \underline{H.B.}$$

↑  
multitone

→ for single tone modulation

$$\text{i.e. } m(t) = A_m \cos 2\pi f_m t.$$

$$\therefore S(t) = A_c \cos \left[ 2\pi f_c t + 2\pi k_f \int A_m \cos 2\pi f_m t \right].$$

$$= A_c \cos \left[ 2\pi f_c t + \frac{2\pi k_f A_m}{2\pi f_m} \sin 2\pi f_m t \right].$$

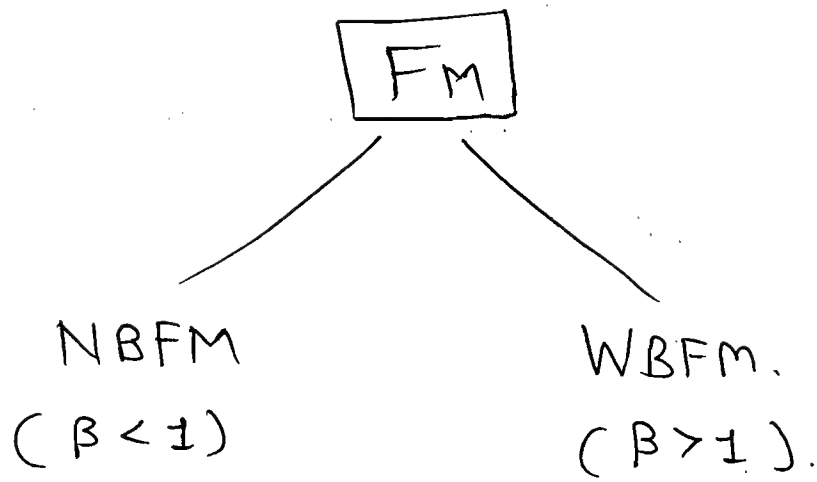
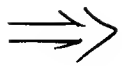
$$\therefore S(t) = A_c \cos \left[ 2\pi f_c t + \left( \frac{k_f A_m}{f_m} \right) \sin 2\pi f_m t \right].$$

$$\therefore \boxed{S(t) = A_c \cos \left[ 2\pi f_c t + \beta \sin 2\pi f_m t \right].}$$

↑  
single tone.

$$\Rightarrow \boxed{\beta = \frac{k_f A_m}{f_m} = \frac{\Delta f}{f_m} = \text{Modulation Index.}}$$

$$= \frac{\text{Freq. deviation}}{\text{message freq.}}$$



→ In AM, modulation index Concept is used for the demodulation and when envelope Detector is used as demodulator.

→ In DSB, SSB and VSB, modulation index Concept is not used because the envelope detector is not used as demodulator.

→ In FM, Modulation index Concept is required for analyzing the Spectrum.

① Narrow Band FM (NBFM) :  $\beta < 1$

$$\rightarrow S(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

$$= A_c \cos 2\pi f_c t \cdot \cos[\beta \sin 2\pi f_m t] \\ - A_c \sin 2\pi f_c t \cdot \sin[\beta \sin 2\pi f_m t]$$

$$\text{let } \theta = \beta \sin 2\pi f_m t$$

$$\therefore S(t) = A_c \cos 2\pi f_c t \cdot \cos \theta - A_c \sin 2\pi f_c t \cdot \sin \theta$$

$$\theta < 1$$

$$\cos \theta \approx 1$$

$$\sin \theta \approx \theta$$

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

$$\Rightarrow \lim_{\theta \rightarrow 0} \sin \theta = \theta$$

$$\therefore S(t) = A_c \cos 2\pi f_c t \cdot \underline{1} - A_c \sin 2\pi f_c t \cdot \underline{\theta}$$

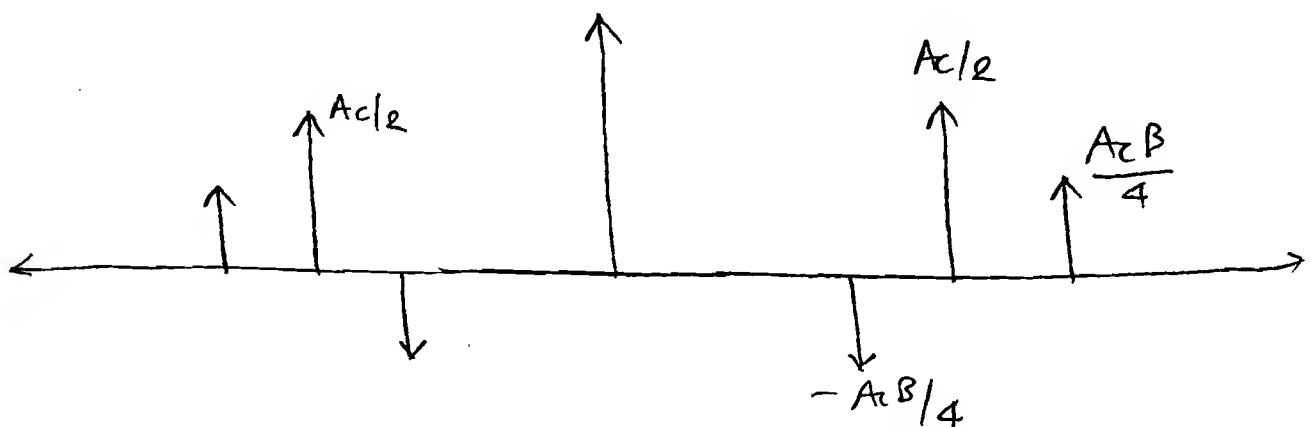
$\downarrow$  H.B.

$$\therefore S(t) = A_c \cos 2\pi f_c t - A_c \beta \sin 2\pi f_c t \cdot \sin 2\pi f_m t$$

$$\therefore S(t) \approx A_c \cos 2\pi f_c t + \frac{A_c \beta}{2} \cos 2\pi (f_c + f_m) t \\ - \frac{A_c \beta}{2} \cos 2\pi (f_c - f_m) t$$

$\leftarrow$  H.B.

$\Rightarrow$



→ The LSB in the spectrum is out of phase by  $180^\circ$ .

Ex: (1) An Am signal and NBFM signal having same modulation index are added, the resultant signal is,

(A) Am

(B) DSB

(C) SSB

✓ (D) SSB with Carrier.

Ans:  $\hookrightarrow$   $LSB = \frac{A_c \mu}{4}$  ,  $LSB = \frac{A_c \beta}{4}$

$\mu = \beta$  so, ans (D) SSB with Carrier.

⇒ The magnitude spectrum of NBFM is same as Am. so, BW and power are also same as Am.

→  $P_t = P_c \left[ 1 + \frac{\beta^2}{2} \right]$   $\leftarrow \text{H.B.}$

NOTE:

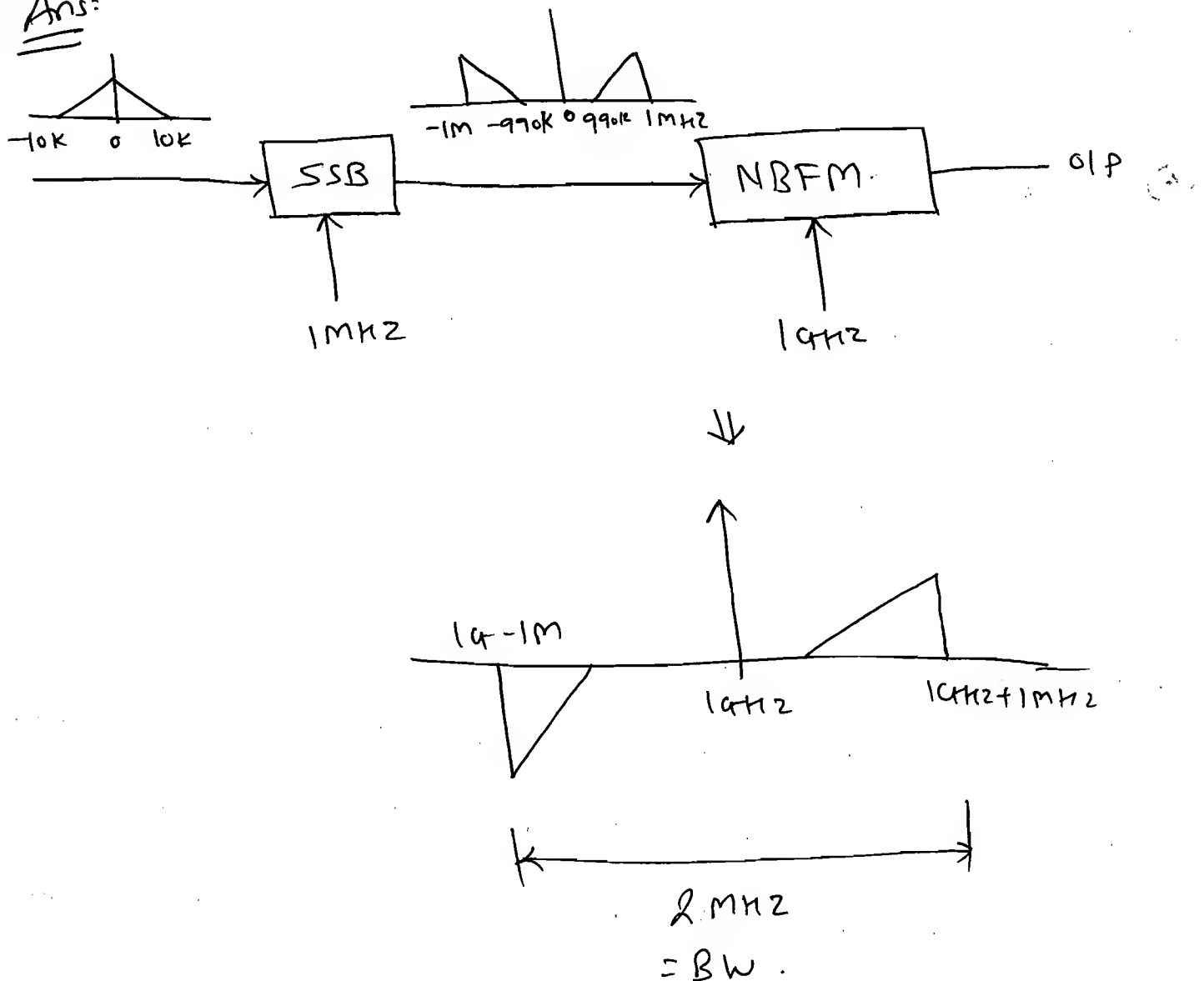
⇒ Practically NBFM is not used because of its similarity to Am.

Gate: 2006:

Ex-2 A message signal bandlimited to 10 kHz is lower sideband SSB modulated with a carrier frequency of 1 MHz. The resultant signal is again passed through NBFM modulator having a carrier freq. of 1 GHz. Determine the BW of signal at O/P.

- (A) 10 kHz      (B) 20 kHz  
(C) 1 MHz      (D) 2 MHz.

Ans:



## ② Wide Band FM : (WBFM) : ( $\beta > 1$ )

$\Rightarrow$  Time domain eq<sup>n</sup> for the single tone modulation is

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

### \* Bessel Function:

$$J_n(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(x \sin \theta - n\theta)} d\theta$$

$\Rightarrow$  Properties:

①  $J_n(x) = (-1)^n J_{-n}(x)$

②  $\sum_{n=-\infty}^{\infty} J_n^2(x) = 1$   $\leftarrow \frac{H.B}{2}$

$\Rightarrow$  Time domain eq<sup>n</sup> of WBFM is  $\downarrow \frac{H.B}{2}$

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t]$$

$$\therefore s(t) = A_c J_0(\beta) \cos 2\pi f_c t$$

$$+ A_c J_1(\beta) \cos 2\pi(f_c + f_m)t + A_c J_{-1}(\beta) \cos 2\pi(f_c - f_m)t$$

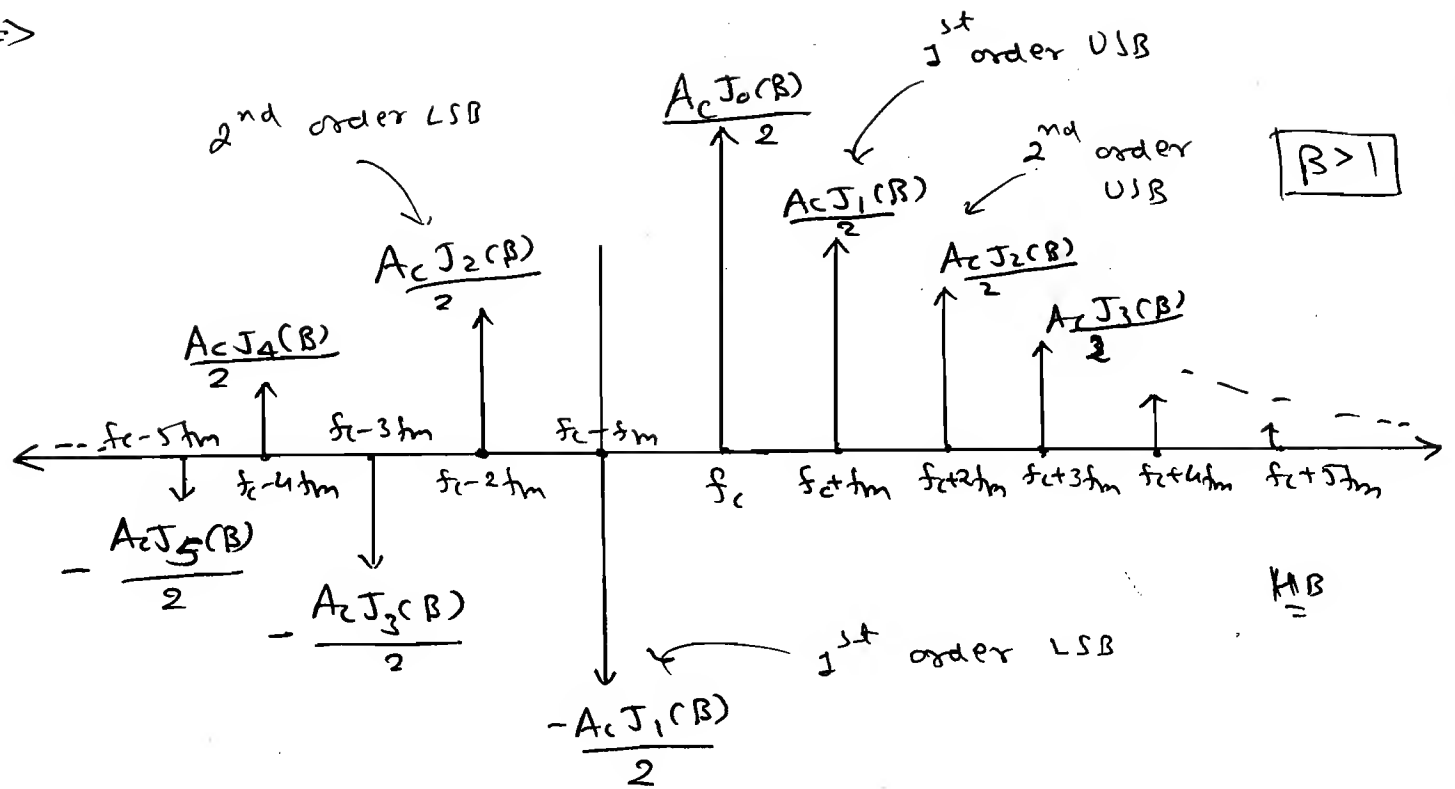
$$+ A_c J_2(\beta) \cos 2\pi(f_c + 2f_m)t + A_c J_{-2}(\beta) \cos 2\pi(f_c - 2f_m)t$$

$$+ A_c J_3(\beta) \cos 2\pi(f_c + 3f_m)t + A_c J_{-3}(\beta) \cos 2\pi(f_c - 3f_m)t$$

+ ...



⇒



### \* Analysis of spectrum:

⇒ The spectrum consist of carrier and infinite no. of upper and lower sideband frequencies.

⇒ Theoretical BW is infinite.

⇒ The magnitude of the spectral component depends on the Bessel function coefficient.

But Bessel function's values gradually decreases as  $n$  increases. So, the magnitude of higher order of frequencies are negligible.

⇒ The carrier magnitude in the spectrum varies with modulation index.

⇒ The Bessel function coefficient  $J_0(B) = 0$

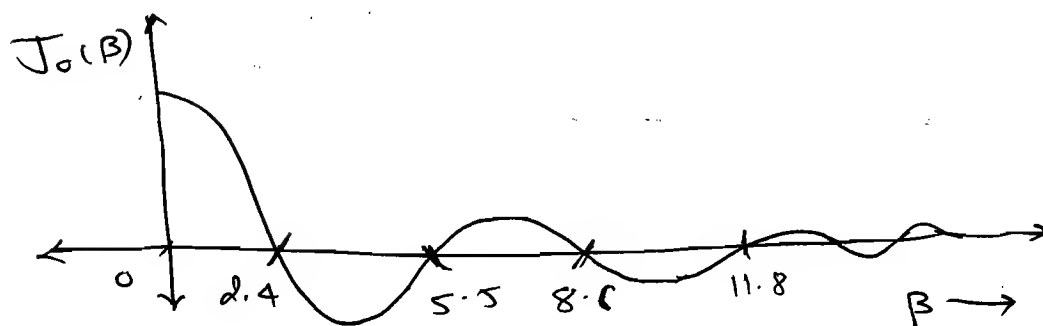
where  $B = 2.4, 5.5, 8.6, \dots$

⇒ The Bessel function coefficient

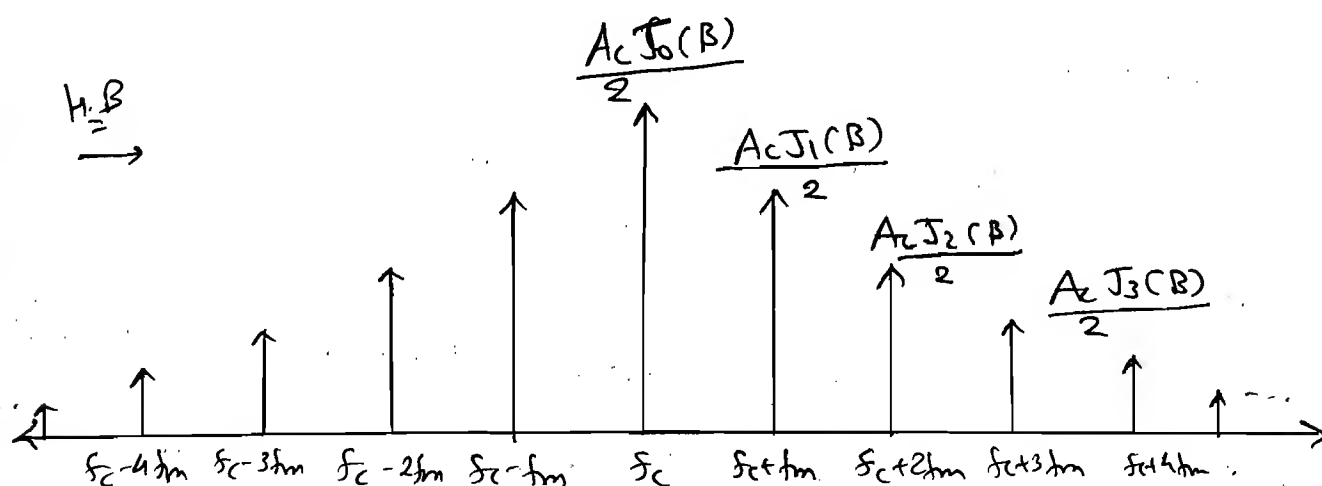
$$J_0(\beta) = 0.$$

←  $\underline{H.B.}$

Where  $\beta = \underline{2.4, 5.5, 8.6, \dots}$



⇒ For these values of  $\beta$ , the carrier magnitude in the spectrum will zero and modulation efficiency is 100%.



[Magnitude Spectrum]

# \* Power Calculation: H.B.

$$\Rightarrow P = \frac{V_{rms}^2}{R}$$

$$\therefore P_{sc} = \frac{\left( \frac{A_c J_0(\beta)}{\sqrt{2}} \right)^2}{R}$$

$$\therefore P_{sc} = \frac{A_c^2 J_0^2(\beta)}{2R}$$

$$\begin{aligned} \Rightarrow P_{sc} + P_{sm} &= \frac{A_c^2}{2R} J_1^2(\beta) \\ P_{sc} - P_{sm} &= \frac{A_c^2}{2R} J_1^2(\beta) \end{aligned} \left. \vphantom{\begin{aligned} \Rightarrow P_{sc} + P_{sm} &= \frac{A_c^2}{2R} J_1^2(\beta) \\ P_{sc} - P_{sm} &= \frac{A_c^2}{2R} J_1^2(\beta) \end{aligned}} \right\} \begin{array}{l} \text{first order} \\ \text{SB power} = \end{array} \frac{A_c^2 J_1^2(\beta)}{R}$$

$$\begin{aligned} \Rightarrow P_{sc} + 2P_{sm} &= \frac{A_c^2}{2R} J_2^2(\beta) \\ P_{sc} - 2P_{sm} &= \frac{A_c^2}{2R} J_2^2(\beta) \end{aligned} \left. \vphantom{\begin{aligned} \Rightarrow P_{sc} + 2P_{sm} &= \frac{A_c^2}{2R} J_2^2(\beta) \\ P_{sc} - 2P_{sm} &= \frac{A_c^2}{2R} J_2^2(\beta) \end{aligned}} \right\} \begin{array}{l} 2^{nd} \text{ order} \\ \text{SB power} = \end{array} \frac{A_c^2 J_2^2(\beta)}{R}$$

$$\therefore P_t = \frac{A_c^2}{2R} \left[ \sum_{n=-\infty}^{\infty} J_n^2(\beta) \right]$$

$$\therefore P_t = \frac{A_c^2}{2R} \cdot 1$$

$$\therefore \boxed{P_t = \frac{A_c^2}{2R}} \rightarrow \text{Same as Unmodulated Carrier Power } [A_c \cos 2\pi f_c t]$$

# \* Transmission B.W. of FM signal using

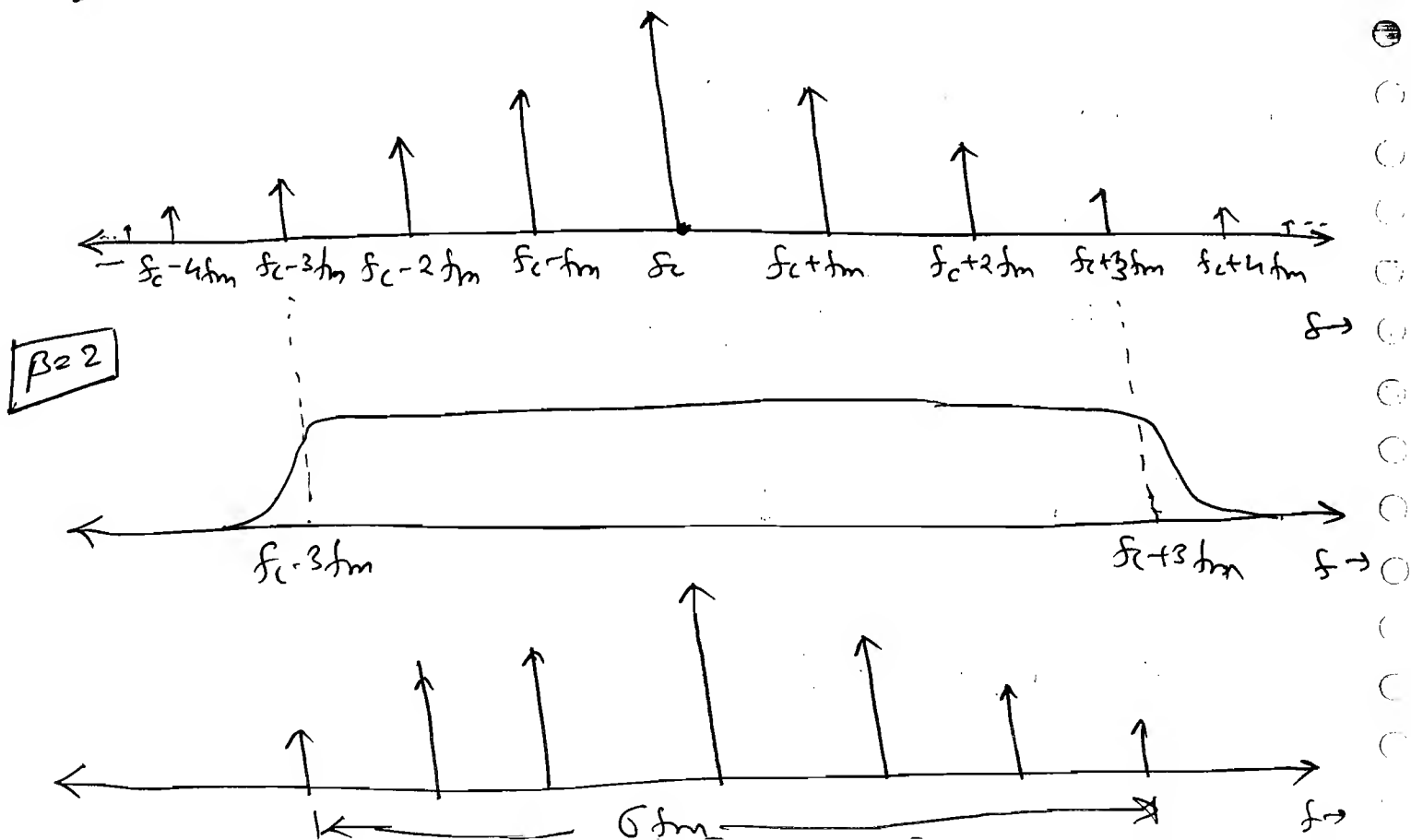
## Carson's rule:

⇒ The theoretical B.W. of FM signal is  $\infty$ .

⇒ Practically BW of the signal should be as minimum as possible. So insignificant frequencies should be eliminated.

⇒ According to Carson's rule B+1 Upper and Lower sidebands will have significant magnitude and contains 99% of the total

Power. So, the FM signal is passed through a Band pass filter to eliminate the insignificant frequencies.



$\Rightarrow$  Carson's rule: H.B.

$$\boxed{B.W. = 2(\beta + 1)f_m} \quad - (1)$$

$$= 2\left(\frac{\Delta f}{f_m} + 1\right)f_m$$

$$\boxed{B.W. = 2\Delta f_m + 2f_m} \quad \text{H.B.} \quad - (2)$$

Ex-1 A carrier signal is frequency modulated by a sinusoidal signal of amplitude 20V and freq. 100 kHz. The freq. sensitivity of modulator is 25 kHz/V.

(i) Determine the freq. deviation, modulation index & BW.

(ii) Repeat the above calculation when the amplitude of the message signal is 40V.

Ans:  $A_m = 20V$ ,  $f_m = 100 \text{ kHz}$ ,  $k_f = 25 \text{ kHz/V}$ .

$$(i) \Delta f = k_f \cdot A_m = 20 \times 25 = 500 \text{ kHz}$$

$$\therefore \beta = \frac{\Delta f}{f_m} = \frac{500 \text{ kHz}}{100 \text{ kHz}} = 5$$

$$\therefore \beta = 5$$

$$BW = 2(\beta + 1)f_m$$

$$\therefore BW = 2(6) \times 100 \text{ kHz}.$$

$$= 1200 \text{ kHz}.$$

$$\therefore \boxed{BW = 1.2 \text{ MHz}}.$$

(ii) When  $A_m = 40 \text{ V}$

$$\Delta f = K_f \cdot A_m = 25 \text{ kHz/V} \times 40 \text{ V}$$

$$\Delta f = 1000 \text{ kHz}$$

$$\therefore \beta = \frac{\Delta f}{f_m} = \frac{1000 \text{ kHz}}{100 \text{ kHz}} = 10.$$

$$\therefore BW = 2(\beta + 1) f_m$$

$$= 2 \times 11 \times 100 \text{ kHz}$$

$$\boxed{BW = 2.2 \text{ MHz}}.$$

Ex-2 Consider the FM signal

$$s(t) = 10 \cos [2\pi 10^6 t + 8 \sin 4\pi 10^3 t]$$

(i) Determine Modulation Index, Freq. deviation,

BW and Power.

(ii) Repeat the above calculation when the message freq. is doubled.

Ans:  $A_c = 10$ ,  $f_c = 10^6$ ,  $\beta = 8$ ,  $f_m = 2 \times 10^3 \text{ Hz}.$

(i)  $\boxed{\beta = 8}$

$$\therefore \beta = \frac{\Delta f}{f_m}$$

$$\therefore \Delta f = \beta \cdot f_m$$

$$\Delta f = 8 \times 2 \text{ kHz} = 16 \text{ kHz}.$$

$$\therefore B_W = 2(\beta+1)f_m = 2(9) \times 2 \text{ kHz} \\ = 36 \text{ kHz.}$$

$$\therefore P_t = \frac{A_c^2}{2\rho} = \frac{100}{2 \times 1} = 50 \text{ W.}$$

(ii)

~~XXXXXXXXXXXX~~~~XXXXXXXXXXXX~~

~~11/10/2~~ ~~11/10/2~~ ~~11/10/2~~

~~PLS X KEY X 25mm~~~~10/2~~ ~~2/3~~

⇒ When,  $f_m' = 2 f_m$ .

$$\therefore \beta = 4 \quad \text{as} \quad \beta = \frac{k_f \cdot A_m}{S_m}$$

So,  $\Delta f = 4 \times 4 \text{ kHz} = 16 \text{ kHz}$ .

$$BW = 2(4 + i) \text{ kHz}$$

$$BW = 40 \text{ kHz}$$

$\therefore P_t = \text{SOW.}$

Ex-3 A Carrier signal is frequency modulated by a sinusoidal signal and the frequency deviation is 8 kHz. The message freq. is 2 kHz.

i) Determine the modulation index and BW.

ii) Repeat the above calculation when the amplitude ~~of the~~ the message signal is increased by a factor of 3 and its freq. is reduced to 1 kHz.

Ans:  $\Delta f = 8 \text{ kHz}$ ,  $f_m = 2 \text{ kHz}$ .

$$\therefore (i) \beta = \frac{\Delta f}{f_m} = \frac{8 \text{ kHz}}{2 \text{ kHz}} = 4.$$

$$\therefore BW = 2(\beta + 1) f_m = 2(5) \times 2 \text{ kHz} = 20 \text{ kHz}.$$

(ii) When  $A_m' = 3A_m$  and  $\Rightarrow A_m' = 3A_m$

$$f_m' = f_m - 1 \text{ kHz} = f_m/2 = 1 \text{ kHz}.$$

$$\therefore \Delta f = k_f \cdot A_m'$$

$$\therefore \Delta f = 3 \times 8 = 24 \text{ kHz}.$$

$$\beta = \frac{24 \text{ kHz}}{1 \text{ kHz}} = 24.$$

$$BW = 2(\beta + 1) f_m = 2(25) \times 1 \text{ kHz} = 50 \text{ kHz}.$$



Ex - 4 \* \* \* A Carrier Signal is freq. modulated by a sinusoidal signal and freq. deviation is 50 kHz. Determine the modulation index and BW when message freq. is (i)  $f_m = 500 \text{ kHz}$  (ii)  $f_m = 500 \text{ Hz}$ .

Ans:  $\Delta f = 50 \text{ kHz}$ ,  $f_m = 500 \text{ kHz}$

(i)  $\beta = \frac{\Delta f}{f_m} = \frac{50 \text{ kHz}}{500 \text{ kHz}}$

$\beta = 0.1$  (NBFM)

So,  $BW = 2 f_m$  (same as AM)

$BW = 1 \text{ MHz}$

(ii)  $\beta = \frac{\Delta f}{f_m} = \frac{50 \text{ kHz}}{500} = 100 \gg 1$  (WBFM).

$\therefore BW = 2(\beta + 1) f_m$   
 $= 2 \times 101 \times 500$

$\therefore BW = 101 \text{ kHz.}$

Ex - 5 A Carrier Signal is freq. Modulated by a sinusoidal signal and freq. deviation is 50 kHz. Det. the Modulation index and BW when the message freq. is

(i)  $f_m = 500 \text{ kHz}$  (ii)  $f_m = 500 \text{ Hz}$ .

Ex-6 A sinusoidal signal of 4 kHz is used as modulating signal for an AM and FM transmitters. Both of the transmitters uses the same carrier. The freq. deviation of the FM transmitter is 4 times the BW of the AM signal. The magnitudes of the freq. at  $f_c + 4 \text{ kHz}$  are same in AM and FM. Determine the modulation index of AM & FM.

$$J_1(2) = 0.577, \quad J_1(4) = 0.06, \quad J_1(8) = 0.235.$$

Ans:  $f_m = 4 \text{ kHz}$

$$\Delta f = 4 \times \text{BW}_{\text{AM}}$$

$$\Delta f = 4 \times 2 f_m$$

$$\therefore \Delta f = 8 f_m$$

$$\therefore \beta = \frac{\Delta f}{f_m}$$

$$\therefore \boxed{\beta = 8}$$

$$\frac{A_c J_1(\beta)}{2} = \frac{A_c \mu}{4}$$

$$J_1(8) = \frac{\mu}{2}$$

$$\therefore \mu = 2 \times 0.235$$

$$\boxed{\mu = 0.47}$$

Ex-7 A carrier signal is modulated by message signal  $m(t) = A_m \cos 2\pi f_m t$ . In a certain experiment conducted with  $f_m = 1 \text{ kHz}$  increasing  $A_m$  (starting from 0V) it was found that the carrier magnitude in a spectrum becomes zero for the first time

When  $A_m$  is 2V.

- ① Det. frequency sensitivity of Modulator.
- ② Det. the Value of  $A_m$  for which the carrier magnitude become zero for the second time.

Ans:

$$(i) \uparrow \beta = \frac{k_f \cdot A_m}{f_m} \uparrow$$

$$\beta = 2.4, 5.5, 8.6, \dots$$

$$\therefore 2.4 = \frac{k_f \cdot 2V}{1 \text{ kHz}}$$

$$\therefore \boxed{k_f = 1.2 \text{ kHz/Volt.}}$$

$$(ii) \quad \beta = \frac{k_f \cdot A_m}{f_m}$$

$$\therefore 5.5 = \frac{1.2 \times A_m}{1 \text{ kHz.}}$$

$$\therefore \boxed{A_m = 4.6 \text{ V}}$$

\*  
\*  
\*

Ex-8 FM transmitter radiates 100W when the carrier is not modulated. the carrier is now modulated and the modulation index is adjusted so that the magnitude of the 1<sup>st</sup> order sidebands is zero in the spectrum under this condition ① calculate the power at carrier freq. ② calculate the power in all the remaining sideband. ③ calculate the power in the second order sideband.

$$J_0(0) = 1$$

$$J_0(3.8) = -0.4$$

$$J_1(2.4) = 0.52$$

$$J_0(2.4) = 0$$

$$J_0(5.1) = -0.16$$

$$J_1(3.8) = 0$$

$$J_1(5.1) = -0.33$$

$$J_2(2.4) = 0.43$$

$$J_2(3.8) = 0.41$$

$$J_2(5.1) = 0$$

Ans:  $P_t = \frac{A_c^2}{2R} = 100W$

$$\therefore A_c^2 = 200$$

$$A_c = 14.14V$$

$\therefore$  MI is adjusted so that the magnitude of the 1<sup>st</sup> order SB is zero

$$\therefore \text{so, } J_1(3.8) = 0 \Rightarrow J_1(\beta) = 0$$

$$\therefore \boxed{\beta = 3.8}$$

$$\textcircled{1} P_{fc} = \frac{A_c^2}{2R} \cdot J_0^2(\beta)$$

$$= \frac{A_c^2}{2R} \cdot J_0^2(3.8)$$

$$= 100 \times (-0.4)^2$$

$$P_{fc} = 16W$$

$$\textcircled{2} P_{SB} = P_t - P_{fc}$$

$$= 100 - 16$$

$$\therefore \boxed{P_{SB} = 84W}$$

$$\textcircled{3} \quad P_{\text{2nd order}} = \frac{A_z^2}{R} J_2^2(3.8)$$

$$= \frac{100}{2} \cdot (0.41)^2$$

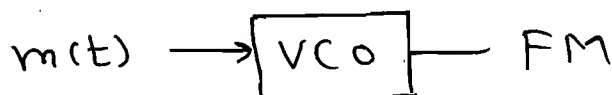
$$\therefore P_{\text{2nd order}} = 33.3 \text{ W}$$

\* Generation of WBFM signal.

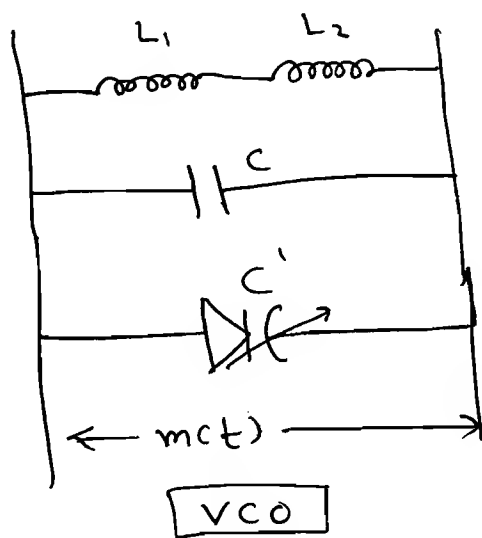
① Direct Method.

② Indirect method (or) Armstrong method  
(Practically it is not used).

① Direct Method:



⇒ FM modulator is voltage to frequency converter. So, VCO (voltage control oscillator) is used to convert voltage variation into freq. variation.



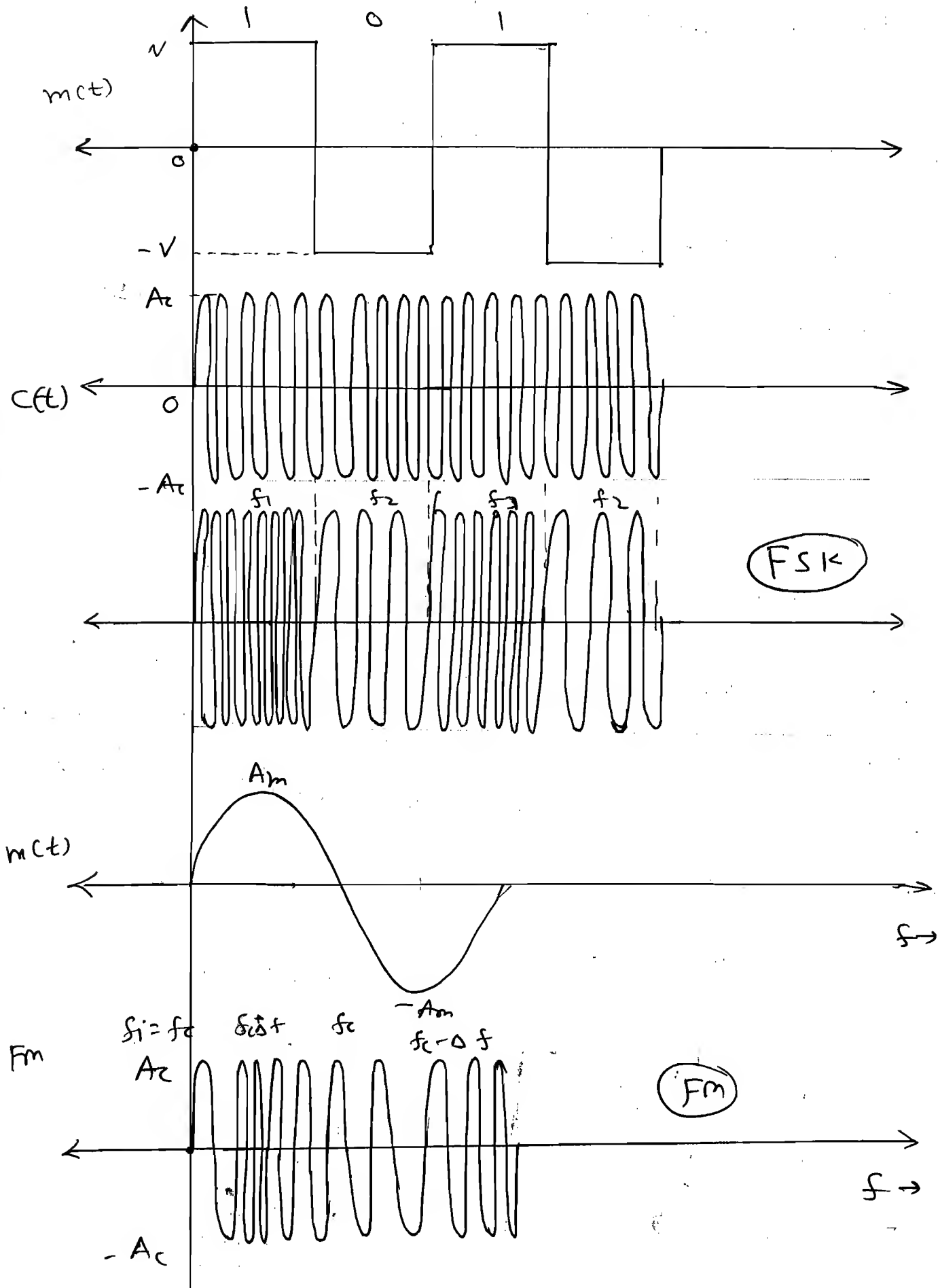
Varactor diode



$$C = \frac{\epsilon A}{W}$$



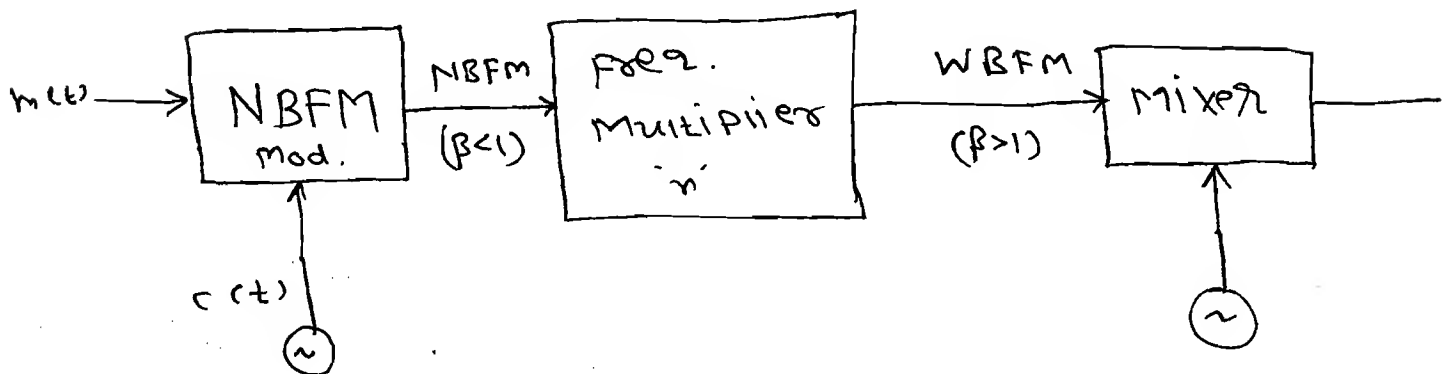
$$f \approx \frac{1}{2\pi \sqrt{(L_1 + L_2)(C + C')}}.$$



$$f_1 = f_c + k_f V = \frac{1}{2\pi \sqrt{(L_1 + L_2)(C + C_1)}}$$

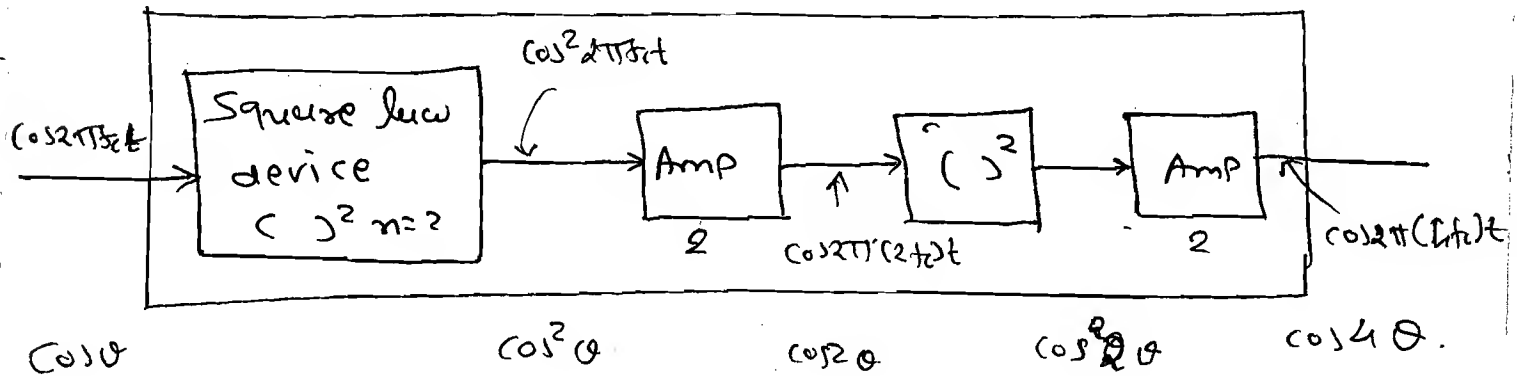
$$f_2 = f_c - k_f V = \frac{1}{2\pi \sqrt{(L_1 + L_2)(C + C_2)}}$$

## ② Armstrong Method:



⇒ Freq Multiplier 'n':

$$n=4$$



$$\rightarrow \cos^2 2\pi f_c t = \frac{1}{2} + \frac{1}{2} \cos 2\pi (2f_c) t.$$

→ Assume that the message signal & carrier signal are applied to NBFM modulator.

The o/p signal is:

$$A_c \cos [2\pi f_c t + \underbrace{\beta \sin 2\pi f_m t}_{\beta < 1}]$$

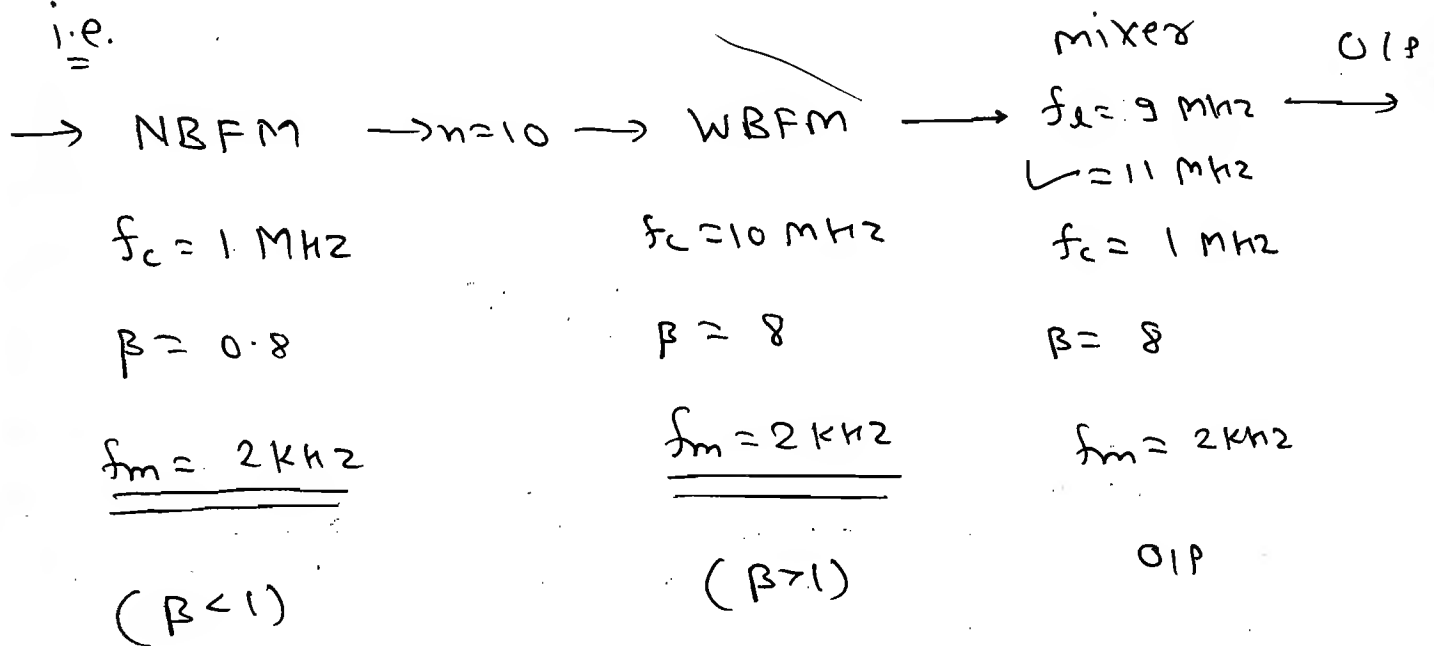
$\Rightarrow$  If the above signal is passed through a 'freq. multiplier by  $n$ ' then O/P is:

$$A_c \cos [n (2\pi f_c t + \beta \sin 2\pi f_m t)]$$

$$= A_c \cos [2\pi (nf_c)t + (n\beta) \sin 2\pi f_m t] \quad \checkmark \text{ u.B}$$

$\Rightarrow$  In a freq. multiplier carrier freq.  $f_c$  &  $\beta$  is increased by a factor of  $n$ , but the message frequency is same.

i.e.



Ex-1 Consider a NBFM signal

$$s(t) = 10 \cos [2\pi 10^6 t + 0.4 \sin 2\pi 10^3 t]$$

The signal is passed through two freq. multipliers connected in cascade  $n_1 = 2$  &  $n_2 = 5$ . Determine the carrier freq., modulation index, & BW at the O/P of first multiplier & second multiplier.



Ans:  $A_c = 10V$ ,  $f_c = 1 MHz$ ,  $f_m = 1 kHz$ ,  $\beta = 0.4$ .



$$f_c = 1 MHz$$

$$f_m = 1 kHz$$

$$\beta = 0.4$$

$$f_c = 2 MHz$$

$$f_m = 1 kHz$$

$$\beta = 0.8 < 1$$

(NB FM)

$$BW = 2 f_m$$

$$\boxed{BW = 2 kHz}$$

$$f_c = 10 MHz$$

$$f_m = 1 kHz$$

$$\beta = 4$$

(WBFM)

$$BW = 2(\beta + 1)f_m$$

$$BW = 2 \times 5 \times 1 kHz$$

$$\boxed{BW = 10 kHz}$$

$\Rightarrow$  We know that,

modulation index  $\beta = \frac{\Delta f}{f_m}$  No change.

$\downarrow$  H.B.

but  $f_m$  is not change

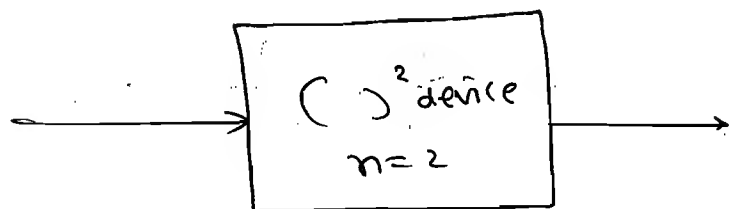
$\therefore \Delta f$  change, during conversion of

NB FM to WBFM.

$\Rightarrow$  Hardware complexity of Armstrong Modulator is very high so it is not practically used.

Ex-1 FM signal is having frequency deviation of 90 kHz is passed through a square law device the message freq. is 5 kHz. Determine  $\beta$ , BW, at the O/P.

Soln:



$$\Delta f = 90 \text{ kHz}$$

$$f_m = 5 \text{ kHz}$$

$$\beta = 18$$

$$\Delta f = \beta f_m = 36 \times 5 = 180 \text{ kHz}$$

$$\beta = 2 \times 18 = 36$$

$$BW = 2(\beta + 1) f_m$$
$$= 2 \times 36 \times 5 \text{ kHz}$$

$$BW = 370 \text{ kHz}$$

(OR)

$$BW = 2 \left( \frac{\Delta f}{f_m} + 1 \right) f_m$$

$$= 2\Delta f + 2f_m$$

$$= 2(180) + 10$$

$$BW = 370 \text{ kHz}$$

\* Demodulation of FM signal:

$\Rightarrow$  FM demodulator is a frequency to voltage converter.

① Frequency discrimination method.

② phase discrimination method.

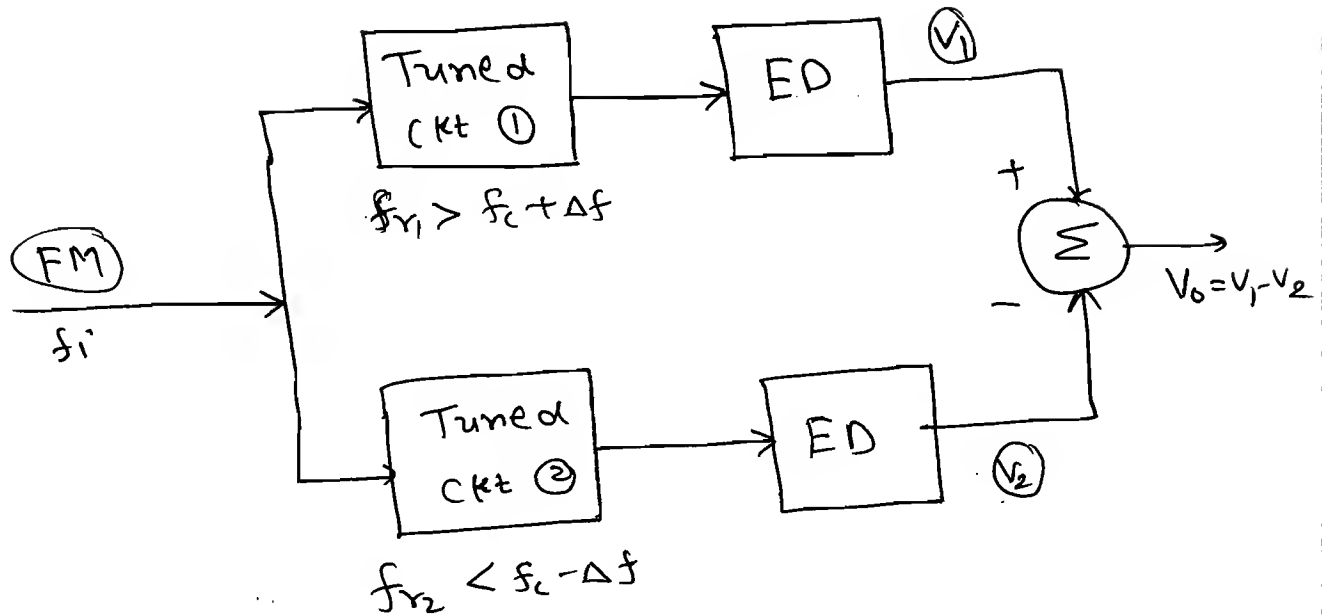
$\rightarrow$  Foster seeley discrimination.

$\rightarrow$  Ratio detector.

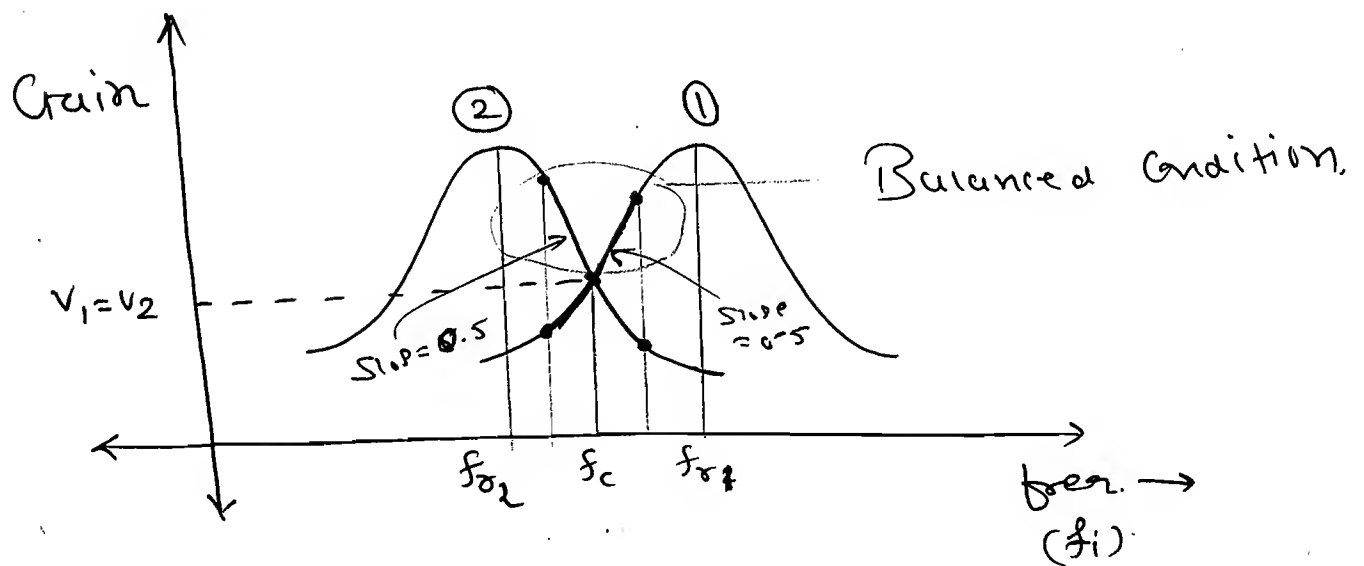
$\rightarrow$  PLL (phase locked loop).

# ① Balanced Slope Detector :-

⇒



⇒ The Freq. Response of the Tuned ckt :-



⇒ Condition:-

①  $f_i = f_c$  :

⇒ The gain of the 1<sup>st</sup> tuned ckt is same as the 2<sup>nd</sup> tuned ckt. So the o/p of the EDs  $V_1 = V_2$ . So,  $V_0 = 0$

②  $f_i > f_c$ :

$\Rightarrow$  The gain of the 1<sup>st</sup> tuned ckt is larger than gain of the 2<sup>nd</sup> tuned ckt.  $\therefore V_1 > V_2$  and  $\therefore$  the output voltage  $V_o = \underline{V_1 - V_2} > 0$  (Positive).

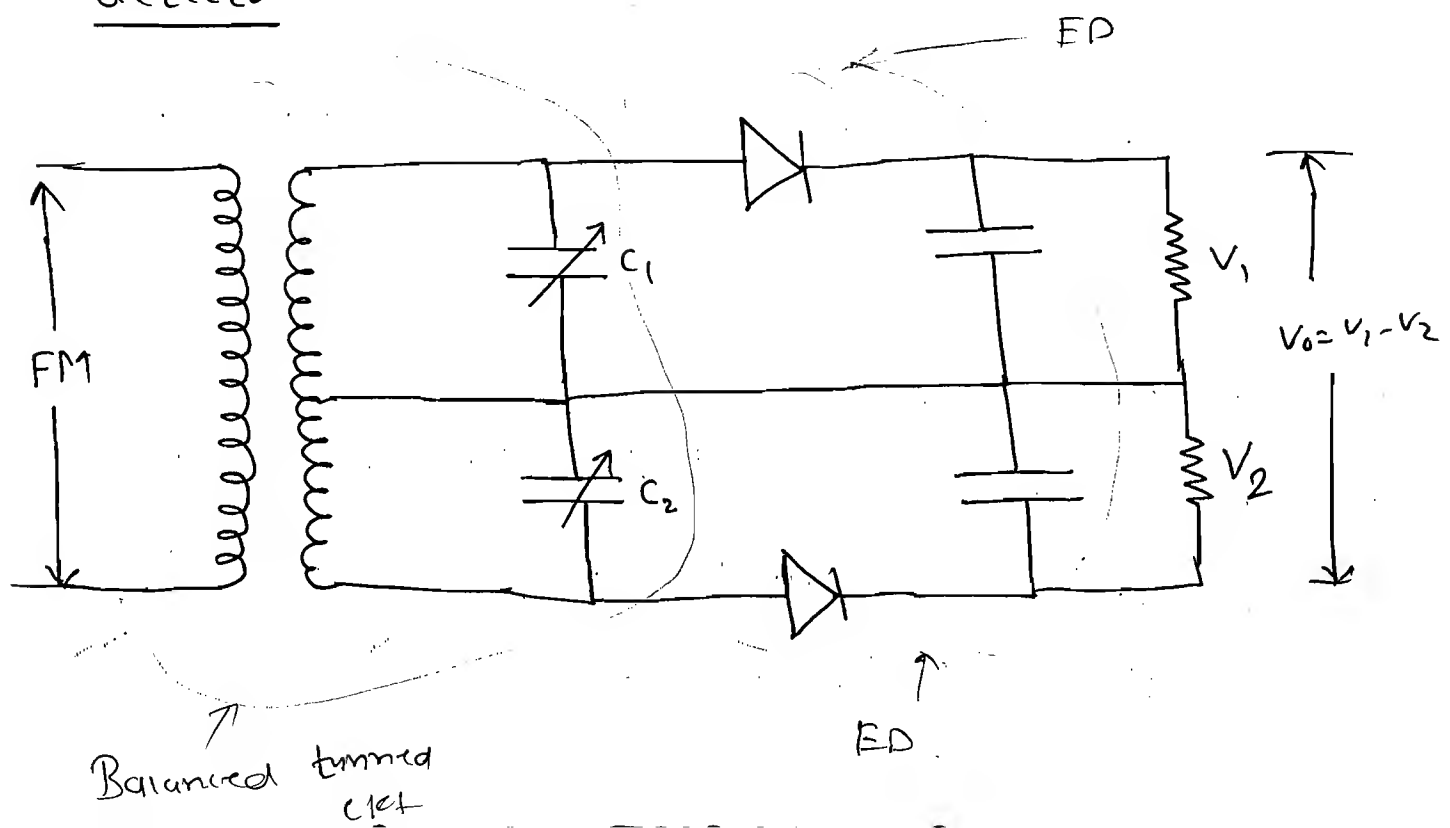
③  $f_i < f_c$ :

$\Rightarrow$  The gain of the 1<sup>st</sup> tuned ckt is less than the 2<sup>nd</sup> tuned ckt.

$\therefore V_1 < V_2$ .

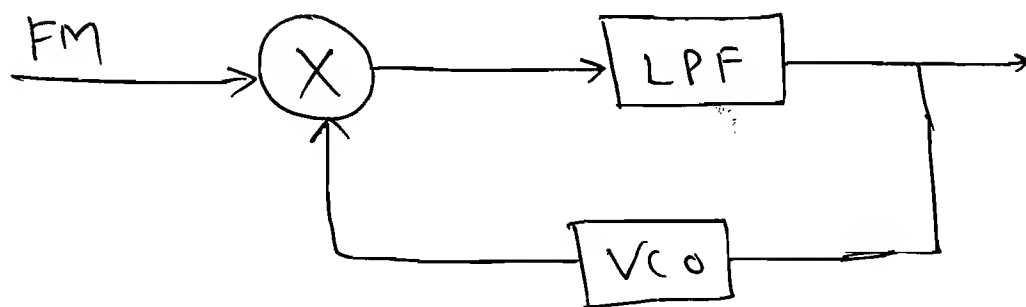
$\therefore$  o/p voltage,  $V_o = V_1 - V_2 = \text{negative} < 0$ .

\* Hardware implementation of Balanced slope detector:



## \* FM demodulation using PLL:

⇒



first order PLL:

⇒ The working principle of PLL is same as synchronously detector.

→ In AM, DSB & SSB the carrier frequency is constant. So, local oscillator is used to generate the same carrier.

→ In fm the carrier freq. is varied according to the message signal using VCO. So, the local oscillator is replaced with VCO to generate the same carrier.

→ If the input to the PLL is  $\cos[2\pi f_c t + \phi]$  then O/P of PLL is:

then the O/P of PLL is:

$$V_o \propto \frac{d[\phi]}{dt}$$

$\Rightarrow$  Time domain eq<sup>n</sup> of Multitone Mod<sup>n</sup> is:

$$A_c \cos \left[ 2\pi f_c t + 2\pi K_f \int m(t) dt \right].$$

$$\therefore \text{O/P } V_o \propto \frac{d}{dt} [\phi].$$

$$\therefore V_o \propto \frac{d}{dt} \left[ 2\pi K_f \int m(t) dt \right].$$

$$\therefore V_o \propto 2\pi K_f m(t).$$

$$\therefore V_o = \frac{2\pi K_f}{2\pi K_v} \cdot m(t).$$

$$\therefore \boxed{V_o = \frac{K_f}{K_v} \cdot m(t).} \quad \leftarrow \text{H.B.}$$

$K_f \Rightarrow$  freq. sensitivity of the VCO at transmitter.

$K_v \Rightarrow$  freq. sensitivity of the VCO at Receiver.

## \* Phase Modulation:

⇒ Time domain eq<sup>n</sup> of the PM signal is

$$S(t) = A_c \cos [2\pi f_c t + K_p m(t)] \quad \leftarrow \text{H.B.}$$

$$\therefore \boxed{\phi(t) = K_p m(t)} \quad (\text{rad}).$$

$K_p$  = phase sensitivity of the modulator  
(rad/volts).

for single tone modulation

$$m(t) = A_m \cos 2\pi f_m t.$$

$$\therefore S(t) = A_c \cos [2\pi f_c t + K_p A_m \cos 2\pi f_m t].$$

$$\text{H.B.} \Rightarrow \boxed{\Delta\phi = K_p \cdot A_m} = \text{Phase deviation.}$$

$$\therefore S(t) = A_c \cos [2\pi f_c t + \Delta\phi \cos 2\pi f_m t].$$

$$\therefore \boxed{S(t) = A_c \cos [2\pi f_c t + \beta \cos 2\pi f_m t]} \quad \leftarrow \text{H.B.} =$$

$$\Rightarrow \boxed{K_p m(t) = \Delta\phi = \beta = \text{modulation index.}} \quad \leftarrow \text{H.B.}$$

⇒ In phase modulation phase deviation and modulation index are same.

$\Rightarrow$  The magnitude Spectrum of the PM signal is same as the FM signal except a phase shift of  $90^\circ$  at message freq. So, the magnitude Spectrum of the PM signal is same as the FM signal.

Ex-1 Consider an angle modulated signal  $s(t) = 10 \cos [2\pi f_c t + 5 \sin 8\pi 10^3 t]$ .

Determine the freq. deviation and phase deviation.

Ans: (i) FM  $\rightarrow \Delta f = \beta \cdot f_m = 5 \times 4 \text{ kHz} = 20 \text{ kHz}$ .

(ii) PM  $\rightarrow \Delta \phi = \beta = 5 \text{ rad}$ .

Ex-2 Consider an angle modulated <sup>signal</sup> ~~index~~

$$s(t) = 10 \cos [2\pi 10^6 t + 8 \sin 2\pi 10^3 t]$$

(i) Assuming the given signal is FM determine the modulation index, freq. deviation, BW & power.

(ii) Repeat the above calculation when the message freq. is doubled.



Sol<sup>n</sup>:  $A_c = 10$ ,  $f_c = 1 \text{ MHz}$ ,  $f_m = 1 \text{ kHz}$

(i)  $\beta = 8$ ,

$$\Delta f = \beta \cdot f_m = 8 \times 1 \text{ kHz} = 8 \text{ kHz}.$$

$$\therefore B_w = 2(\beta + 1) f_m = 2 \times 9 \times 1 \text{ kHz} = 18 \text{ kHz}.$$

$$P = \frac{A_c^2}{2R} = \frac{100}{2} = 50 \text{ W}.$$

(ii) When  $f_m' = 2 f_m = 2 \text{ kHz}$ .

$$\therefore \beta = \frac{\Delta f}{f_m} = \frac{8 \text{ kHz}}{2 \text{ kHz}} = 4, \quad \Delta f = 8 \text{ kHz}$$

$$\therefore B_w = 2(\beta + 1) f_m = 2(5) \times 2 = 20 \text{ kHz}.$$

Ex-3 Repeat the above numerical problem  
assume that the given signal is PM.

Sol<sup>n</sup>:

(i)  $\beta = 8$ .

Phase deviation  $\Delta \phi = \beta = 8 \text{ rad}.$

$$B_w = 2(\beta + 1) f_m$$

$$= 2 \times 9 \times 10 \text{ kHz}$$

$$B_w = 18 \text{ kHz}$$

$$P_t = \frac{A_c^2}{2R} = \frac{100}{2} = 50 \text{ W}$$

(ii) When  $f_m' = 2 f_m = 2 \times 10 \text{ kHz} = 20 \text{ kHz}.$

$$\beta = 8 = \Delta \phi.$$

$$P_t = 50 \text{ W}$$

$$B_w = 2(\beta + 1) f_m'$$

$$= 2(9) \times 20 \text{ kHz} = 36 \text{ kHz}.$$

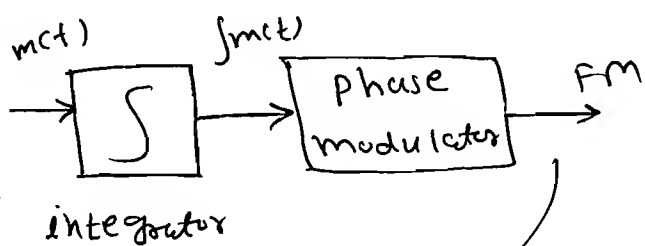
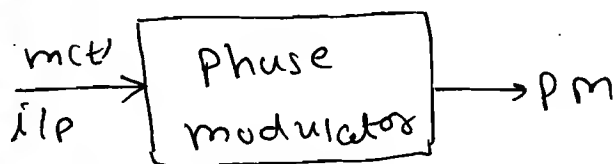
## \* Conclusion:

- ① In AM & PM, the BW of the modulated signal is double when the message freq. is double. But in FM, BW is not double.
- ② In PM, modulation index  $\beta$  is independent of modulating frequency.

## \* Time domain eq<sup>n</sup> for multitone modulation:

**PM**

$$S(t) = A_c \cos [2\pi f_c t + k_p m(t)]$$

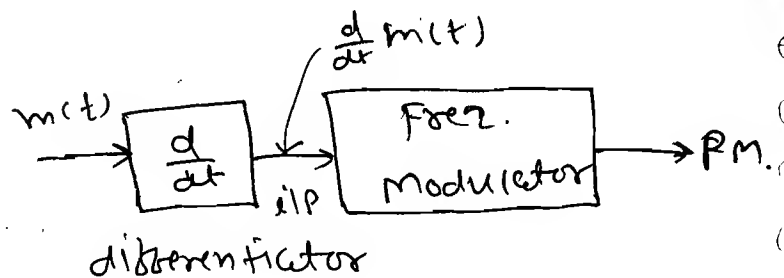
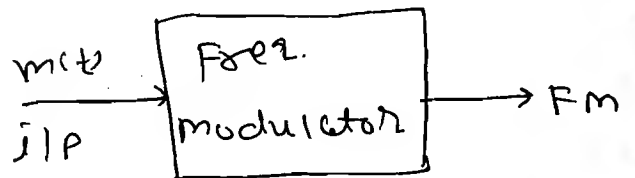


$$A_c \cos [2\pi f_c t + K_f \int m(t) dt]$$

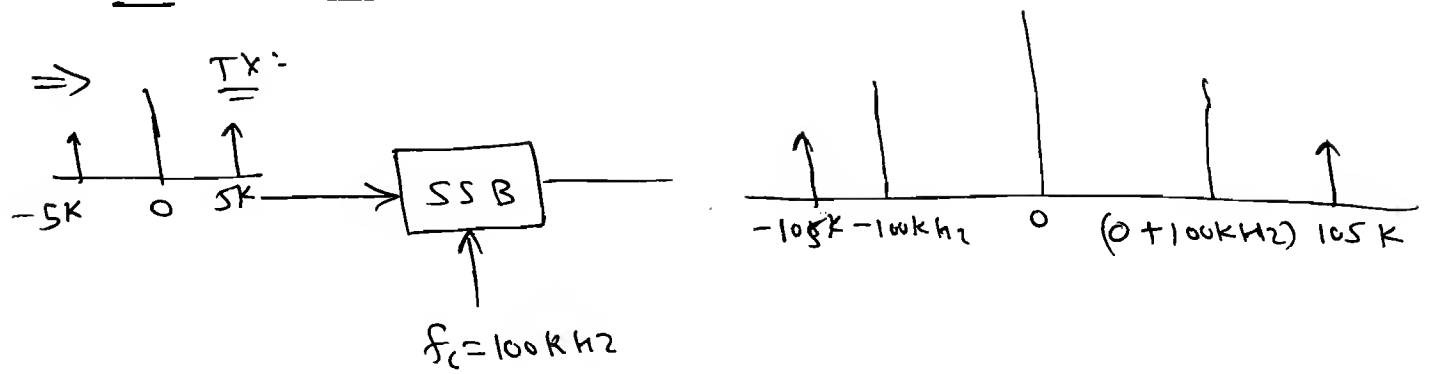
$$K_f = 2\pi K_s$$

**FM**

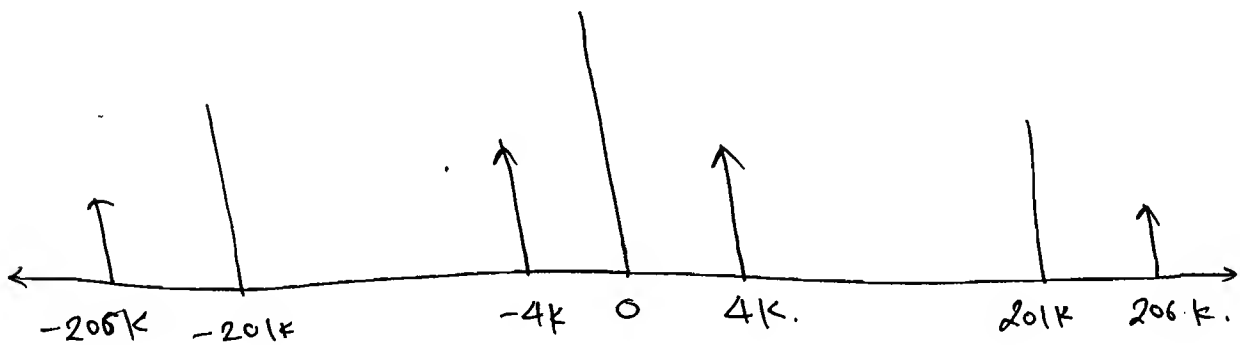
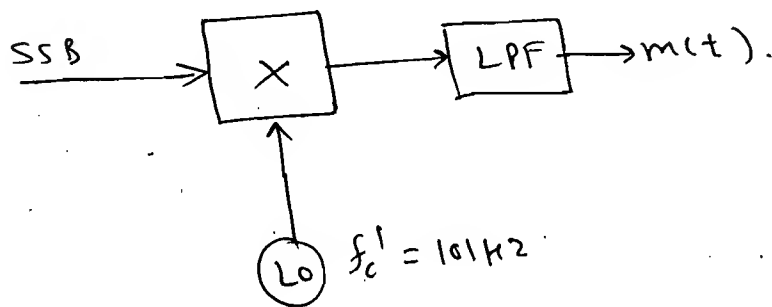
$$S(t) = A_c \cos [2\pi f_c t + 2\pi K_f \int m(t) dt]$$



# \* IMP Concept: $\leftarrow H.B$



$\Rightarrow$  Review: Using coherent/synchronous detection.



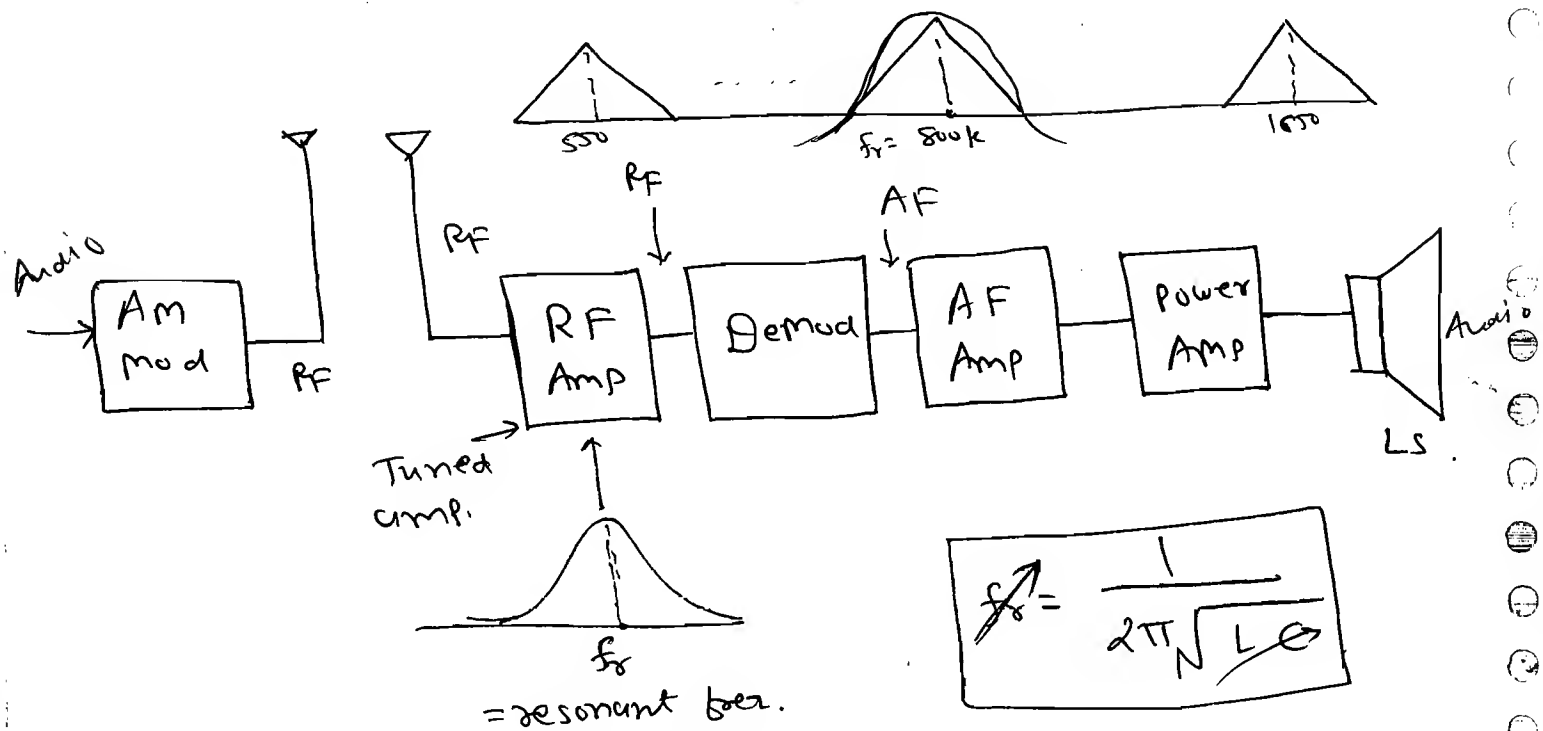
## Conclusion:

<u><math>T_x</math></u> :	$f_m = 5KHz$	$f_c = 100KHz$	USB $\rightarrow 105KHz$ / LSB.
<u><math>H_x</math></u> :	Lo $\rightarrow 100KHz$	$\rightarrow$ O/P:	$f_m = 5KHz$ . 5KHz.
	Lo $\rightarrow 101KHz$	$\rightarrow$ O/P:	$f_m = 4KHz$ . 6KHz.
	Lo $\rightarrow 102KHz$	$\rightarrow$ O/P:	$f_m = 3KHz$ . 7KHz.
	Lo $\rightarrow 99KHz$	$\rightarrow$ O/P:	$f_m = 6KHz$ . 4KHz.
	Lo $\rightarrow 98KHz$	$\rightarrow$ O/P:	$f_m = 7KHz$ . 3KHz.
for LSB	$f_m' = sub.$		

# ★ AM Receivers :-

- ① Tuned Radio Freq. Receiver, (TRF Rx).
- ② Superhetrodyne Receiver.

## ① Tuned Radio Frequency Rx (TRF Rx) :-



$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

FM : 88M - 108MHz  
 Am : 550 K - 1050 KHz.

←  $H.F.$

⇒ In order to select the 800 KHz signal, we have to be set resonant freq. at 800 KHz. by changing capacitor value.

⇒ So, tuning in TRF Rx is nothing but the setting a  $f_r$  in order to get desired signal.

⇒ International standard BW of signal = 10 KHz  
 (including Guard Band)

⇒ After Receiving the signal from antenna RF Amp. is used to increase the signal strength. The RF Amp. is also a tuned amp. which is used to select the required signal by adjusting the resonant freq.

⇒ Demodulator Converts RF signal into AF.

⇒ AF amp. and Power amp. is used to increase the signal strength to the required level.

\* Characteristic Parameters of a Receiver:

① Sensitivity.

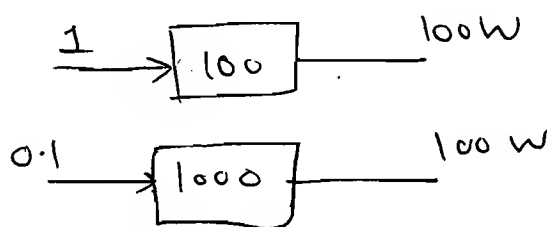
② Selectivity.

③ Fidelity.

① Sensitivity:

⇒ It is defined as the minimum signal strength that should be maintained at the input of a receiver to get a standard output.

⇒ Sensitivity depends on the overall gain of the receiver.



## ② Selectivity:

⇒ Selectivity is defined as the ability of the receiver to select the required freq only.

⇒ Assume that a receiver is tuned to 800 kHz the tuned ckt at the RF amplifier has to select the freq from 795 kHz to 805 kHz. This is possible only when the resonant freq. is adjusted to 800 kHz and  $Q$  should be 80.

$$(BW = \frac{f_0}{Q}) \Rightarrow Q = 80. \text{ But simultaneously}$$

Variation of resonant freq. and  $Q$  is not possible in tuned ckt. and  $Q$  is not

⇒ If the BW of the tuned ckt is greater than 10 kHz adjacent signal freq. are selected - If the BW is less than 10 kHz some of the required freq are attenuated.

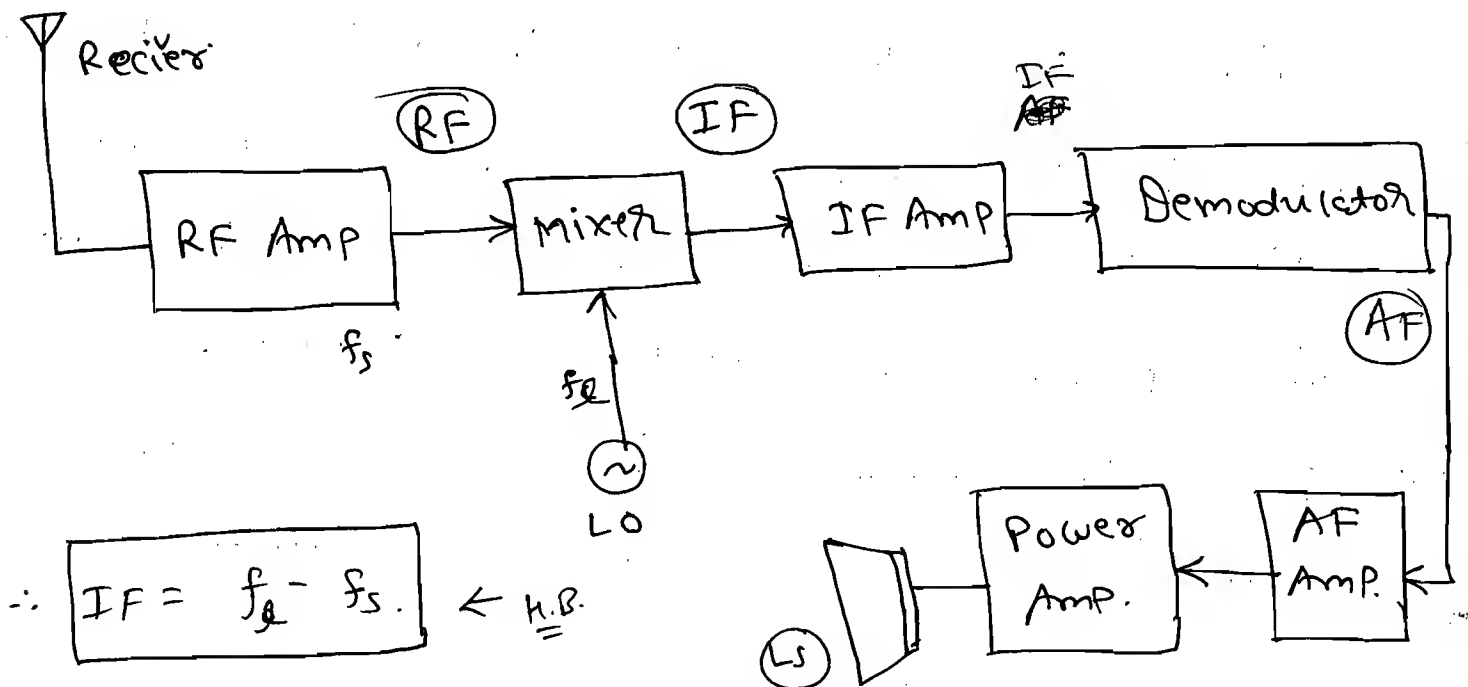
## ③ Fidelity:

⇒ Fidelity is defined as the ability of the receiver to reproduce all audio freq. at the o/p.

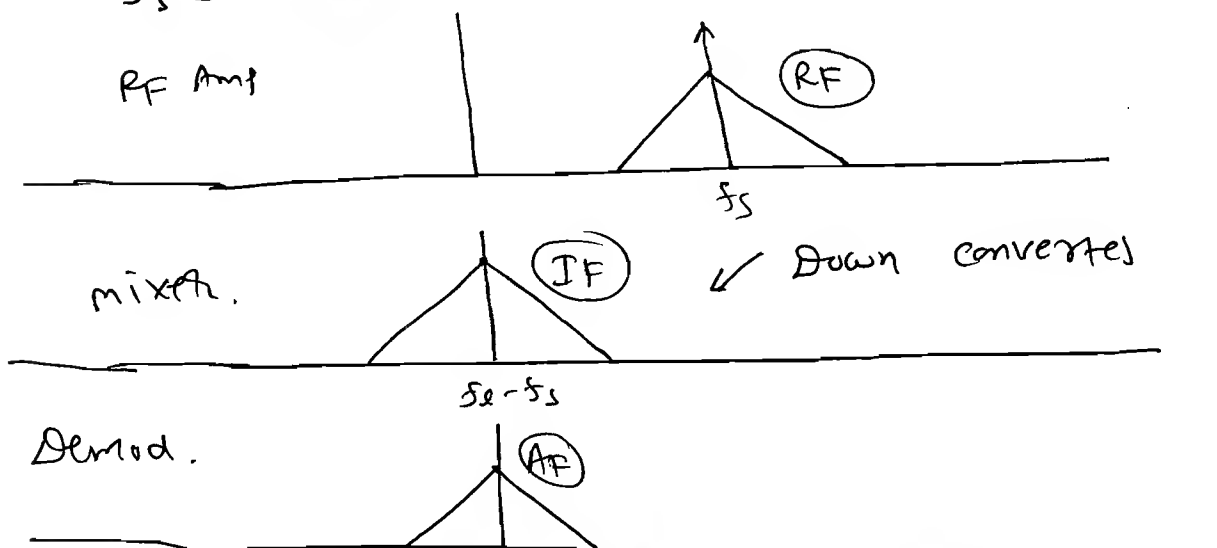
⇒ In AM radio the audio signal is Band limited to 5 kHz as the Bandwidth allotted to each broadcasting station is 10 kHz.

⇒ The highest audio freq. at o/p of the receiver is 5 kHz. So, the signal fidelity is very low in AM receivers.

## ② Superhetrodyne Receiver:



$f_s$  = Carrier freq. of the tuned station.



→ After receiving signal from the antenna, RF Amp is used to increase the signal strength.

→ Mixer is used to down convert the RF signal into IF.

→ The IF is a constant value and depends on the Application.

→ In Am radio the IF used is 455 KHz.

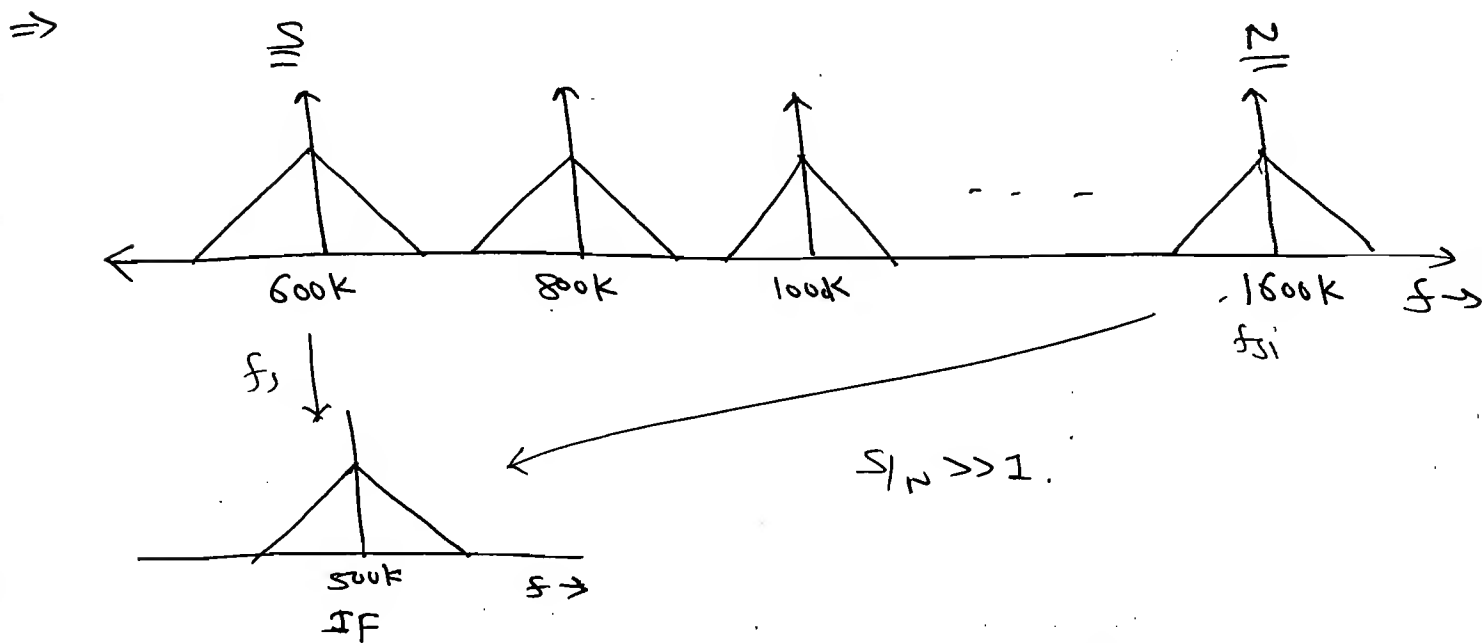
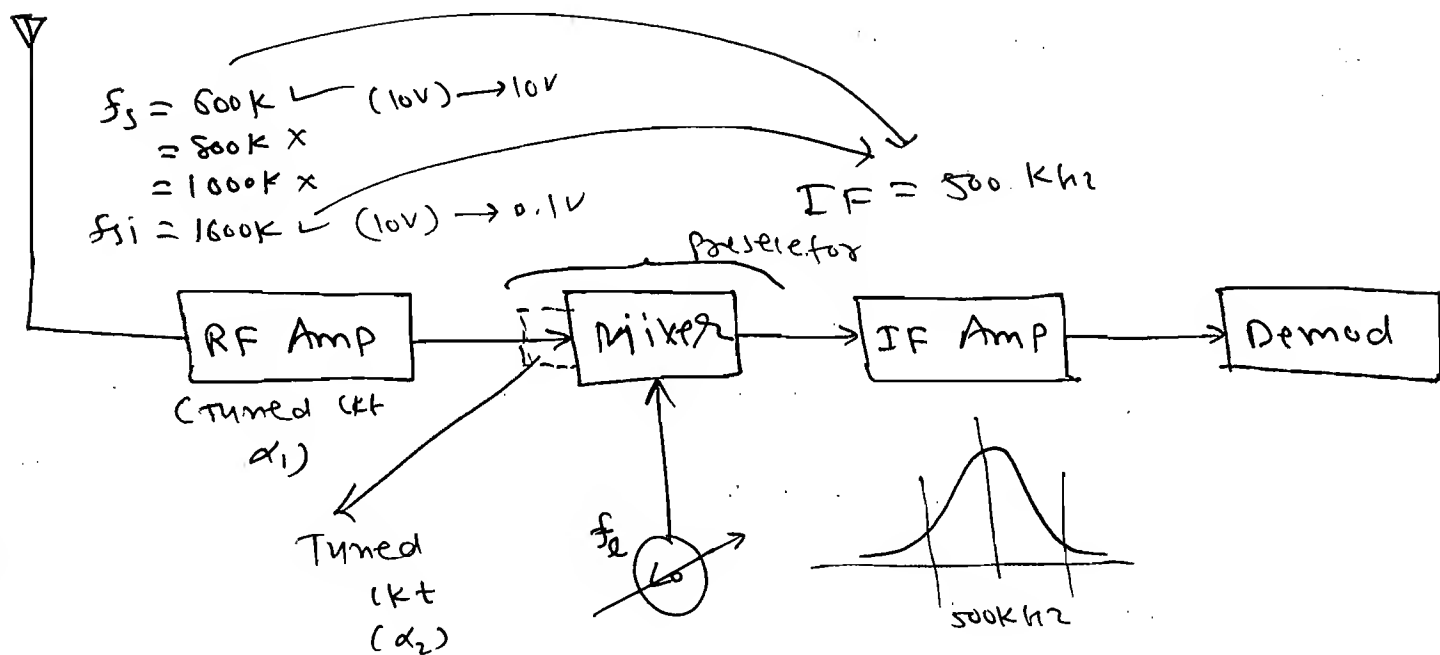
The local oscillator freq. should be adjusted so that  $f_c - f_s$  is 455 KHz.

This process is called as tuning.

→ The IF amplifier is a tuned amplifier which is always tuned 455 KHz.



# \* Image freq. and its Suppression:



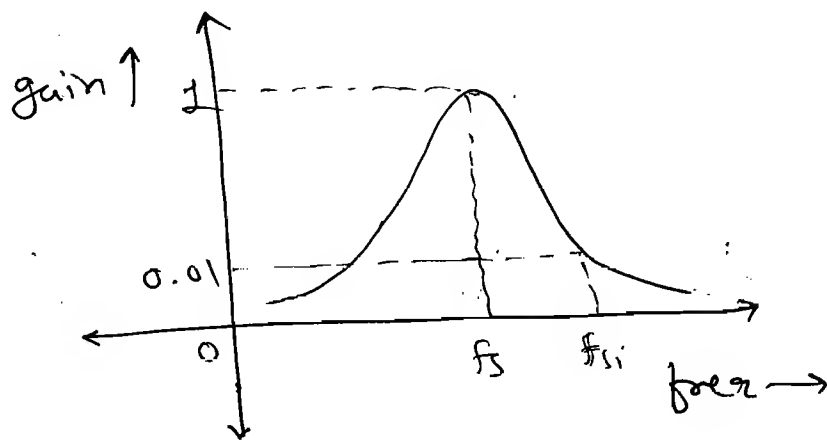
$\Rightarrow$  Consider a receiver with IF 500 kHz and assume that receiver is tuned at 600 kHz. The Local oscillator freq. is adjusted to 1100 kHz to down convert signal to 500 kHz. Assume that another signal received from antenna having a carrier freq. of 1600 kHz. This signal is also down converted to 500 kHz.

and causes interference to the required signal, the signal which is causing interference is called as the image freq.

$$f_{si} = f_s + 2IF \leftarrow \text{N.B.}$$

$$1600\text{K} = 600\text{K} + (2 \times 500\text{K})$$

→ To reduce the image frequency signal strength of a tuned ckt is used at the i/p of the mixer, the freq. response of tuned ckt as shown in fig.



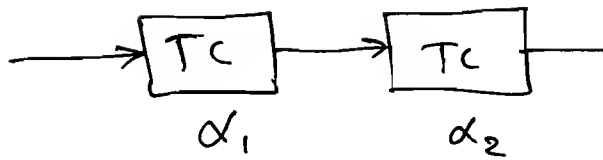
⇒ The gain of tuned ckt at  $f_{si}$  should be as minimum as possible.

⇒ To determine the suppression factor of the tuned ckt, Image Rejection Ratio is used.

$$IRR = \alpha = \frac{G_{fs}}{G_{f_{si}}} = \sqrt{1 + \alpha^2 p^2}$$


---

Where,  $p = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$



$$\alpha = \alpha_1 \cdot \alpha_2$$

Q-1 A Superhetrodyne receiver having no RF Amp. is tuned to 555 kHz. The Local oscillator freq. is adjusted to 1010 kHz & Q is 50. Determine the image freq. and image rejection ratio.

Ans:

$$f_s = 555 \text{ kHz} \quad Q = 50$$

$$f_l = 1010 \text{ kHz}$$

$$\therefore \text{IF} = f_l - f_s = 1010 - 555 = 455 \text{ kHz.}$$

$$\therefore f_{si} = f_s + 2\text{IF}$$

$$\therefore f_{si} = 555 + (2 \times 455) = 555 + 910$$

$$\therefore \boxed{f_{si} = 1465 \text{ kHz.}}$$

$$\therefore \rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$$

$$\therefore \rho = \frac{1465}{555} - \frac{555}{1465}$$

$$\therefore \rho = 2.26$$

$$\therefore \alpha = \sqrt{1 + Q^2 \rho^2} = \sqrt{1 + (2500)(2.26)^2}$$

$$\therefore \boxed{\alpha = 113.}$$

Ex-2 A Superhetrodyne receiver having RF Amp. is tuned to 1200 kHz, the IF 450 kHz. The  $Q$  of the tuned ckt at the RF Amp. and also at the mixer are same and is equal to 65. Calculate the image freq & IRR.

Ans:

$$f_s = 1200 \text{ kHz.}$$

$$IF = 450 \text{ kHz.}$$

$$Q = Q_2 = 65.$$

$$\therefore f_{si} = f_s + 2IF$$

$$= 1200 + 900$$

$$\boxed{f_{si} = 2100 \text{ kHz}}$$

$$\therefore f = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$$

$$= \frac{2100}{1200} - \frac{1200}{2100}$$

$$\therefore \boxed{f = 1.178}$$

$$\therefore \alpha = \alpha_1 = \alpha_2$$

$$\alpha = \sqrt{1 + Q^2 f^2} \cdot \sqrt{1 + Q_2^2 f^2}$$

$$\therefore \alpha = (1 + Q^2 f^2) \quad (\because Q = Q_2 = 65)$$

$$\therefore \alpha = 1 + (65)^2 (1.178)^2$$

$$\therefore \boxed{\alpha = 5870.}$$

Ex-3 A Superhetrodyne receiver having no RF Amp. is tuned to 1000 KHz the IF is 455 KHz &  $Q$  is ~~1000~~ 100. (1) Determine the Image freq. & IRR. (2) Repeat the above calculation when receiver is tuned to 25 MHz.

Ans: (i)  $f_s = 1000 \text{ KHz}$   
 $IF = 455 \text{ KHz}$   
 $Q = 100$

$$\therefore f_{si} = f_s + 2IF$$

$$\therefore f_{si} = 1000 + 910 = 1910 \text{ KHz.}$$

$$\therefore S = \frac{1910}{1000} - \frac{1000}{1910}$$

$$\therefore S = 1.3864$$

$$\alpha = \sqrt{1 + Q^2 S^2} = \sqrt{1 + 1.38^2 \cdot (100)^2}$$

$$\therefore \boxed{\alpha = 138.64}$$

(ii)  $f_s = 25 \text{ MHz}$

$$\therefore f_{si} = 25 \text{ M} + 2 \times 455 \text{ K}$$

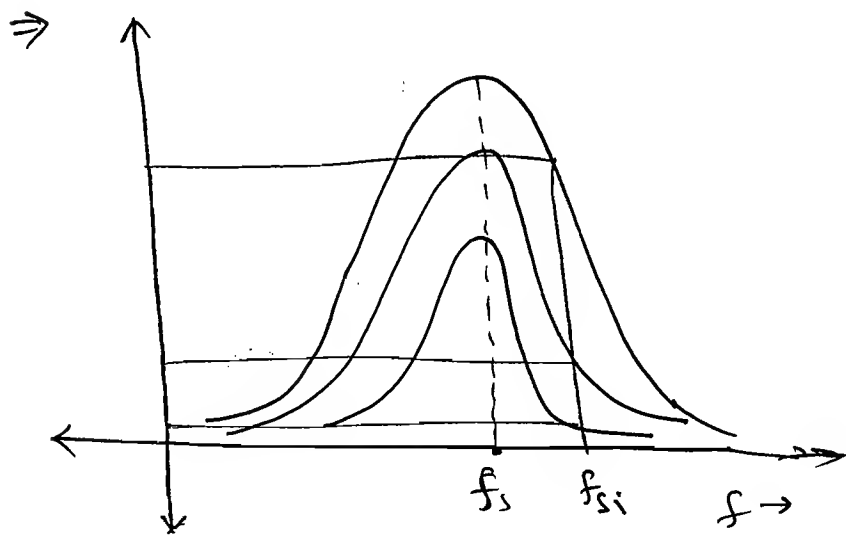
$$f_{si} = 25.910$$

$$\therefore S = \frac{25.910}{25} - \frac{25}{25.910}$$

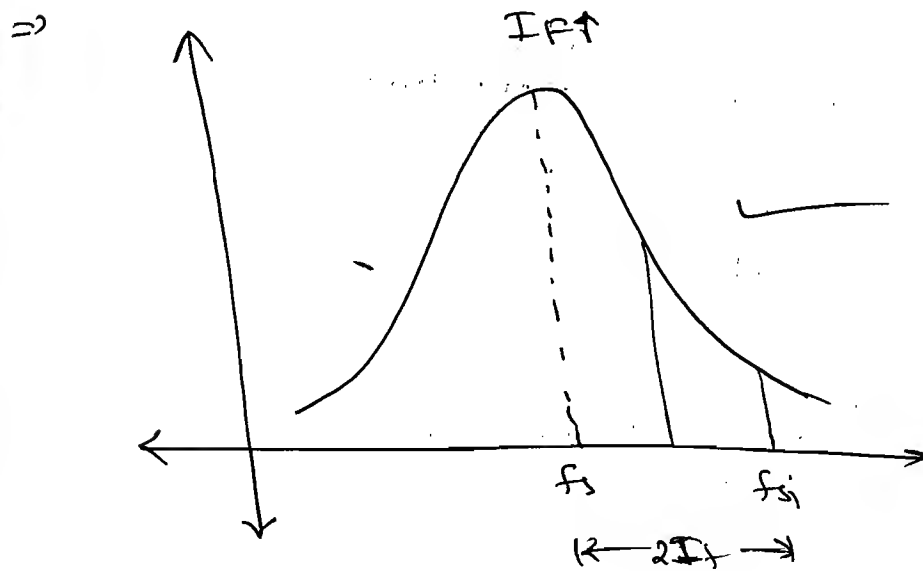
$$\therefore S = 0.036$$

$$\boxed{\alpha = 7.22}$$

⇒ To improve the Image Rejection Ratio, either a (or) 'IF' should be increased.



Practically  
not possible  
because BW must 10kHz



Ex-1 Determine the value of IF Required in the above numerical problem so that the IRR at 25 MHz is also equal to 138.6.

Ans:

$$f_s = 25 \text{ MHz}$$

$$Q = 100$$

$$\alpha = \sqrt{1 + (100)^2 (1.386)^2}$$

$$= 138.6$$

$$\therefore \alpha = 138.6 = \sqrt{1 + (100)^2 \cdot f^2}$$

⇒

$$\boxed{f = 1.38}$$

$$\therefore S = 1.386 = \frac{25 + 2If}{25} - \frac{25}{25 + 2If}$$

$$\Rightarrow \boxed{If = 11.4 \text{ MHz}}$$

\* In FM:

$$88 \text{ M} - 108 \text{ M Hz}$$

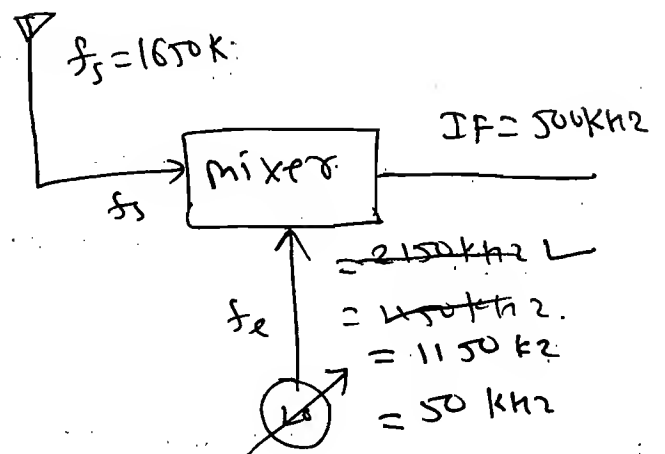
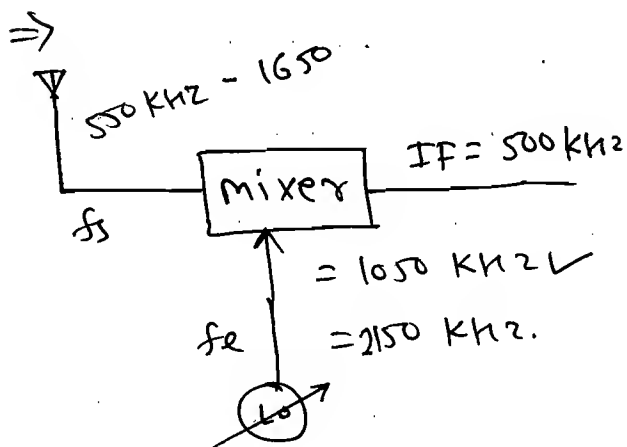
$$\boxed{If = 10.7 \text{ MHz}}$$

In Am

$$\boxed{If = 455 \text{ kHz}}$$

IMP:

\* Why should Local oscillator freq. should always larger than signal freq.?



$$\frac{1}{2\pi\sqrt{(L_1+L_2)C}}$$

$$\textcircled{1} f_L > f_S$$

$$1050 \text{ k} = \frac{1}{2\pi\sqrt{(L_1+L_2)C_{\text{max}}}}$$

$$2150 \text{ k} = \frac{1}{2\pi\sqrt{(L_1+L_2)C_{\text{min}}}}$$

$$\therefore \frac{C_{\text{max}}}{C_{\text{min}}} = \left(\frac{2150}{1050}\right)^2$$

$$\approx 4$$

$$\textcircled{2} f_L < f_S$$

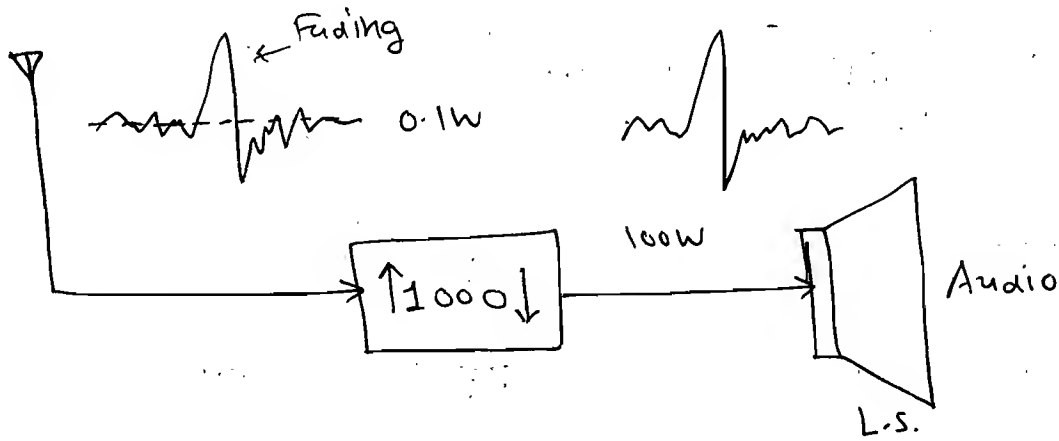
$$50 \text{ k} = \frac{1}{2\pi\sqrt{(L_1+L_2)C_{\text{max}}}}$$

$$1150 \text{ k} = \frac{1}{2\pi\sqrt{(L_1+L_2)C_{\text{min}}}}$$

$$\therefore \frac{C_{\text{max}}}{C_{\text{min}}} = \left(\frac{1150}{50}\right)^2 = 529$$

It is impossible to get such a large ratio.

## ★ AGC Circuit



⇒ The signal strength from the antenna in any wireless communication system is not constant is called fading.

⇒ Signal fading occurs due to various propagation losses. If overall gain of the receiver is constant the o/p signal strength also changes.

⇒ AGC circuit is used to maintain a constant audio o/p irrespective of the variations at the i/p of the receiver.

⇒ AGC circuit will control the overall gain of the receiver according to variation in the signal strength at the i/p.



# \* FM Receiver :

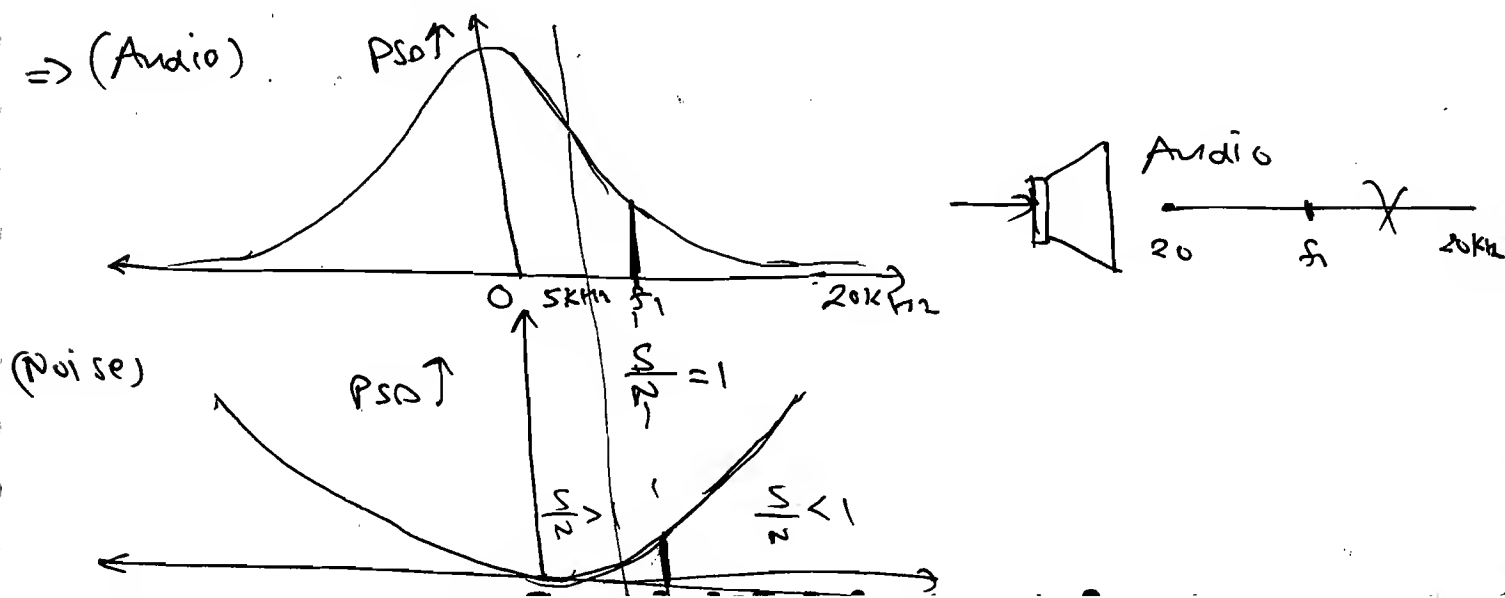
## \* International Standards:

- ①  $88 \text{ MHz} = 108 \text{ MHz} \leftarrow f_c$
- ①  $I_F = 10.7 \text{ MHz}$
- ①  $B.W. = 200 \text{ kHz} = 2\Delta f + 2f_m = 150\text{K} + 30\text{kHz} + \underline{\underline{20\text{kHz}}}$   
Guard Band
- ①  $\Delta f = 75 \text{ kHz}$
- ①  $f_m = 15 \text{ kHz}$
- ①  $\beta = 5$

## \* Preemphasis (Tx) and Deemphasis (Rx):

$\Rightarrow$  These Concept are used in FM Radio not in AM Radio because Am signal is Band limited to  $5 \text{ kHz}$  and upto  $5 \text{ kHz}$  signal power is very more than the noise power and hence,  $S/N \gg 1$ .

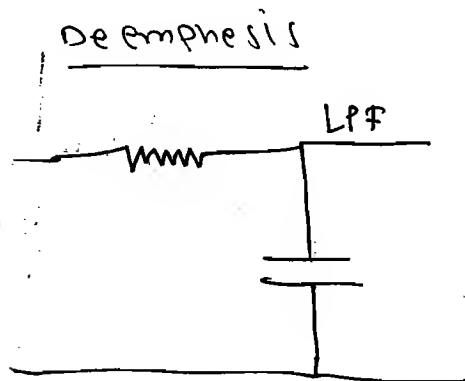
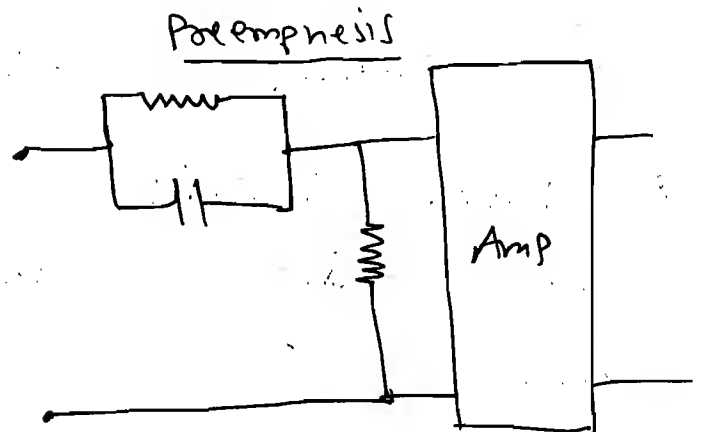
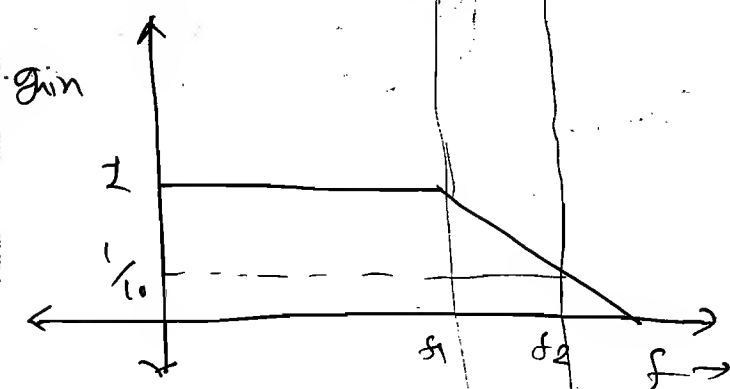
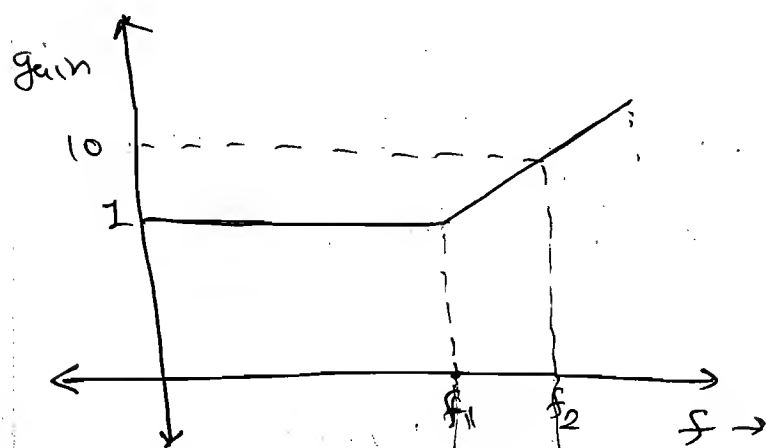
$\Rightarrow$  These techniques are used to improve the fidelity of an Audio signals.



→ In Audio Signal transmission  $\frac{S}{N} > 1$  at low frequencies and  $\frac{S}{N} < 1$  at high freq. So, high freq of the Audio signal will not be reproduced & signal Fidelity decreases.

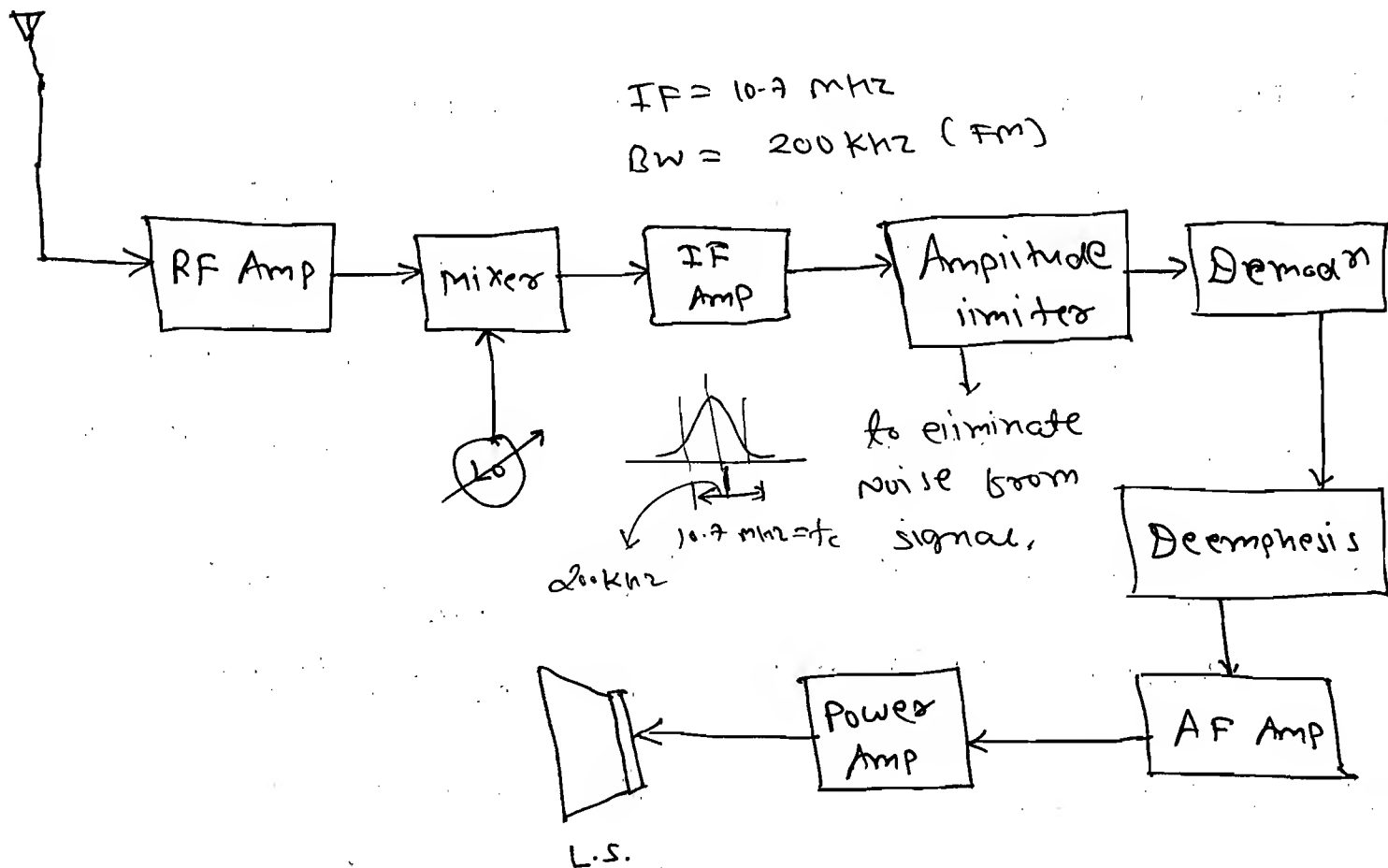
→ To improve the fidelity preemphasis technique is used at Tx before modulator.

⇒ "Preemphasis is defined as the process of boosting the high freqs of the Audio signal so that the signal to noise ratio is greater than '1' ( $S/N > 1$ ) at all Audio frequencies."



⇒ Whenever Preemphasis and deemphasis used practically in music system, it is called as Dolby Noise Reduction System and Dolby is one of the laboratory of USA.

\* Block Diagram of FM Radio:



⇒ FM demodulator is a freq. to Voltage Converter.

→ Frequency variation will be converted back into Voltage variation only if the envelope of the FM signal is constant.

→ Due to noise, Amplitude distortion occurs and it is not possible to convert freq. into Voltage. so Amplitude limiter is used to eliminate the noise or to maintain a constant

envelope.

→ In AM Amplitude limiter is not used as the information is in the form of amplitude.

\* Major differences in AM & FM:

AM

FM

①  $P_t = P_c + \frac{P_c \mu^2}{2}$

② AM requires more power.

③ Power varies with modulation index.

④  $\eta_{\max} = 33.33\%$   
(single tone modulation)

⑤  $BW = 2f_m$

⑥ BW is very low.

⑦ BW is independent of modulation index.

⑧ AM receiver is less complex.

⑨ The effect of noise is more.

①  $P_t = P_c$

② FM requires less power.

③ Power is independent of modulation index.

④  $\eta_{\max} = 100\%$ ;  
 $\beta = 2.4, 5.5, 8.6, \dots$

⑤  $2(\beta + 1)f_m$

⑥ BW is ~~more~~ very high.

⑦ B.W. varies with modulation index.

⑧ FM receiver is more complex.

⑨ The effect of noise is very less.

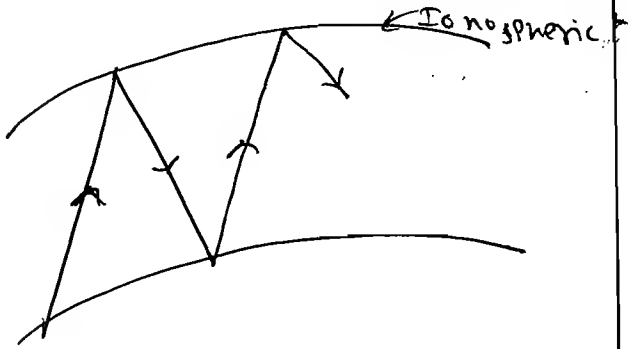
⑩  $550\text{K} - 1650\text{ KHz}$

$IF = 455\text{ KHz}$

$B.W. = 10\text{ KHz}$

$\mu = 1$

⑪ Ionospheric Propagation



⑫  $\rightarrow$  Area coverage is more

⑬ Freq. reuse is not possible.

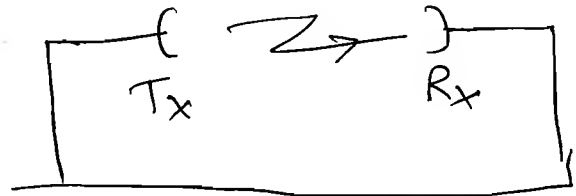
⑩  $88\text{ M} - 108\text{ MHz}$

$IF = 10.7\text{ MHz}$

$B.W. = 200\text{ KHz}$

$\beta = 5$

⑪ Line of sight Propagation (LOS)



$\rightarrow$  microwave freq. 'G' Hz

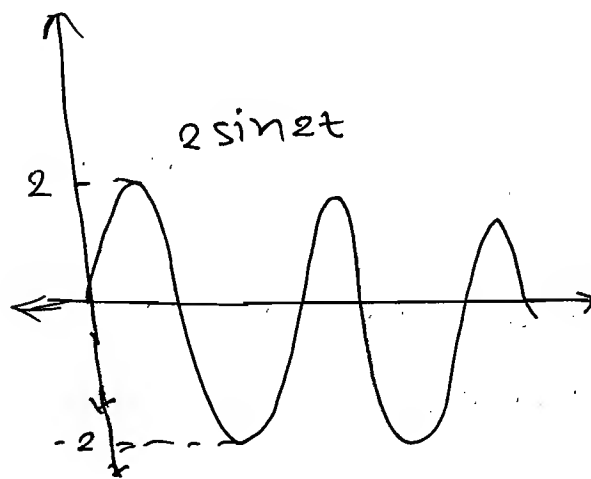
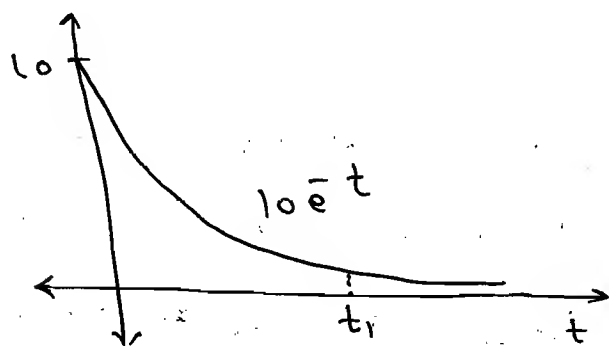
⑫ Area covered by antenna is restricted due to LOS.

⑬ Freq. reuse is possible.

# ★ Random Signal Theory

⇒ The i/p to the Communication receiver is signal & noise. The signal is completely deterministic but the noise voltage changes randomly w.r.t. time. So Probability concepts are used to analyze a random signal.

⇒ Deterministic signals:



→ At any instant of time it can be determine.

⇒ Random signal:



Impossible to represent Random ~~signal~~ Variations mathematically.

# ★ ~~Random~~ ~~Staged~~ Theory:

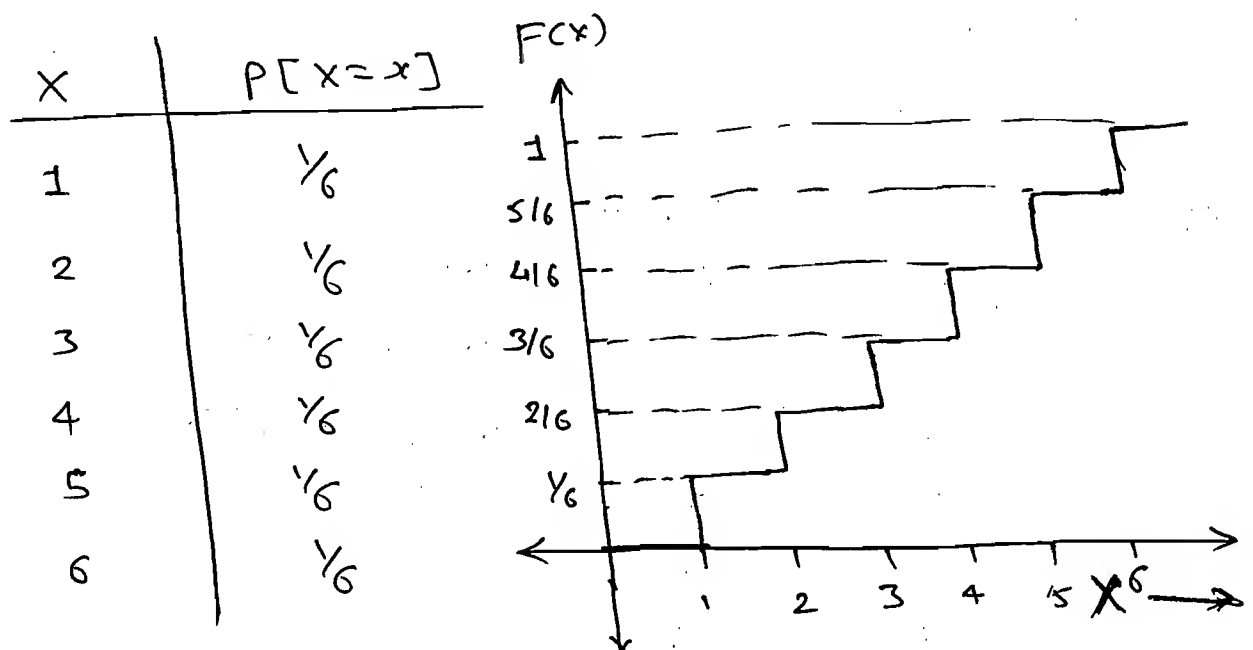
\* Probability Distribution Function (PDF)  
(or) Cummulative Distribution Function (CDF).

⇒ PDF mathematically defined as,

$$F_x(x_i) = P[X \leq x_i]$$

$$F_x(5) = P[X \leq 5]$$

Dice:



$$F(0.99) = P[X \leq 0.99] = 0$$

$$F(1) = P[X \leq 1] = \frac{1}{6}$$

$$F(1.8) = P[X \leq 1.8] = \frac{1}{6}$$

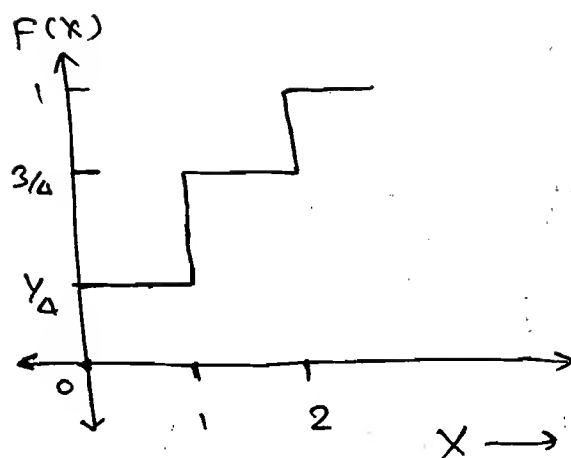
$$F(2) = P[X \leq 2] = \frac{2}{6}$$

Q A coin is tossed twice and the Random Variable  $X$  represent the number of Head.  
Sketch the PDF.

Sol<sup>n</sup>:

		X
T	T	0
T	H	1
H	T	1
H	H	2

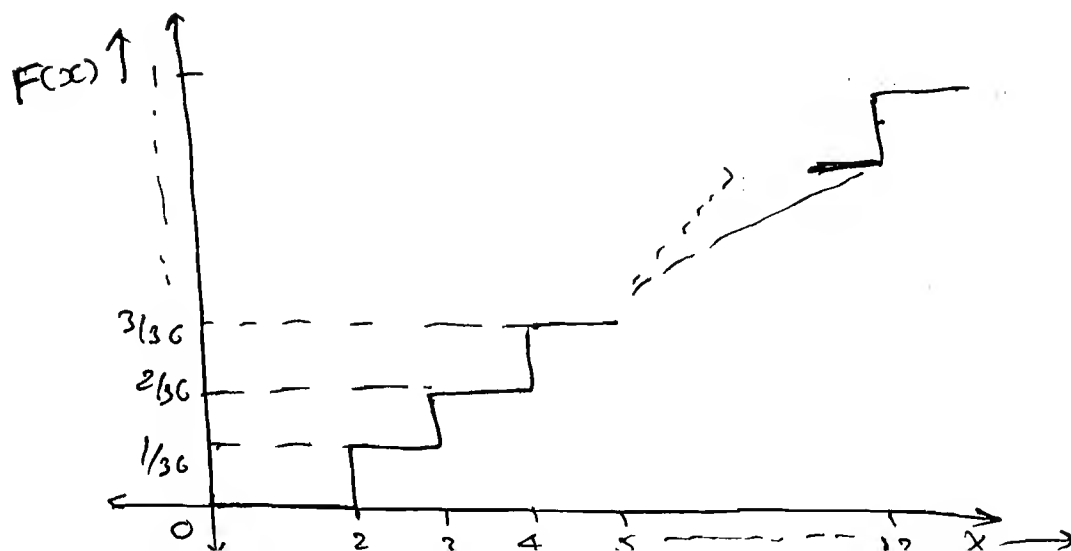
X	$P[X=x]$
0	$\frac{1}{4}$
1	$\frac{1}{2}$
2	$\frac{1}{4}$
	<hr/>
	1



Q Two dice are thrown simultaneously and Random Variable  $X$  represent the sum of the two outcomes. sketch the probability Dis. f<sup>n</sup>?

Ans:

X	$P[X=x]$	
2	$\frac{1}{36}$	(1,1)
3	$\frac{2}{36}$	(1,2), (2,1)
4	$\frac{3}{36}$	(1,3), (3,1), (2,2)
5	$\frac{4}{36}$	
6	$\frac{5}{36}$	
7		
8		
9		
10		
11		
12	$\frac{1}{36}$	(6,6)





## \* Properties of PDF:

①  $0 \leq F(x) \leq 1$

②  $F(x)$  is a increasing function of  $x$ .

③  $F(\infty) = 1$  &  $F(-\infty) = 0$ .

$$P[X \leq \infty] = 1 \quad \& \quad P[X \leq -\infty] = 0.$$

④  $P[X > x_1] = 1 - P[X \leq x_1] = 1 - F[x_1]$ .

⑤  $P[x_1 < X \leq x_2] = F[x_2] - F[x_1]$

e.g.  $P[3 < X \leq 5] = F[5] - F[3]$   
 $= 5/6 - 3/6$   
 $= 2/6 = 1/3.$

⑥  $P[X = x_1] = F[x_1] - F[\underbrace{x_1^-}_{\text{Small decremental value}}]$

e.g.  $P[X = 2] = F[2] - F[2^-]$   
 $= F[2] - F[1.99]$   
 $= 2/6 - 1/6$   
 $= 1/6.$

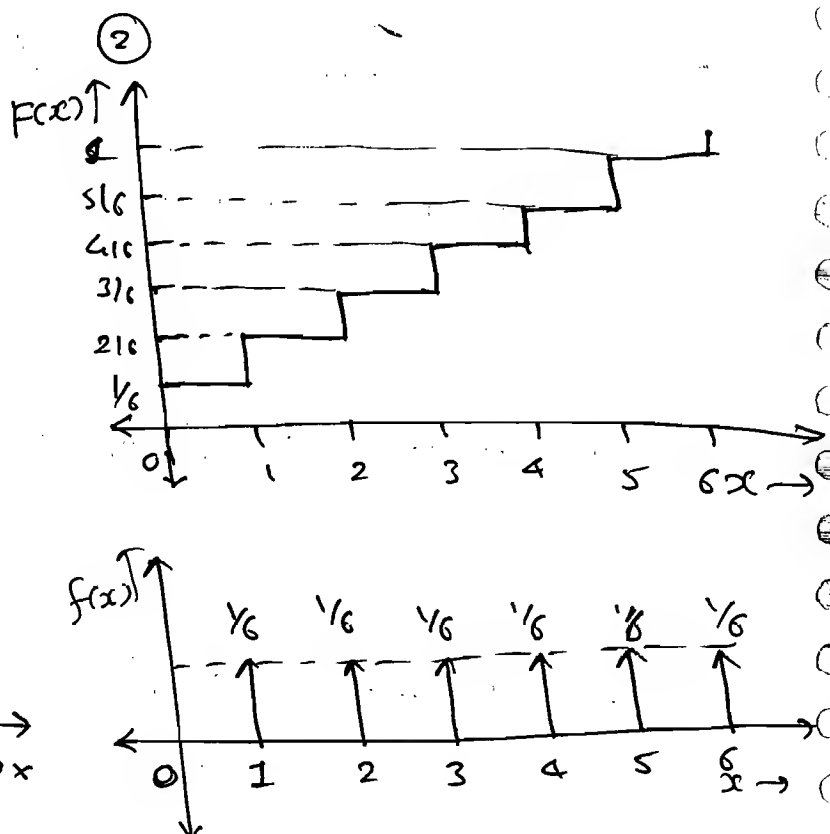
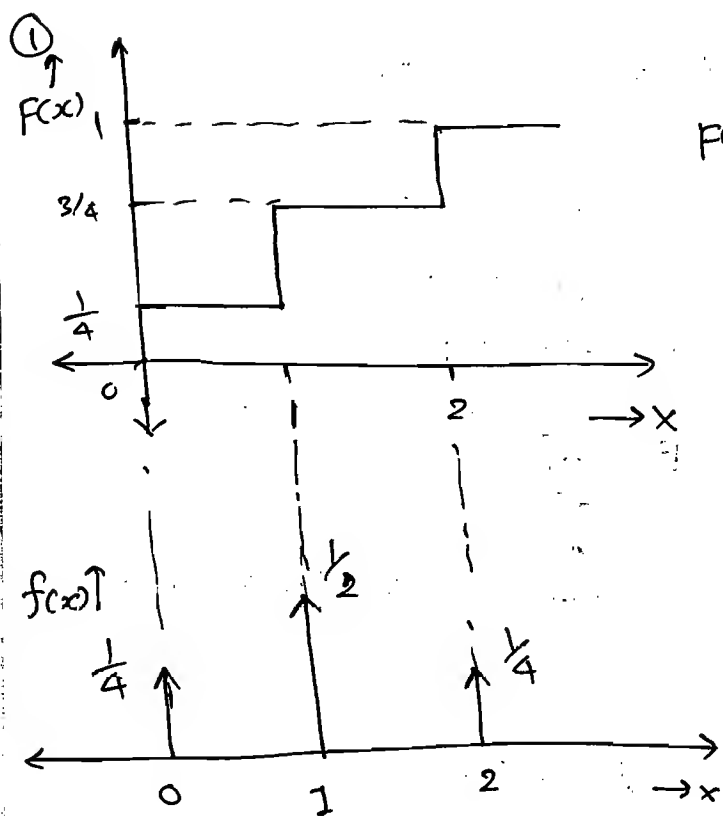
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$$\begin{aligned} P[X = 3.4] &= F[3.4] - F[3.4^-] \\ &= F[3.4] - F[3.399] \\ &= 3/6 - 3/6 \\ &= 0. \end{aligned}$$

# \* Probability density function (pdf)

→ pdf indicates the distribution of lower probability to various random variables.

$$f(x) = \frac{d}{dx} F(x).$$



⌊

## \* Properties of pdf:

①  $\int_{-\infty}^{\infty} f(x) \cdot dx = 1$  or Total area = 1 ⌊ Imp.

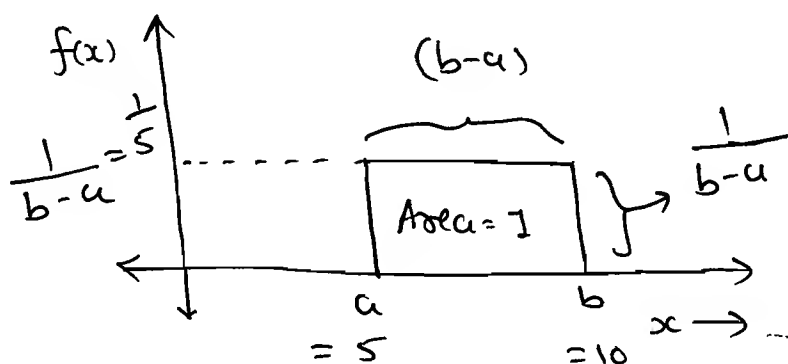
②  $P[X \leq x_1] = F(x_1) = \int_{-\infty}^{x_1} f(x) \cdot dx$  ⌊

③  $P[X > x_1] = 1 - F(x_1) = \int_{x_1}^{\infty} f(x) \cdot dx.$

$$(4) \quad P[x_1 < x \leq x_2] = \int_{x_1}^{x_2} f(x) dx.$$

### ① Uniform Density Function:

$\Rightarrow$  If pdf is constant within specified range, then the Random Variable is said to be uniformly distributed.

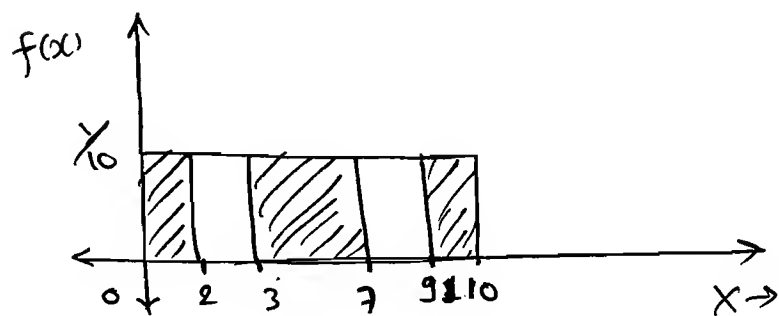


$$\begin{aligned} \text{Area} \\ A &= (b-a) \times \frac{1}{(b-a)} \\ &= 1 \end{aligned}$$

Q A Continuous R.V. is uniformly distributed in the  $[0, 10]$  ① Sketch the pdf & determine the following probability.

- i)  $P[x \leq 2]$
- ii)  $P[x > 9]$
- iii)  $P[3 < x \leq 7]$ .

② Sketch the PDF.



$$\text{① } P[x \leq 2] = \int_{-\infty}^2 f(x) dx = \frac{1}{10} \times 2 = 0.2$$

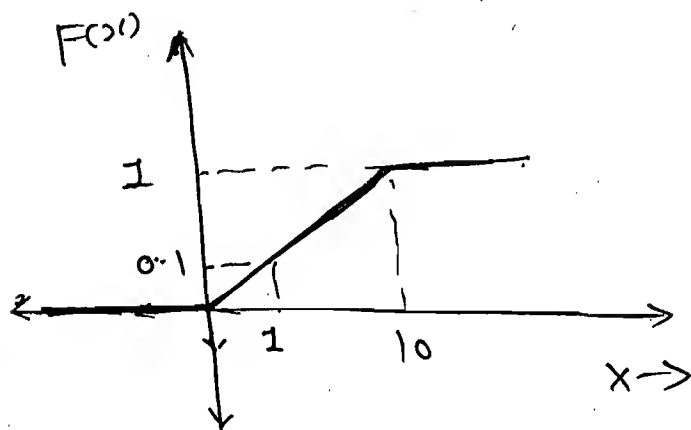
$$(ii) \quad P[X > 9] = \int_9^{\infty} f(x) \cdot dx = (10-9) \times \frac{1}{10} = 0.1.$$

$$(iii) \quad P[3 < X \leq 7] = \int_3^7 f(x) \cdot dx = \frac{1}{10} \times [7-3] = 0.4.$$

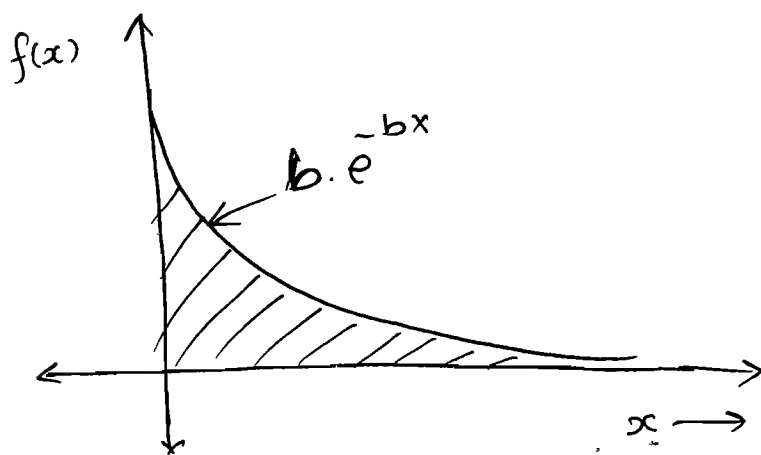
②  $F(x) = \int_{-\infty}^x f(x) \cdot dx$

$$= \int_0^x \frac{1}{10} \cdot dx.$$

$$F(x) = \frac{x}{10}.$$



## ② Exponential Density Function:



$$f(x) = b \cdot e^{-bx}, \quad x \geq 0$$

$$= 0, \quad x < 0.$$

$$\int_0^{\infty} a \cdot e^{-bx} \cdot dx = 1.$$

$$\therefore \frac{a}{b} \cdot [0 - e^{-bx}]_0^{\infty} = 1.$$

$$\Rightarrow \frac{a}{b} = 1 \Rightarrow \boxed{a=b} \text{ (Cond}^n \text{ for valid Pdf.)}$$

\* Identity Valid Pdf:

$$\rightarrow f(x) = 10e^{-10x}, x \geq 0 \quad \checkmark \quad [\because a=b] \quad (\text{valid})$$

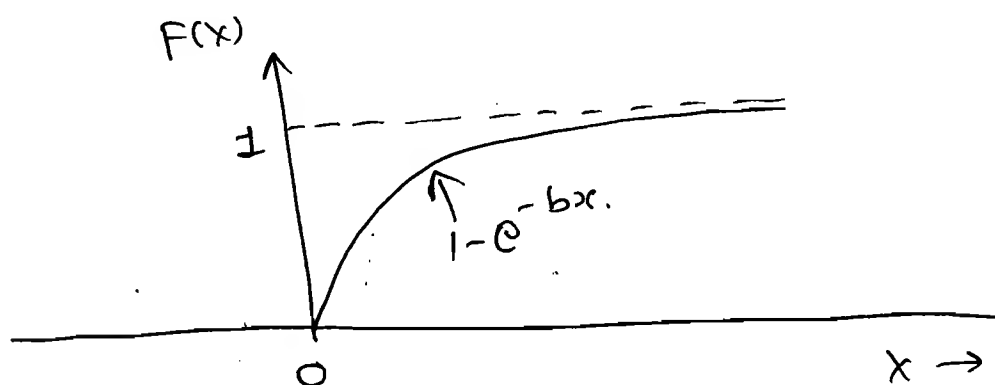
$$\rightarrow f(x) = 5e^{-10x}, x \geq 0 \quad \times \quad (\because a \neq b \Rightarrow 10 \neq 5) \quad (\text{Not valid})$$

$\Rightarrow$

pdf



PDF



$$\begin{aligned} \Rightarrow F(x) &= \int_{-\infty}^x f(x) \cdot dx \\ &= \int_0^x be^{-bx} \cdot dx \\ &= -\frac{b}{b} [e^{-bx}]_0^x \end{aligned}$$

$$F(x) = [1 - e^{-bx}]$$

### ③ Laplacian Density Function:

$$\rightarrow f(x) = a \cdot e^{-b|x|}, \quad -\infty < x < \infty$$

$$= a \cdot e^{-bx}, \quad x > 0$$

$$= a \cdot e^{bx}, \quad x < 0.$$

Cond<sup>n</sup> for

valid pdf:

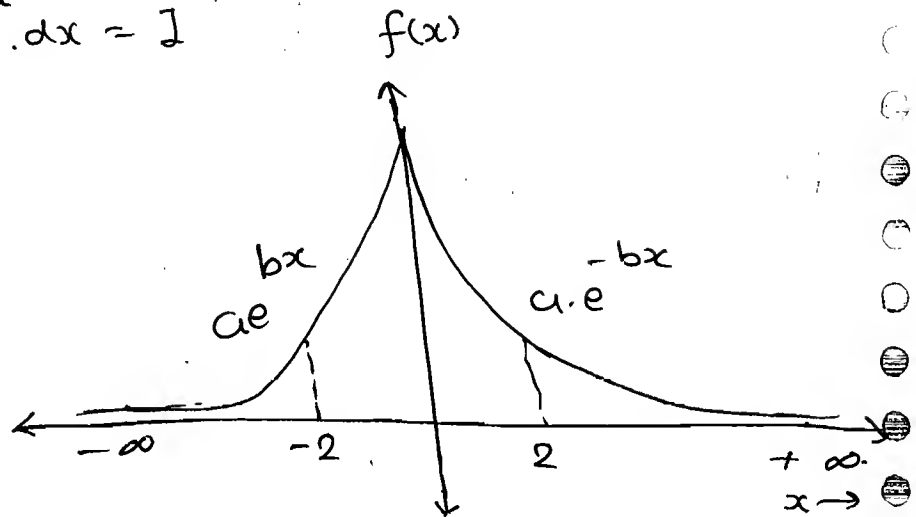
$$\Rightarrow \int_{-\infty}^{\infty} f(x) = 1$$

$$\int_{-\infty}^0 e^{-bx} \cdot dx + \int_0^{\infty} e^{bx} \cdot dx = 1$$

$$\therefore \frac{a}{b} + \frac{a}{b} = 1$$

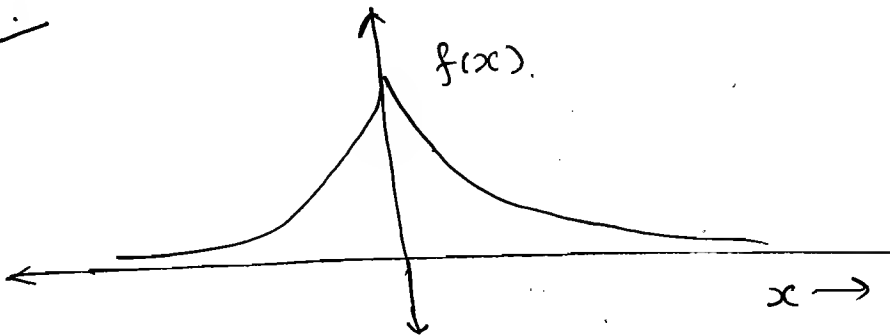
$$\frac{2a}{b} = 1.$$

$$\Rightarrow \boxed{a = b/2}$$

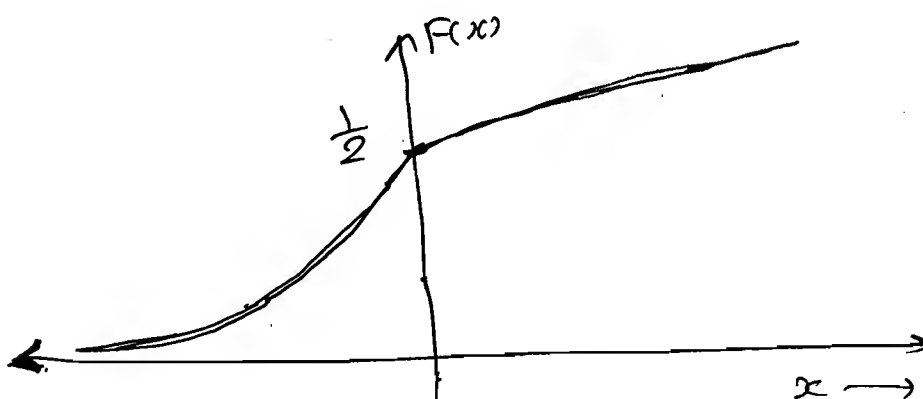


☑ Sketch  $F(x)$  from  $f(x)$ , given.

Crte ✓



Soln:



# \* Statistical Averages of a Random Variable.

## ① Mean (or) Average Value:

$$\rightarrow \bar{x} = E[x] = m_1 = \int_{-\infty}^{\infty} x \cdot f(x) dx \rightarrow \text{dc Component only.}$$

## ② Mean Square Value:

$$\rightarrow \overline{x^2} = E[x^2] = m_2 = \int_{-\infty}^{\infty} x^2 \cdot f(x) dx \rightarrow \text{Total Power.}$$

## ③ Variance:

$$\begin{aligned} \rightarrow \sigma^2 &= E[(x - \bar{x})^2] = m_2 - m_1^2 = \text{ac Power} \\ &= (\text{mean square} - \text{square of mean}). \end{aligned}$$

## ④ Standard Deviation:

$$\begin{aligned} \rightarrow \text{SD} &= \sqrt{\text{Variance}} = \sigma = \sqrt{m_2 - m_1^2} \\ &= \sqrt{\text{ac power}} \\ &= \text{rms value of ac Component.} \end{aligned}$$

## \* Properties of Mean:

①  $E[K] = K$ ,  $K = \text{constant}$

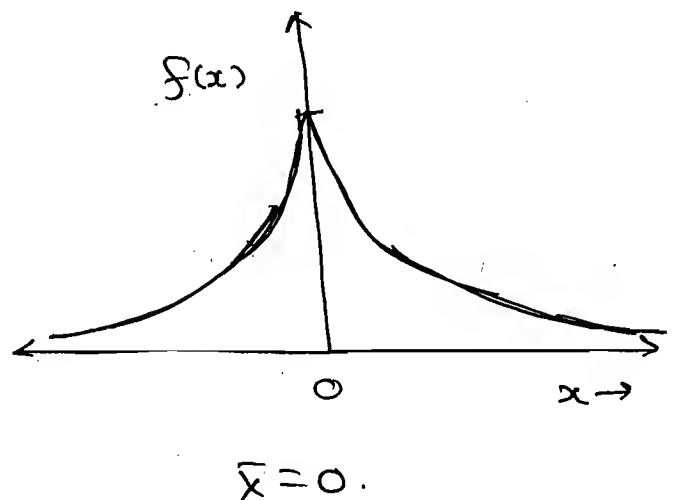
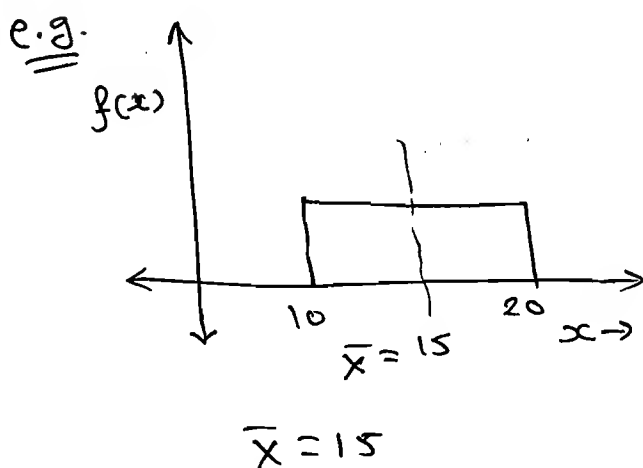
②  $E[Kx] = K \cdot \bar{x}$

③  $E[x+K] = E[x] + E[K] = \bar{x} + K$

④  $E[x+y] = E[x] + E[y]$

⑤  $E[xy] = E[x] \cdot E[y]$  provided that  $x$  &  $y$  are independent R.V.

⑥ If the Probability density function is symmetrical w.r.t. a  $x$ -axis then the average value (mean) is equal to the symmetry point.



Q-1 Determine the statistical avg. of  $\exp. b^n$ .

Ans:  $f(x) = b \cdot e^{-bx}$



① Mean

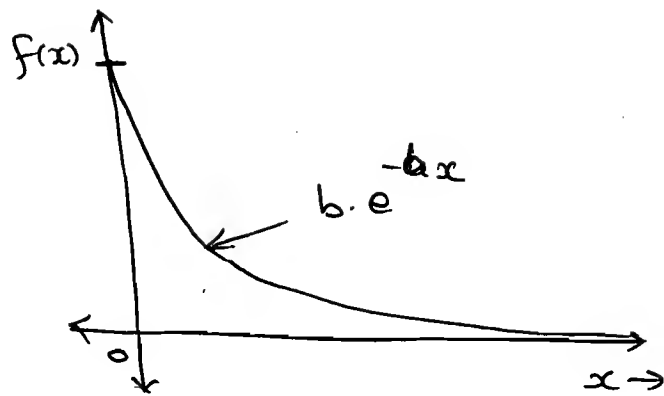
$$\bar{X} = E[X] = \int_0^{\infty} x \cdot f(x) \cdot dx$$

$$= \int_0^{\infty} x \cdot b \cdot e^{-bx} \cdot dx$$

$$= b \cdot \left[ (x) \left( \frac{e^{-bx}}{-b} \right) - (1) \cdot \left( \frac{e^{-bx}}{b^2} \right) \right]_0^{\infty}$$

$$= b \left[ 0 - 0 + \frac{1}{b^2} \right]$$

$$\boxed{\bar{X} = \frac{1}{b}} = m_1$$



② Mean square,

$$m_2 = \overline{X^2} = E[X^2] = \int_0^{\infty} x^2 \cdot b \cdot e^{-bx} \cdot dx$$

$$= b \left[ (x^2) \cdot \left( \frac{e^{-bx}}{-b} \right) - (2x) \left( \frac{e^{-bx}}{b^2} \right) + (2) \left( \frac{e^{-bx}}{b^3} \right) \right]_0^{\infty}$$

$$= b \left[ 0 + \frac{2}{b^3} \right]$$

$$\boxed{m_2 = \frac{2}{b^2}}$$

③ Variance =  $\sigma^2 = m_2 - m_1^2$

$$\sigma^2 = \frac{2}{b^2} - \frac{1}{b^2} = \frac{1}{b^2}$$

④ Standard deviation

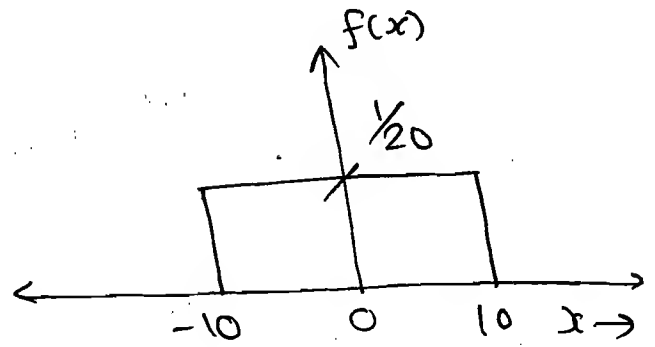
$$S.D. = \sqrt{m_2 - m_1^2}$$

$$\Rightarrow \boxed{S.D. = \frac{1}{b}}$$

**Q-2** A continuous R.V. is uniformly disth. in the  $[-10, 10]$ . Determine the statistical average.

Sol<sup>n</sup>:

①  $f(x) = \frac{1}{20}$



① Mean  
 $m_1 = \bar{x} = E[X] = \int_{-10}^{10} x \cdot \frac{1}{20} \cdot dx$

$m_1 = 0$

② Mean square,

$$m_2 = \bar{x^2} = E[x^2] = \int_{-10}^{10} x^2 \cdot \frac{1}{20} \cdot dx$$

$$m_2 = \frac{2}{20} \int_0^{10} x^2 \cdot dx = \frac{1}{10} \times \frac{1000}{3} = \frac{100}{3}$$

$m_2 = \frac{100}{3}$

③ Variance,  $\sigma^2 = (\sqrt{m_2 - m_1^2})^2$

$$\sigma^2 = m_2 - m_1^2$$

$$= \frac{100}{3} - 0$$

$\sigma^2 = \frac{100}{3}$

④ S.D. =  $\sqrt{\text{variance}}$

$S.D. = \frac{10}{\sqrt{3}}$

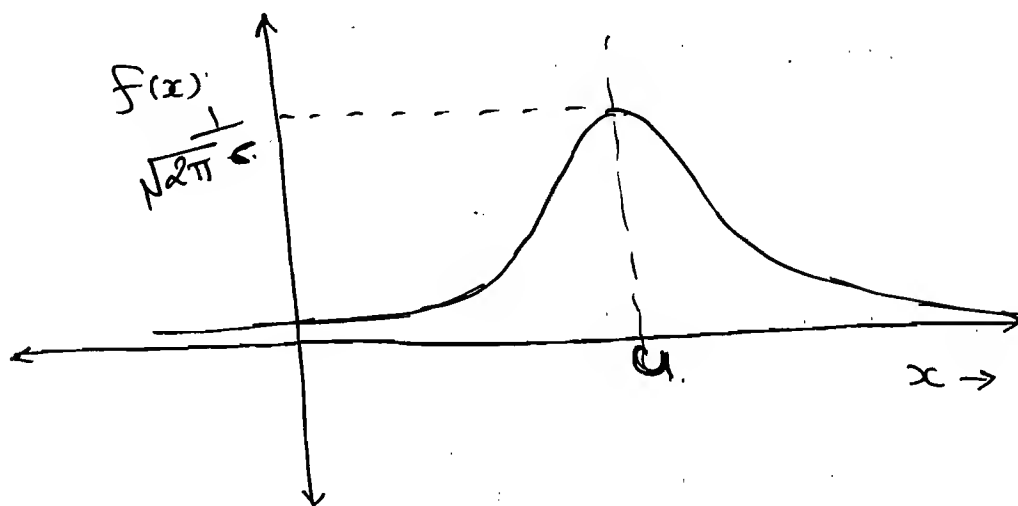
#### ④ Gaussian Density Function:

⇒ Pdf

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-a)^2}{2\sigma^2}}$$

$$-\infty < x < \infty$$

①



⇒ ① Mean:

$f(x)$  is symmetric w.r.t.  $x$  axis at  $x=a$ .

$$\text{So, mean} = m_1 = E[x] = a \Rightarrow \boxed{m_1 = a}$$

$$\textcircled{2} \quad m_2 = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx = \sigma^2 + a^2$$

$$\Rightarrow \boxed{m_2 = \sigma^2 + a^2}$$

$$\textcircled{3} \quad \text{Variance } \sigma^2 = m_2 - m_1^2$$

$$\text{Variance} = \cancel{\sigma^2 + a^2} - a^2 = \sigma^2$$

$$\therefore \boxed{\text{Variance} = \sigma^2}$$

④ S.D. =  $\sigma = \sqrt{\text{Variance}}$

$\boxed{\text{S.D.} = \sigma}$

Q find  $m_2$  of  $f(x) = \frac{1}{\sqrt{(\quad)}} \cdot e^{-\frac{(x-2)^2}{100}}$

Soln:

Here,  $a=2$ ,  $2\sigma^2=100$

$\sigma^2 = \frac{100}{2}$

$\boxed{\sigma^2 = 50}$

$\therefore m_2 = \sigma^2 + a^2$   
 $= 50 + 4$

$\boxed{m_2 = 54}$

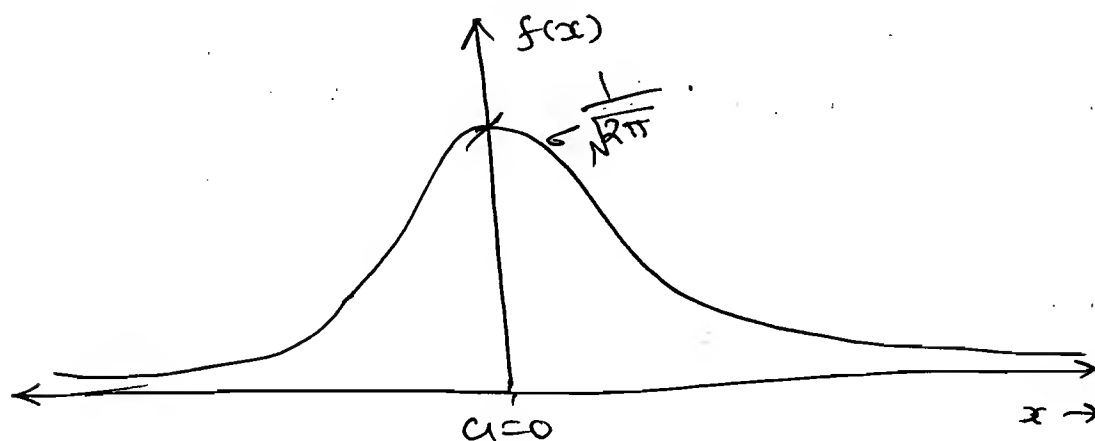
$\boxed{m_1 = a = 2}$

S.D. =  $\sqrt{\sigma^2}$

$\boxed{\text{S.D.} = 5\sqrt{2}}$

$\Rightarrow$  if  $a=0$ , then

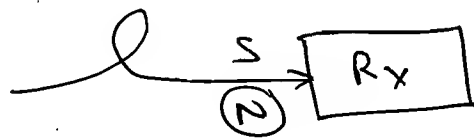
$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{x^2}{2\sigma^2}}$



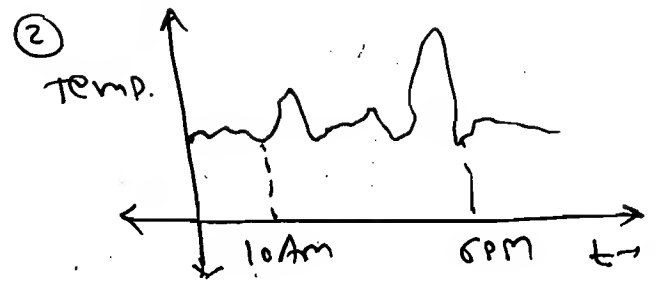
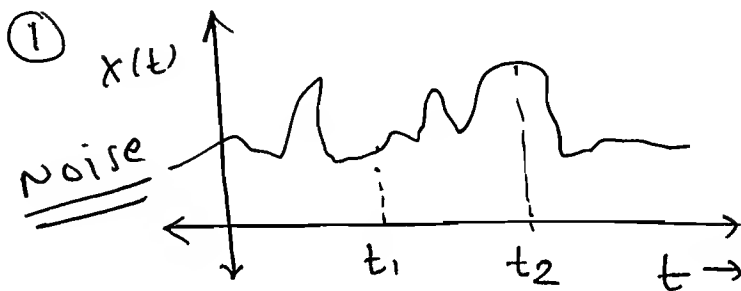
# ★ Random Process:

$\Rightarrow$  A random variable which is function of time is called as the Random Process.

$\Rightarrow$  Random Process  $= X(t)$ .



eg.



## \* PDF of a Random Process:

$$\rightarrow X \rightarrow F[x] = P[X \leq x_1]$$

$$\rightarrow X(t) \rightarrow F[x_1, t_1] = P[X(t) \leq x_1]$$

$\rightarrow$  1<sup>st</sup> order PDF.

$$F[x, t_1] = P[X(t_1) \leq 2\text{mV}]$$

$$= P[X(2\text{PM}) \leq 25^\circ\text{C}]$$

$$= P[X(2\text{AM}) \leq 25^\circ\text{C}]$$

$$\rightarrow F[x_1, x_2, t_1, t_2] = P[X(t_1) \leq x_1 \& X(t_2) \leq x_2]$$

$$\rightarrow 2^{\text{nd}} \text{ order PDF.}$$

\* Pdf of a Random Process:

$$\rightarrow f(x) = \frac{d}{dx} F(x).$$

$$\rightarrow f(x, t_1) = \frac{\partial}{\partial x} F(x, t_1) \rightarrow 1^{st} \text{ order.}$$

$$\rightarrow f(x_1, x_2, t_1, t_2) = \frac{\partial^2}{\partial x_1 \partial x_2} F(x_1, x_2, t_1, t_2) \rightarrow 2^{nd} \text{ order.}$$

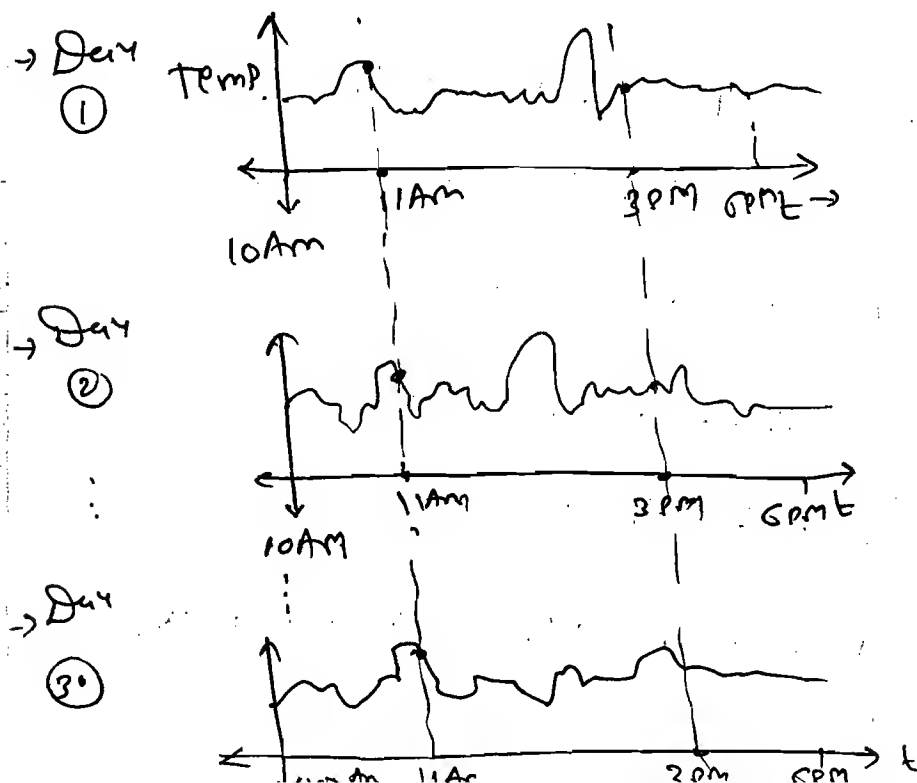
\* Statistical averages of a Random Process:

$\Rightarrow$  ① Ensemble averages.

② Time averages.

$\Rightarrow$  Ensemble means collection of data.

① Ensemble averages:



① Mean,

$$m_1(t) = E[x(t)] = \int_{-\infty}^{\infty} x(t) \cdot f(xt) \cdot \underline{\underline{dx}}$$

② Mean Square,

$$m_2(t) = E[x^2(t)] = \int_{-\infty}^{\infty} x^2(t) \cdot f(xt) \cdot \underline{\underline{dx}}$$

③ Variance,

$$= m_2(t) - m_1^2(t).$$

④ Standard Deviation,

$$S.D. = \sqrt{\text{Variance}}$$

$$S.D. = \sqrt{m_2(t) - m_1^2(t)}.$$

⑤ Auto Correlation Function: ✓

$$ACF = E[x(t_1) \cdot x(t_2)].$$

→ ACF is used to determine the similarity of a Random Process at two instant of time,

$$\boxed{ACF = E[x(t) \cdot x(t+\tau)]}$$

$$\begin{aligned} t_1 &\rightarrow t \\ t_2 &\rightarrow t+\tau \end{aligned}$$

## ② Time averages:

⇒ ① Mean,

$$\langle x(t) \rangle = m_1 = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} x(t) \underline{\underline{dt}}$$

② Mean Square,

$$\langle x^2(t) \rangle = m_2 = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} x^2(t) \underline{\underline{dt}}$$

③ Variance,  $= m_2 - m_1^2$

④ S.D.  $= \sqrt{m_2 - m_1^2}$

⑤ ACF  $= \langle x(t) \cdot x(t+\tau) \rangle$

$t_1 \rightarrow t$   
 $t_2 \rightarrow t+\tau$

$$ACF = \frac{1}{\tau} \int_t^{t+\tau} x(t) \cdot x(t+\tau) dt$$

⇒ If the Ensemble args and Time args are equal then the Random process is called Ergodic Random process.

⇒ A random process is said to be stationary of order one if the following condition is satisfied,



$$\Rightarrow f(x, t) = f(x, t + \Delta t).$$

↑ Stationary of order one.

$$\Rightarrow f(x_1, x_2, t_1, t_2) = f(x_1, x_2, t_1 + \Delta t, t_2 + \Delta t).$$

↑ Stationary of order two.

\* Wide Sense Stationary:

⇒ A random process is said to be wide sense stationary if the following two conditions are satisfied:

- ①  $E[X(t)] = \text{constant}$
- ② The ACF should depend only on  $t_2 - t_1$ , i.e.  $\tau$ .

Q Consider a Random Process

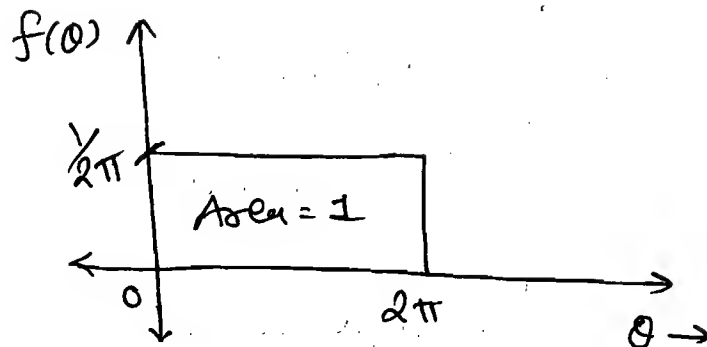
$X(t) = A \cos[\omega_0 t + \theta]$ , where  $A$  &  $\omega_0$  are constant and  $\theta$  is a R.V. uniformly distributed in  $[0, 2\pi]$ . Determine the mean & ACF. and show that Random Process is WSS.

Ans:

$$X(t) = A \cos[\omega_0 t + \theta].$$

$$\Rightarrow f(\theta) = \frac{1}{2\pi}$$

$$\rightarrow E[X] = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$$



$$E[X(\theta)] = \int_0^{2\pi} X(\theta) \cdot f(\theta) \cdot d\theta$$

$$= \int_0^{2\pi} A \cos[\omega_0 t + \theta] \cdot \frac{1}{2\pi} \cdot d\theta$$

$$\Rightarrow \boxed{E[X(t)] = 0} \leftarrow \text{Ensemble mean.}$$

$$\Rightarrow ACF = E[X(t_1) \cdot X(t_2)]$$

$$= E[A \cos[\omega_0 t_1 + \theta] \cdot A \cos[\omega_0 t_2 + \theta]]$$

$$= E\left[ \frac{A^2}{2} \cos[\omega_0 (t_1 + t_2) + 2\theta] + \frac{A^2}{2} \cos[\omega_0 (t_1 - t_2)] \right]$$

$$= E\left[ \frac{A^2}{2} \cdot \cos[\omega_0 (t_2 + t_1) + 2\theta] \right]$$

$$+ E\left[ \frac{A^2}{2} \cdot \cos[\omega_0 (t_2 - t_1)] \right]$$

$$= 0 + \frac{A^2}{2} \cdot \cos[\omega_0 (t_2 - t_1)]$$

$\Rightarrow$  which is independent of R.V. i.e here  $\theta$ .

So, ①  $E[x(t)] = 0 = \text{Constant}$ .

② ACF is depends on  $(t_2 - t_1)$ .

So, Random Process in WSS.

$$\text{ACF} = \frac{A^2}{2} \cos \omega_0 \tau$$

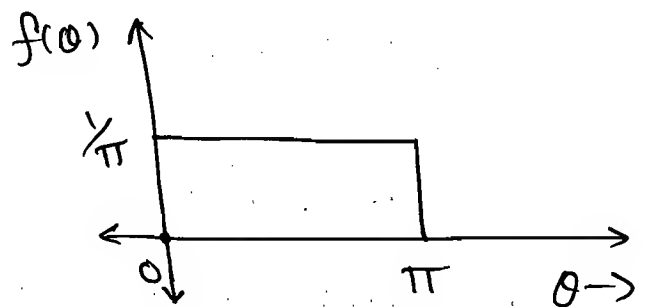
$$\boxed{\text{ACF} = \frac{A^2}{2} \cos 2\pi f_0 \tau = R(\tau)}$$

Q-2 Consider a Random Process

$x(t) = A \cos[\omega_0 t + \theta]$ ,  $A$  &  $\omega_0$  are constants and  $\theta$  is a R.V. uniformly distributed in the  $[0, \pi]$ . Check whether the R.P. is WSS or not?

Sol<sup>n</sup>:  $x(t) = A \cos[\omega_0 t + \theta]$ .

$$\rightarrow f(\theta) = \frac{1}{\pi}$$



① Ensemble mean,

$$E[x(t)] = \int_0^{\pi} A \cos[\omega_0 t + \theta] \cdot \frac{1}{\pi} d\theta.$$

$$= \int_0^{\pi/2} A \cos(\omega_0 t + \theta) d\theta - \int_{\pi/2}^{\pi} A \cos(\omega_0 t + \theta) d\theta.$$

$$= A \left[ \sin(\omega_0 t + \theta) \right]_0^{\pi/2} - A \left[ \sin(\omega_0 t + \theta) \right]_{\pi/2}^{\pi}$$

$$= A \left[ \cos \omega_0 t - \cancel{\sin \omega_0 t} + \cancel{\sin \omega_0 t} + \cos \omega_0 t \right]$$

$$= 2A \cos \omega_0 t$$

$$\boxed{ACF = \frac{A^2}{2} \cos 2\pi f_0 z}$$

$$\Rightarrow E[X(t)] = 2A \cos \omega_0 t$$

which is not constant.

So, Random Process is not WSS.

\* Properties of ACF:

$$\Rightarrow ACF = R(\tau) = E[X(t) \cdot X(t+\tau)]$$

$$\textcircled{1} R(0) = E[X^2(t)]$$

$$\Rightarrow ACF = R(0) = \text{Mean square} = \text{Power.}$$

$\textcircled{2}$  The Fourier Transform of ACF is nothing but the Power spectral Density (PSD).

$$ACF \xleftrightarrow{FT} PSD \quad (\text{W/Hz})$$

$$\Rightarrow \boxed{R(\tau) \xleftrightarrow{FT} S(f)}$$

$\Rightarrow$  PSD indicates how the total Power is distributed to various freq.

⇒

$$S(f) = \int_{-\infty}^{\infty} R(\tau) \cdot e^{-j2\pi f\tau} \cdot d\tau$$


---


$$R(\tau) = \int_{-\infty}^{\infty} \underline{S(f)} \cdot e^{j2\pi f\tau} \cdot \underline{df}$$

✓

← H.B.

$$\omega = 2\pi f$$

$$f = \omega / 2\pi$$

Imp

✓

$$R(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\underline{\omega}) \cdot e^{j2\pi f\tau} \cdot d\underline{\omega}$$

\*

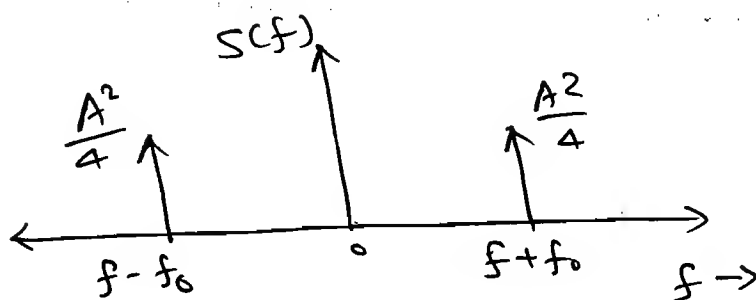
$$x(t) = A \cos(\omega_0 t + \theta)$$

$$\Rightarrow \& ACF = R(\tau) = \frac{A^2}{2} \cos 2\pi f_0 \tau$$

$$\rightarrow S(f) = \frac{A^2}{4} [\delta(f - f_0) + \delta(f + f_0)]$$

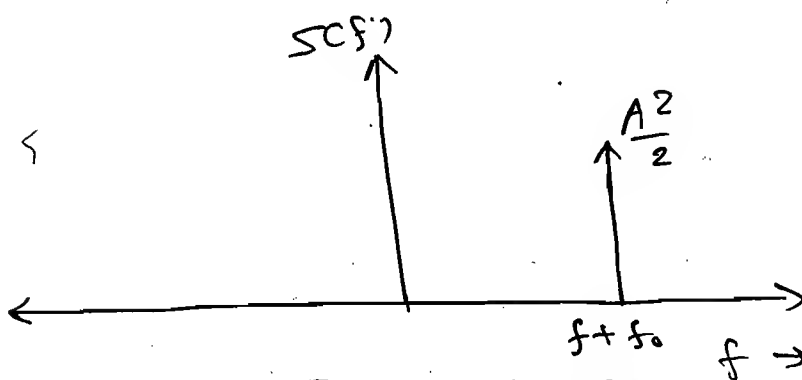
✓ PSD

⇒



(a) Two Sided PSD.

⇒



(b) Single Sided PSD.

$$\Rightarrow \textcircled{3} \quad R(0) = \int_{-\infty}^{\infty} S(f) \cdot df = \text{Power}$$

→ Area under the PSD should give power.

→ Noise Power = Mean square

$$= \int x^2 \cdot f(x) dx.$$

$$= R(0).$$

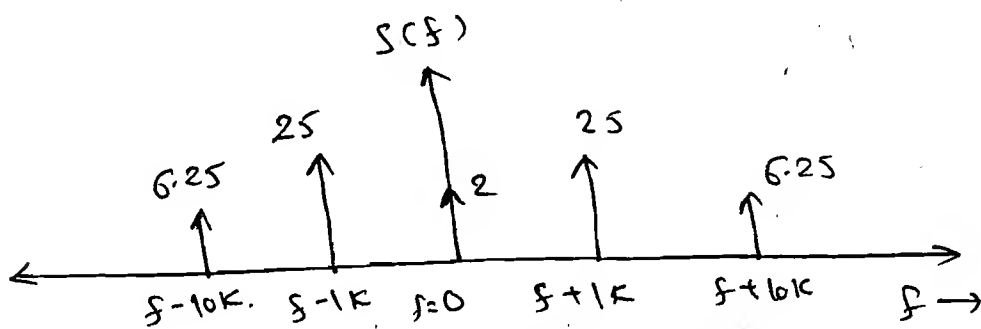
$$= \int (PSD) \underline{df}$$

$$= \frac{1}{2\pi} \int (PSD) \underline{d\omega}.$$

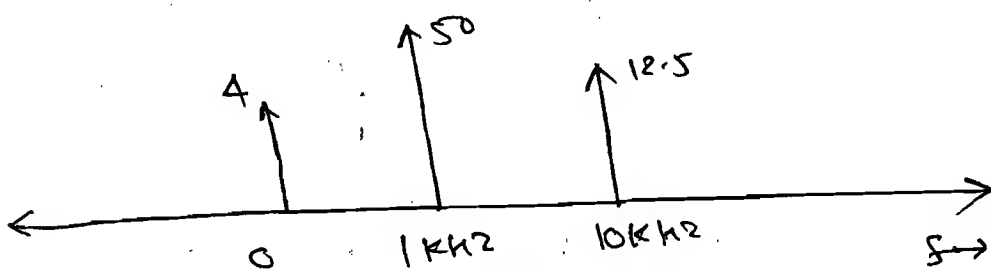
$$= \text{Area under PSD.}$$

$$* \quad 2 + 10 \cos 2\pi 3t + 5 \cos 2\pi 10^4 t$$

DC.  
 $f = 0 \text{ Hz}$

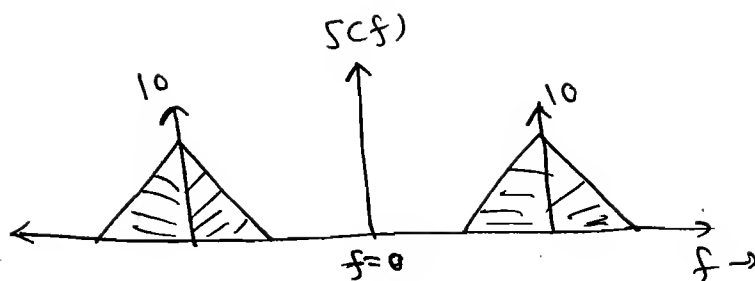


∴ (Two sided PSD)



(Single sided PSD)

⊛ IMP:



Mean =  $\mathcal{D}c$  = impulse at  $f=0$ .  
, otherwise mean = 0.

## ☆ Noise:-

⊛ Generalized def<sup>n</sup>:

⇒ Any unwanted signal interfering with the required signal is called as noise.

⇒ Noise are classified into two types:

① External Noise.

② Internal Noise.

① External Noise:

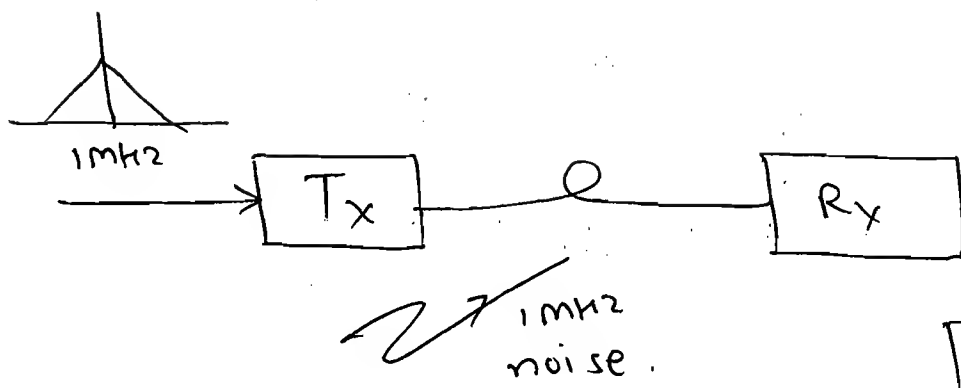
⇒ Noise which is generated outside the system is called as the External Noise.

e.g. ① Atmospheric noise

→ Sudden electrical disturbances.



⇒



② Solar noise

→ radiation from SUN.

③ Industrial Noise.

② Internal Noise:

⇒ The Noise which is generated within the system is called Internal Noise.

e.g.: Radiations from electrical appliances.

⇒ Whenever the noise is transmitted along with the signal intentionally to disturb the communication system then it is called signal Jamming.

for e.g. Mobile Phone Jammer.

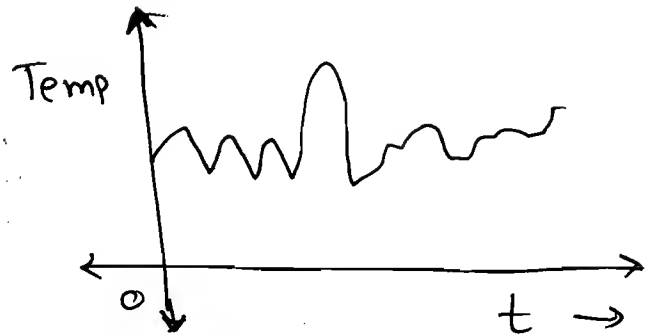
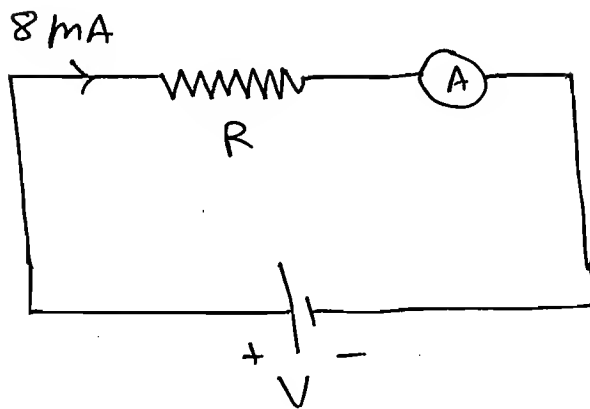
→ Transmit noise signal at signal freq. having noise power greater than signal power. to Jam the desired signal.



# ★ Internal Noise:

## ① Thermal Noise (or) White noise:

⇒



I ~~8mA~~

⇒ Thermal Noise is given by following empirical formula.

$$P = KT B \text{ watts.} \quad \leftarrow \text{H.B.}$$

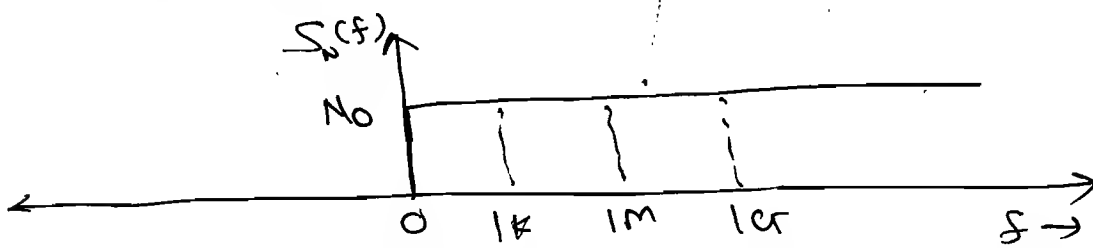
where,  $K = \text{Boltzman Constant}$   
 $= 1.38 \times 10^{-23} \text{ J/K.}$

$T = \text{Temp.}$

$B = \text{Bandwidth.}$

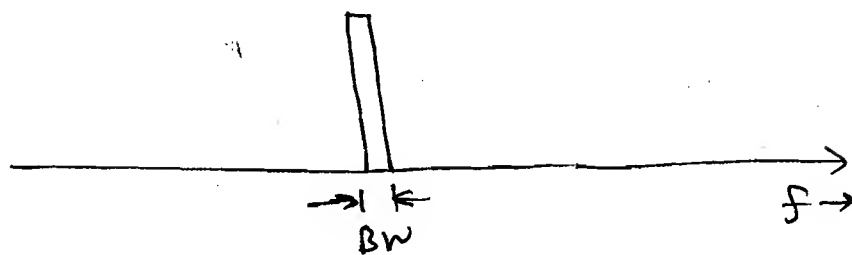
$$KT = \frac{\text{watts}}{\text{Hz}} = \text{PSD.} \quad \leftarrow \text{H.B.}$$

→  $KT = N_0 = \text{PSD of Thermal Noise.}$

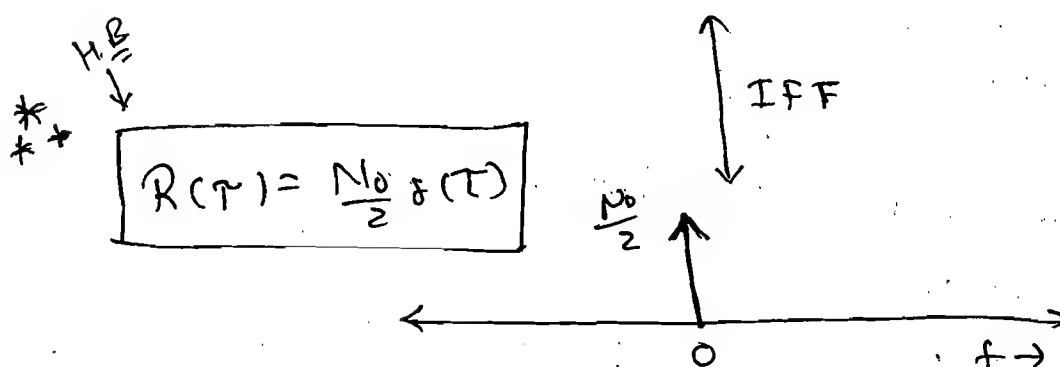
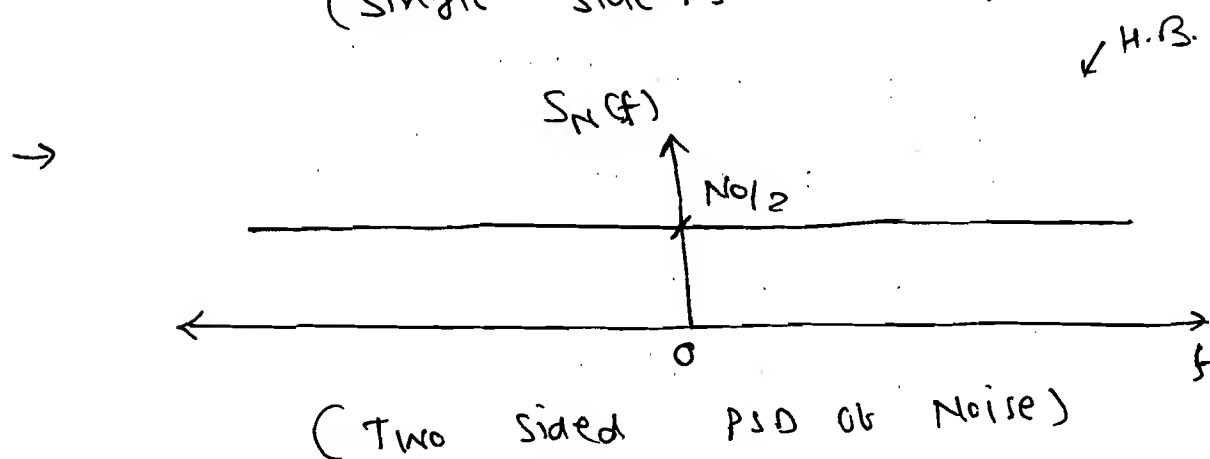
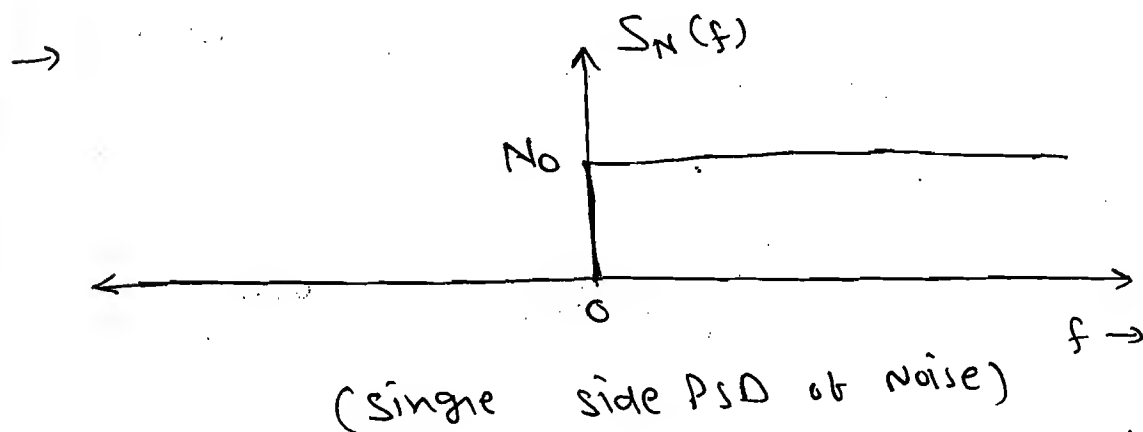


# \* Coloured Noise:

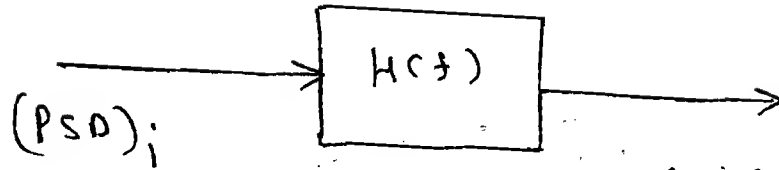
$\Rightarrow$  When noise affecting only narrow band of freqs then it is called Coloured noise.



$\Rightarrow$  Noise is affecting all freqs. equally.  
( $\because N_0$  is independent of freq.).



Most  
Imp  
topic

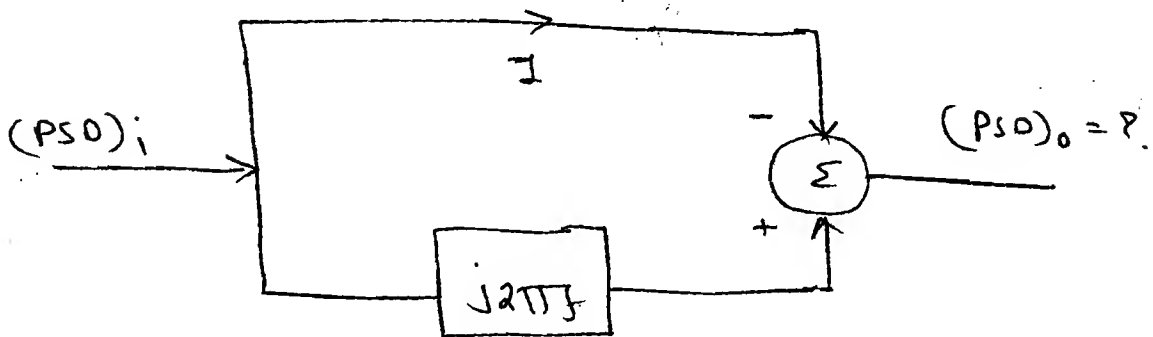


$$(PSD)_o = (PSD)_i \cdot |H(f)|^2$$

$$\Rightarrow \boxed{PSD_o = (PSD)_i \cdot |H(f)|^2} \leftarrow \text{H.B.}$$

Crute:

Q



Soln:

$$H(f) = -1 + j2\pi f$$

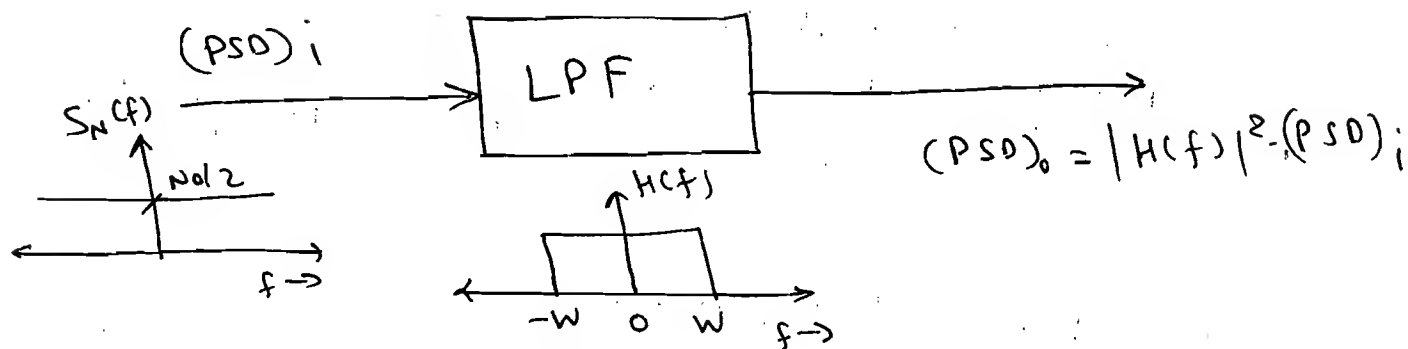
$$|H(f)| = \sqrt{1 + (2\pi f)^2}$$

$$|H(f)|^2 = 1 + (2\pi f)^2$$

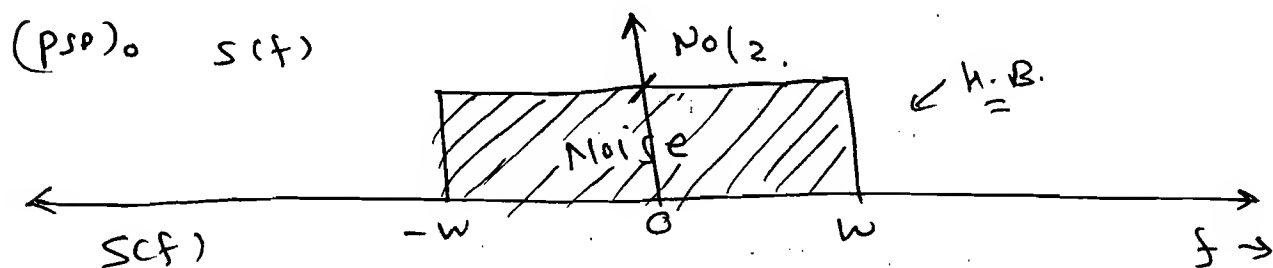
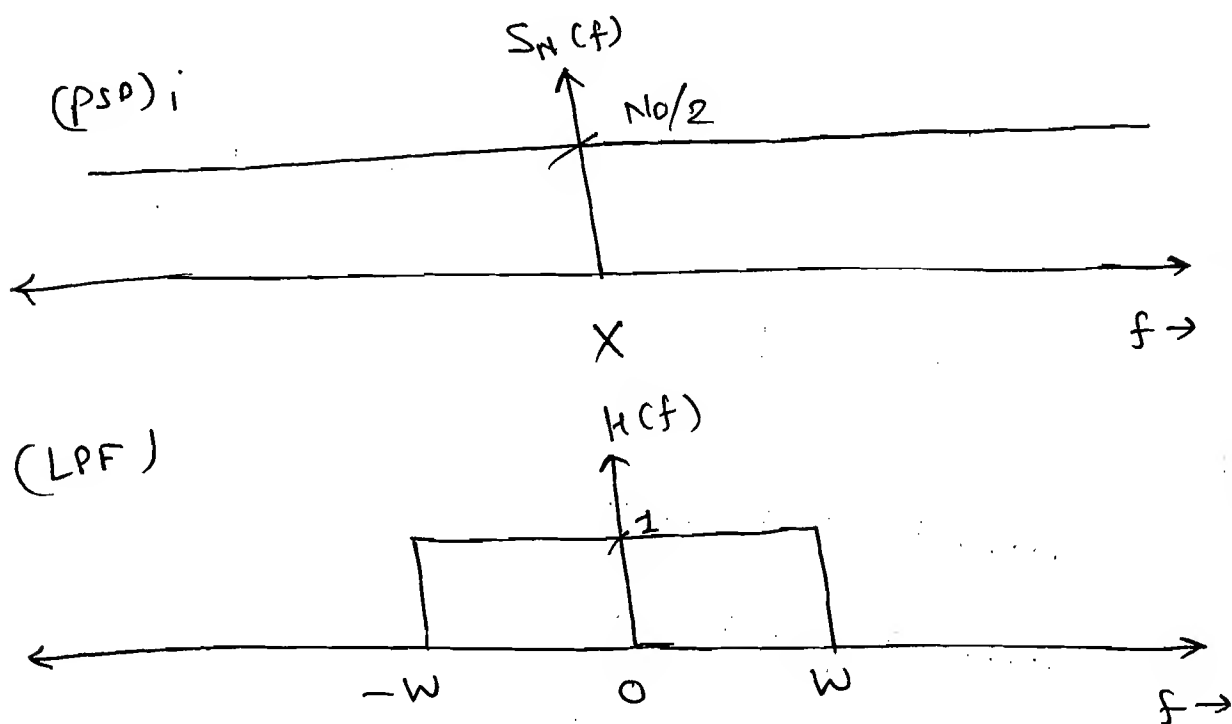
$$(PSD)_o = |H(f)|^2 \cdot (PSD)_i$$

$$\therefore \boxed{(PSD)_o = (1 + 4\pi^2 f^2) (PSD)_i}$$

① When white noise Passed through LPF:

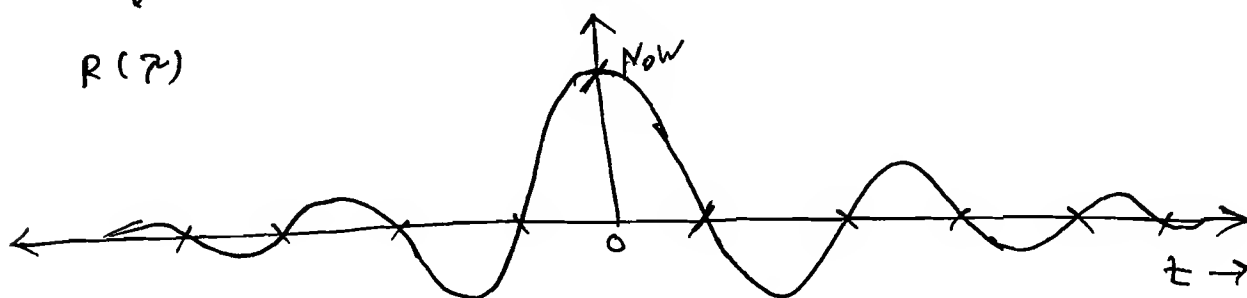


$$|H(f)| = 1 \Rightarrow |H(f)|^2 = 1.$$



IFT  
↓  
 $R(\tau)$

IFT



$$\Rightarrow \text{Noise Power} = \int_{-W}^W \frac{N_0}{2} \cdot df$$

$$\boxed{\text{Noise Power} = 'N_0 \cdot W' \text{ watts.}} \leftarrow \underline{H.B.}$$

$$\underline{\text{ACF}} \Rightarrow R(\tau) = \frac{N_0}{2} \times 2W \cdot \text{sinc}[2W\tau]$$

$$\boxed{R(\tau) = N_0 W \cdot \text{sinc}(2W\tau).} \leftarrow \underline{H.B.}$$

$$\Rightarrow R(0) = \text{Mean Square} = \text{Power} = N_0 W \text{ watts.}$$

Q White noise having a two sided spectral density of  $4 \times 10^{-3}$  watts/Hz is passed through an ideal LPF having a cutoff freq 2 kHz. Determine the noise power and ACF at the o/p of the filter.

Soln:

$$\frac{N_0}{2} = 4 \times 10^{-3} \text{ watts/Hz}$$

$$\Rightarrow N_0 = 8 \times 10^{-3} \text{ watts/Hz.}$$

$$W = 2 \text{ kHz} = 2000 \text{ Hz.} = f$$

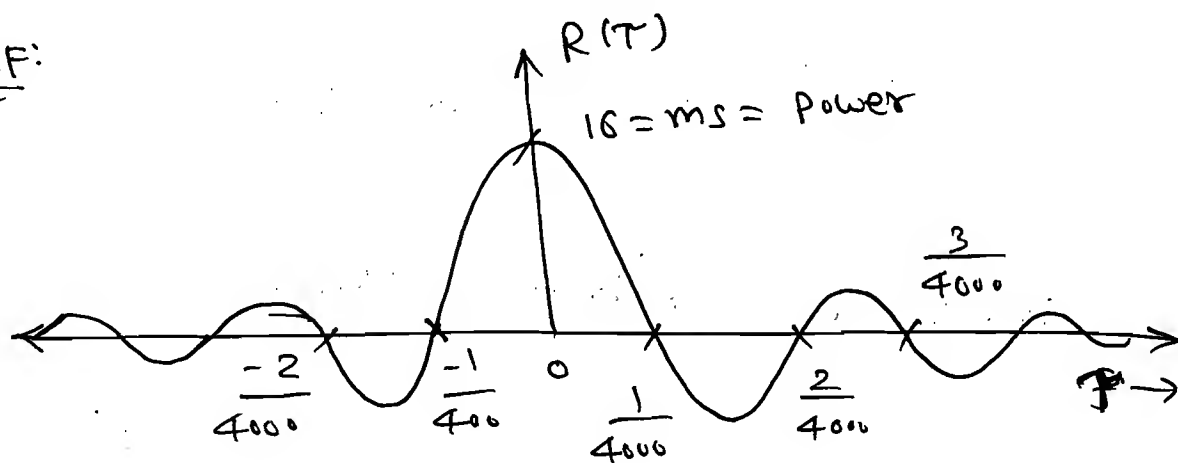
$$\text{Noise Power} = N_0 W$$

$$= 8 \times 10^{-3} \times 2 \times 10^3$$

$$\boxed{\text{Noise Power} = 16 \text{ watts.}}$$

$$\Rightarrow \boxed{\text{ACF} = R(\tau) = 16 \sin[4000\tau]}$$

$\Rightarrow$  ACF:



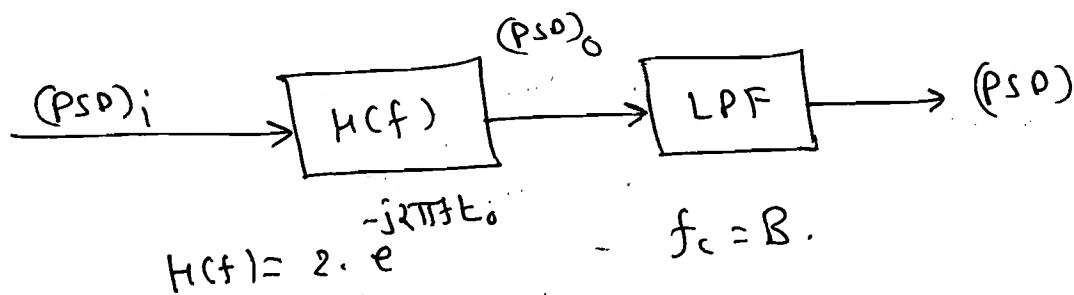
$\rightarrow$  Spacing is  $\left( \frac{2}{4000} - \frac{1}{4000} \right) = \frac{1}{4000} = \frac{1}{2W}$ .

$\rightarrow$  If the ACF is sampled at  $\frac{1}{4000}, \frac{2}{4000}, \frac{3}{4000}$  at this, the samples are zero (or) this these samples are said to be uncorrelated.

Gate-2005 Q <sup>\*\*</sup> White Noise having a uniform spectrum density of  $N_0$  watts/Hz is

Passed through a system having Transfer function  $H(f) = 2 \cdot e^{-j2\pi f t_0}$ . The o/p an ideal LPF having a cut-off freq of 'B' Hz. Determine Noise power at the o/p of the filter.

Sol<sup>n</sup>:



$\Rightarrow |H(f)| = 2$

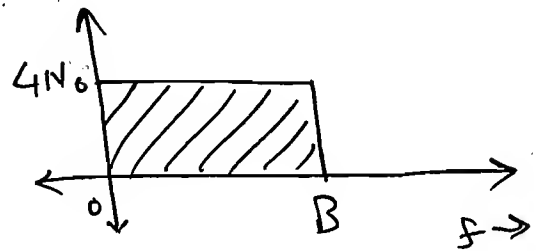
$\Rightarrow |H(f)|^2 = 4$

$$\Rightarrow (PSD)_i = N_0$$

$$(PSD)_o = |H(f)|^2 \cdot (PSD)_i$$

$$\therefore (PSD)_o = 4N_0$$

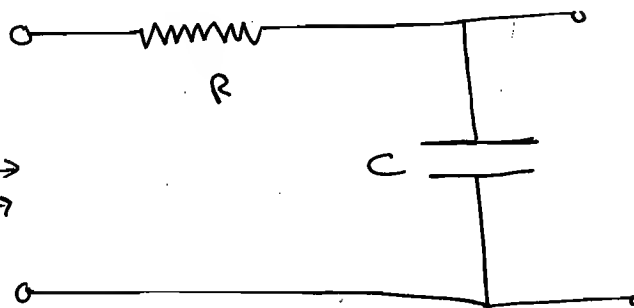
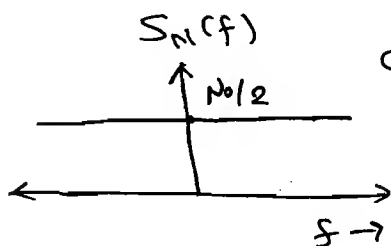
$$\Rightarrow \text{Noise Power} = \int_0^B 4N_0 \cdot df$$



$$\boxed{\text{Noise Power} = 4N_0 B \text{ watts.}}$$

\* When white noise Passed through RC LPF.

$\Rightarrow$



$$(PSD)_o = |H(f)|^2 (PSD)_i$$

$$V_o = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} \cdot V_{in}$$

$$\therefore H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1}{1 + RSC}$$

$$s = j\omega$$

$$\therefore H(j\omega) = \frac{1}{1 + j\omega RC}$$

$$\omega = 2\pi f$$

$$\therefore H(f) = \frac{1}{1 + j2\pi f RC}$$

$$\therefore |H(f)|^2 = \frac{1}{1 + (2\pi f RC)^2}$$

$$\therefore (PSD)_0 = \frac{N_0}{2} \cdot \frac{1}{1 + (2\pi f RC)^2}$$

$$\therefore (PSD)_0 = \frac{\frac{N_0}{2}}{1 + (\omega RC)^2} = \frac{\frac{N_0}{2(RC)^2}}{\omega^2 + \left(\frac{1}{RC}\right)^2}$$

$$\begin{array}{ccc} \underline{\underline{ACF}} & \underline{\underline{S(\omega)}} & \underline{\underline{R(\tau)}} \\ & \frac{2a}{\omega^2 + a^2} & e^{-a|\tau|} \\ & \xleftarrow{FT} & \end{array}$$

$$\Rightarrow \frac{N_0}{4RC} \cdot \frac{2}{\omega^2 + \left(\frac{1}{RC}\right)^2} \xleftarrow{FT} \frac{N_0}{4RC} \cdot e^{-\frac{|\tau|}{RC}}$$

$$\boxed{R(\tau) = \frac{N_0}{4RC} \cdot e^{-\frac{|\tau|}{RC}}} \leftarrow \text{H.B.}$$

Noise Power = mean square =  $R(0)$

$$\boxed{\text{Noise Power} = \frac{N_0}{4RC}} \leftarrow \text{H.B.}$$

**Q** The white noise having a two sided PSD of  $4 \times 10^{-3} \text{ W/Hz}$  is passed through simple RC LPF ckt. having 3db cut off freq of the op of RC ckt the 2KHz. Determine the noise Power at the o/p of RC ckt.



Sol<sup>n</sup>:

$$\frac{N_0}{2} = 4 \times 10^{-3}$$

$$\therefore N_0 = 8 \times 10^{-3} \text{ W/Hz.}$$

$$\therefore \text{Now, } f_{3dB} = \frac{1}{2\pi RC} \text{ for RC LPF.}$$

$$\therefore 2000 = \frac{1}{2\pi RC}$$

$$\Rightarrow \frac{1}{RC} = 4\pi \times 1000$$

$$\text{Now, Noise Power} = \frac{4 N_0}{4RC} \text{ Watts}$$

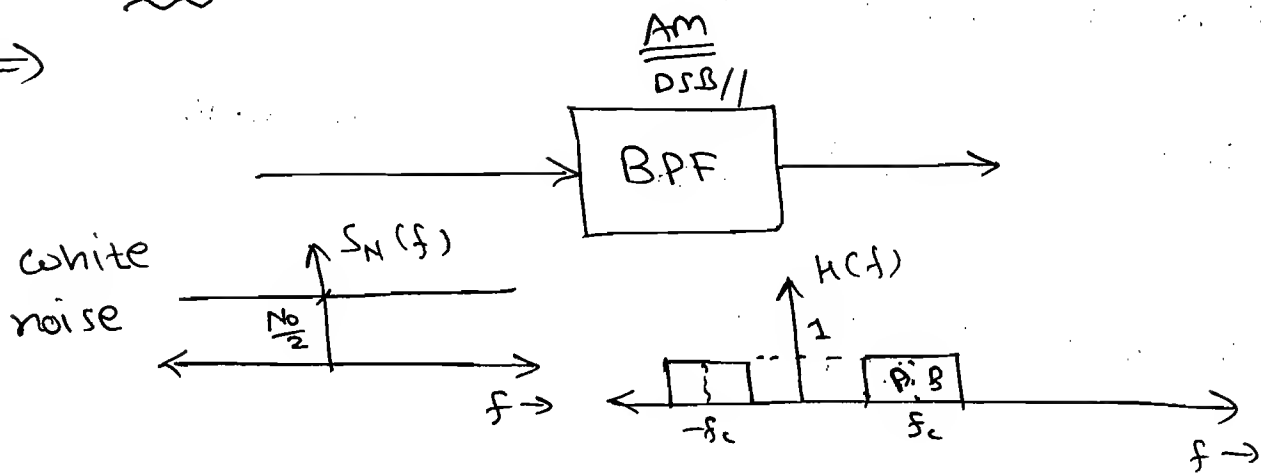
$$= \frac{8 \times 10^{-3}}{4} \times 4\pi \times 1000$$

$$= 25.12 \text{ Watts.}$$

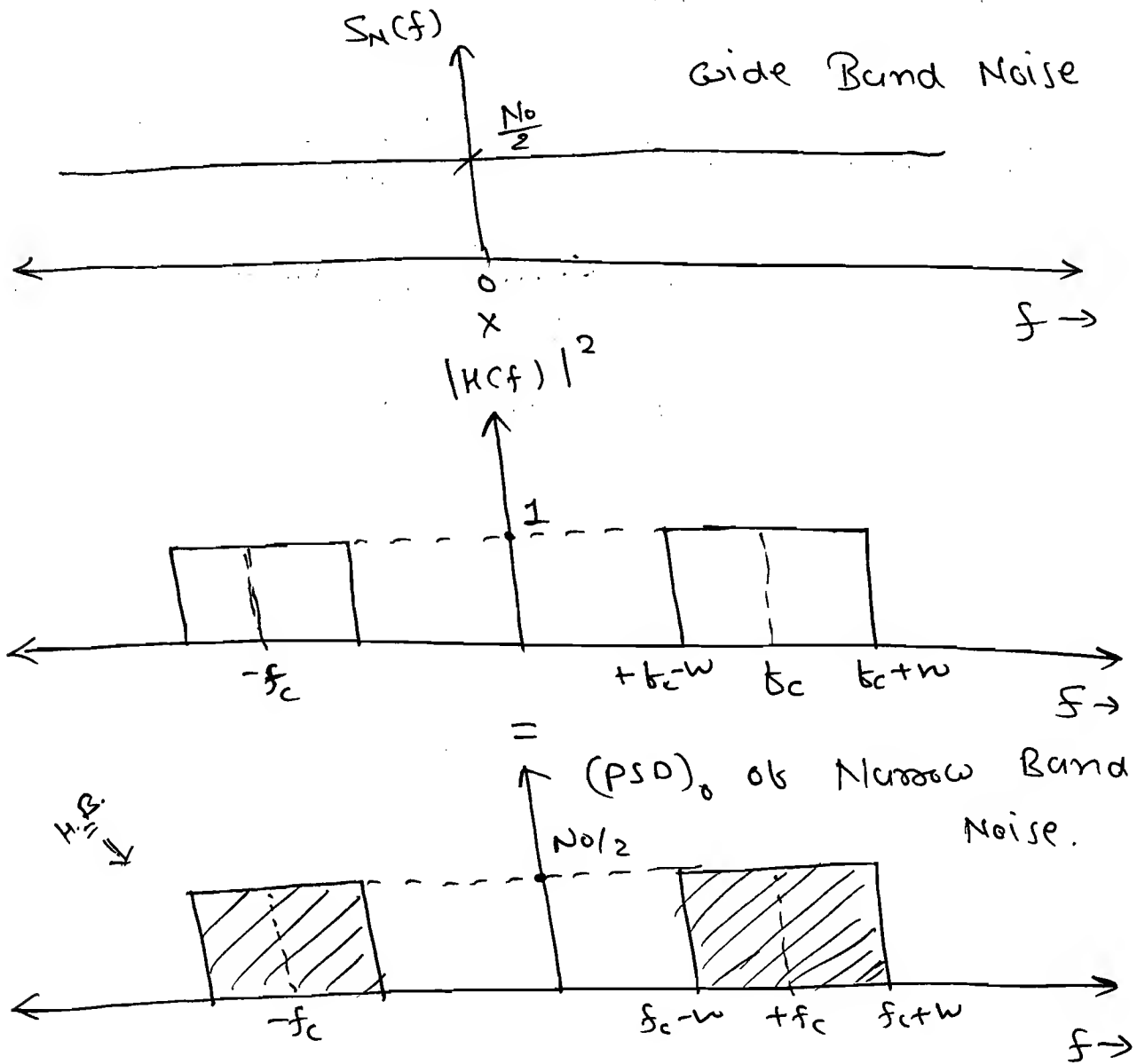
② When White noise Passed through

BPF:

⇒



⇒



⇒ Noise Power = Area under  $(PSD)_o$

$$= 2 \times \frac{N_0}{2} \times 2W$$

$$\text{Noise Power} = 2N_0W$$

$$\Rightarrow \boxed{\text{Noise Power} = 2N_0W.} \quad \leftarrow H.B.$$

$$\rightarrow \boxed{R(z) = 2N_0W \operatorname{sinc}(2Wz) \cdot \cos 2\pi f_c t}$$

$\Rightarrow \cos 2\pi f_c t$  terms is come because (pso) shifted left as well as right.

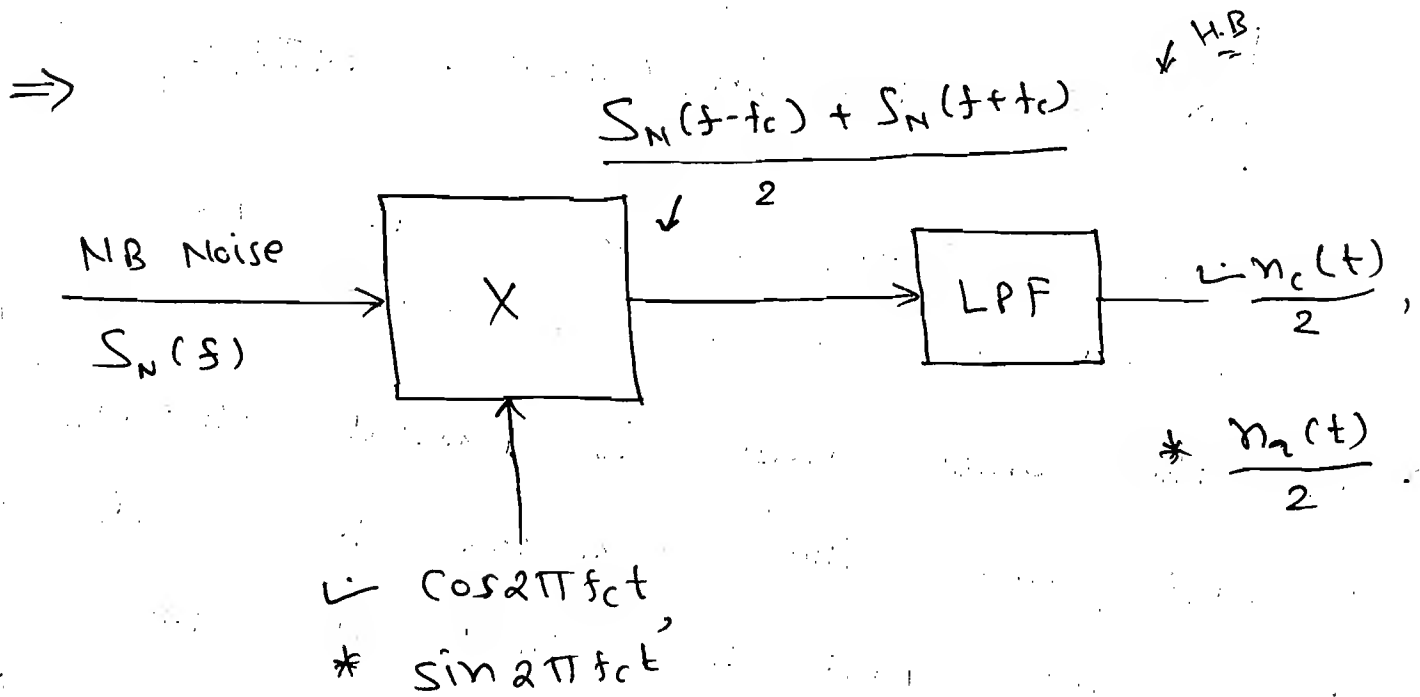
$\Rightarrow$  If the white noise is passed through a BPF, then the resultant noise at o/p of the BPF is called as the Narrow Band Noise.

$\Rightarrow$  Narrow Band Noise represent mathematically as,

$$\text{Ans} \quad \boxed{\begin{array}{cc} \underline{n_c(t) \cdot \cos 2\pi f_c t} & + \quad \underline{n_q(t) \cdot \sin 2\pi f_c t} \\ \downarrow & \downarrow \\ \text{Inphase Component} & \text{Quadrature Component} \end{array}} \quad \leftarrow H.B.$$

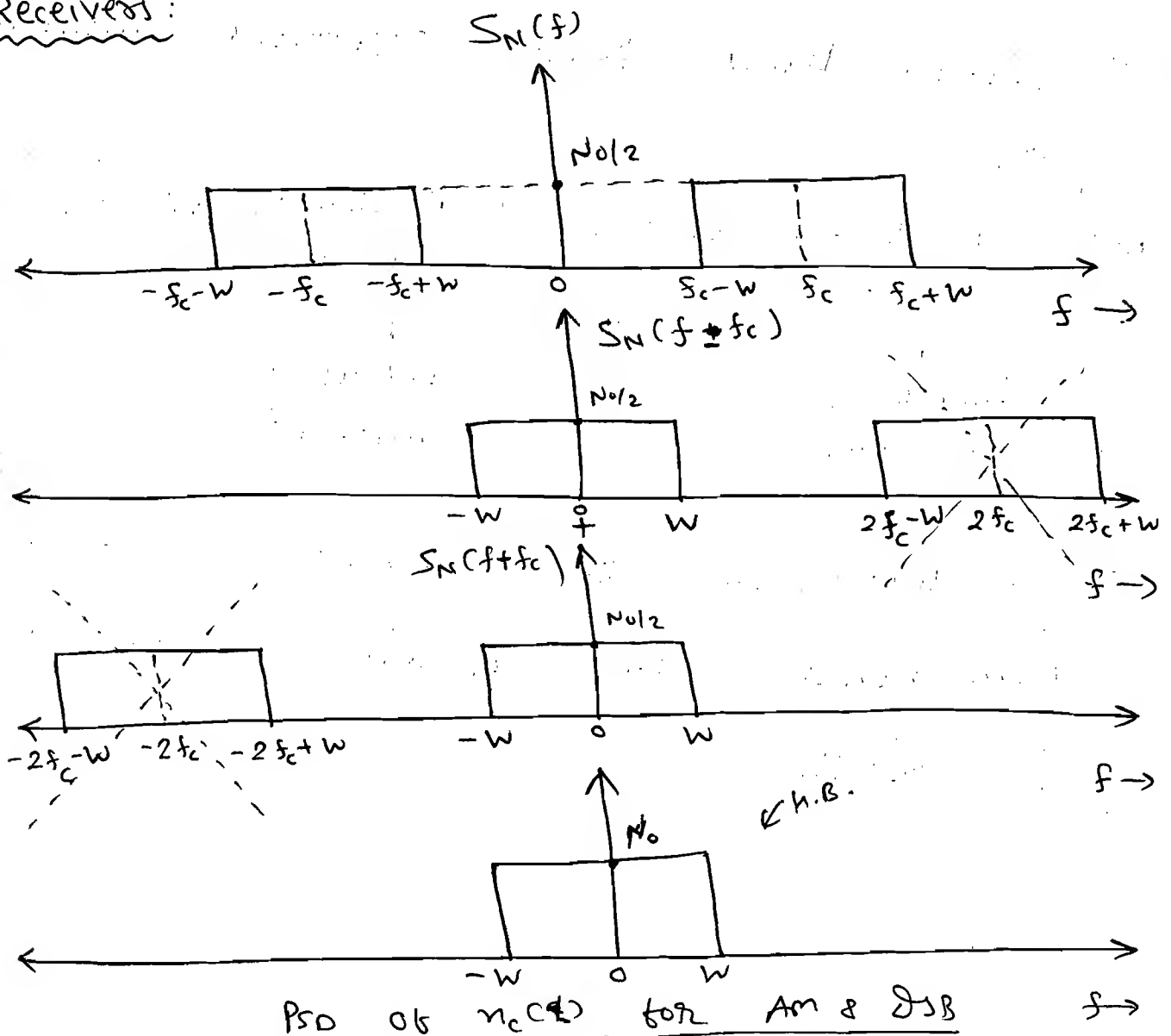
$\Rightarrow$  To determine the inphase (or) Quadrature Component, the following method is used.

=> Method to determine  $n_c(t)$  &  $n_s(t)$ :



\* PSD of Inphase Component in Am & DSB

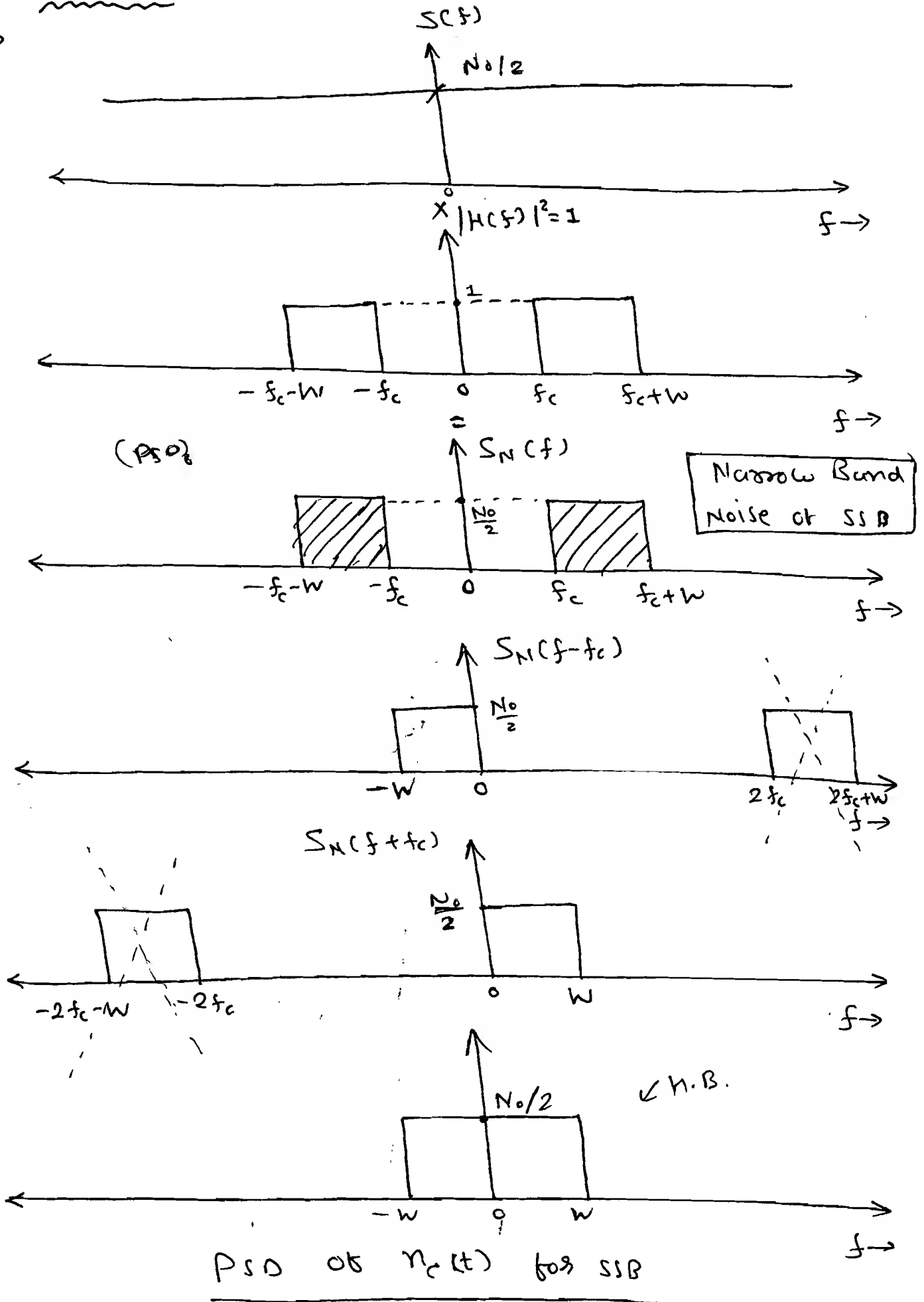
Receivers:



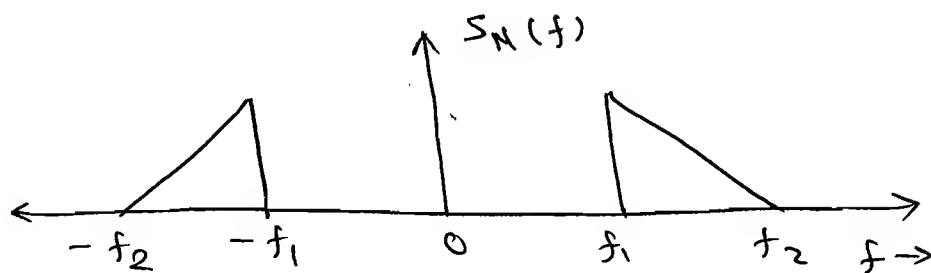
# \* PSD of Inphase Component in SSB

Receivers:

$\Rightarrow$



Q The Power spectral density of NB noise is shown in the fig.



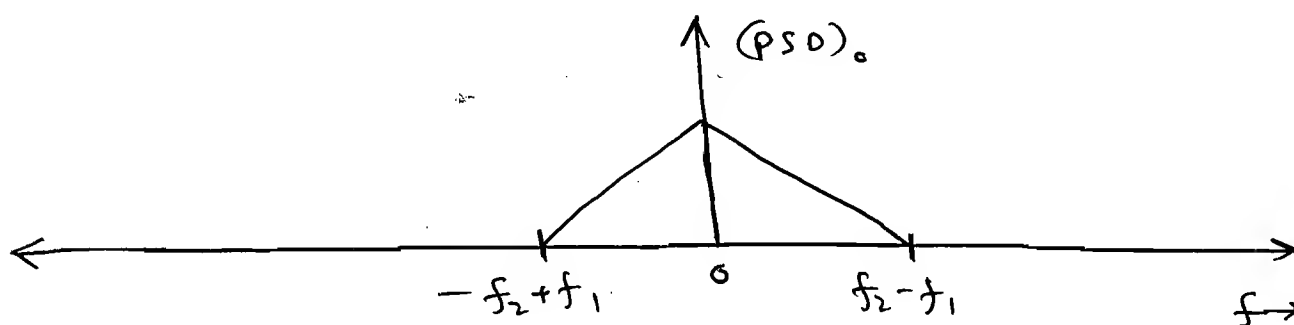
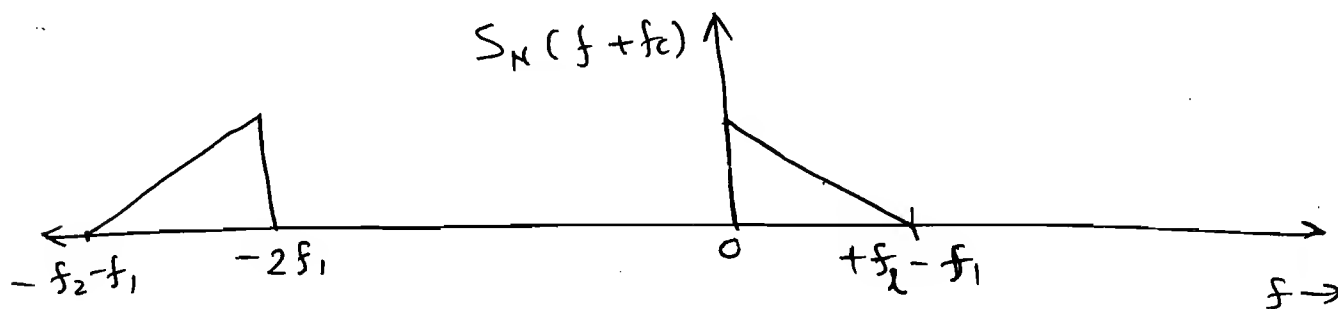
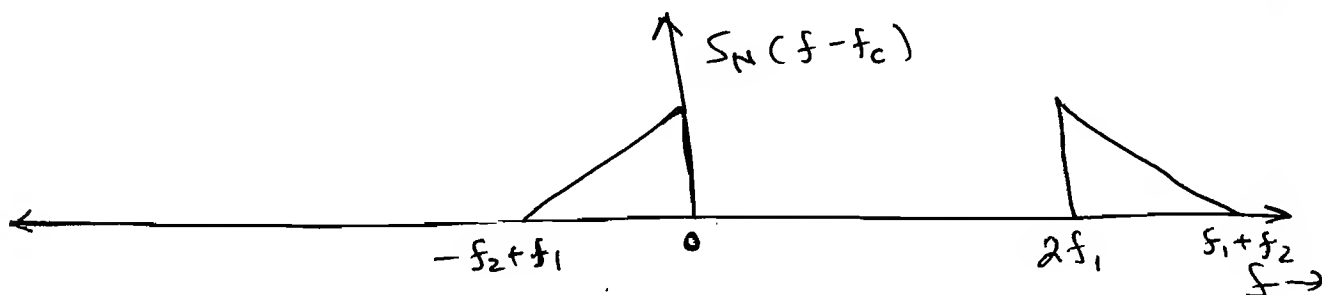
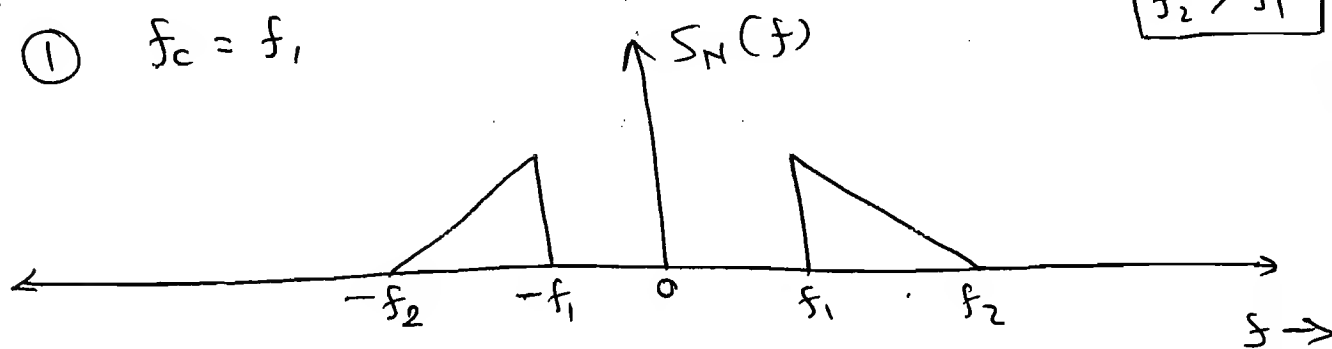
Sketch the PSD of inphase component when the carrier freq is

- ①  $f_c = f_1$  ②  $f_c = f_2$  ③  $f_c = \frac{f_1 + f_2}{2}$ .

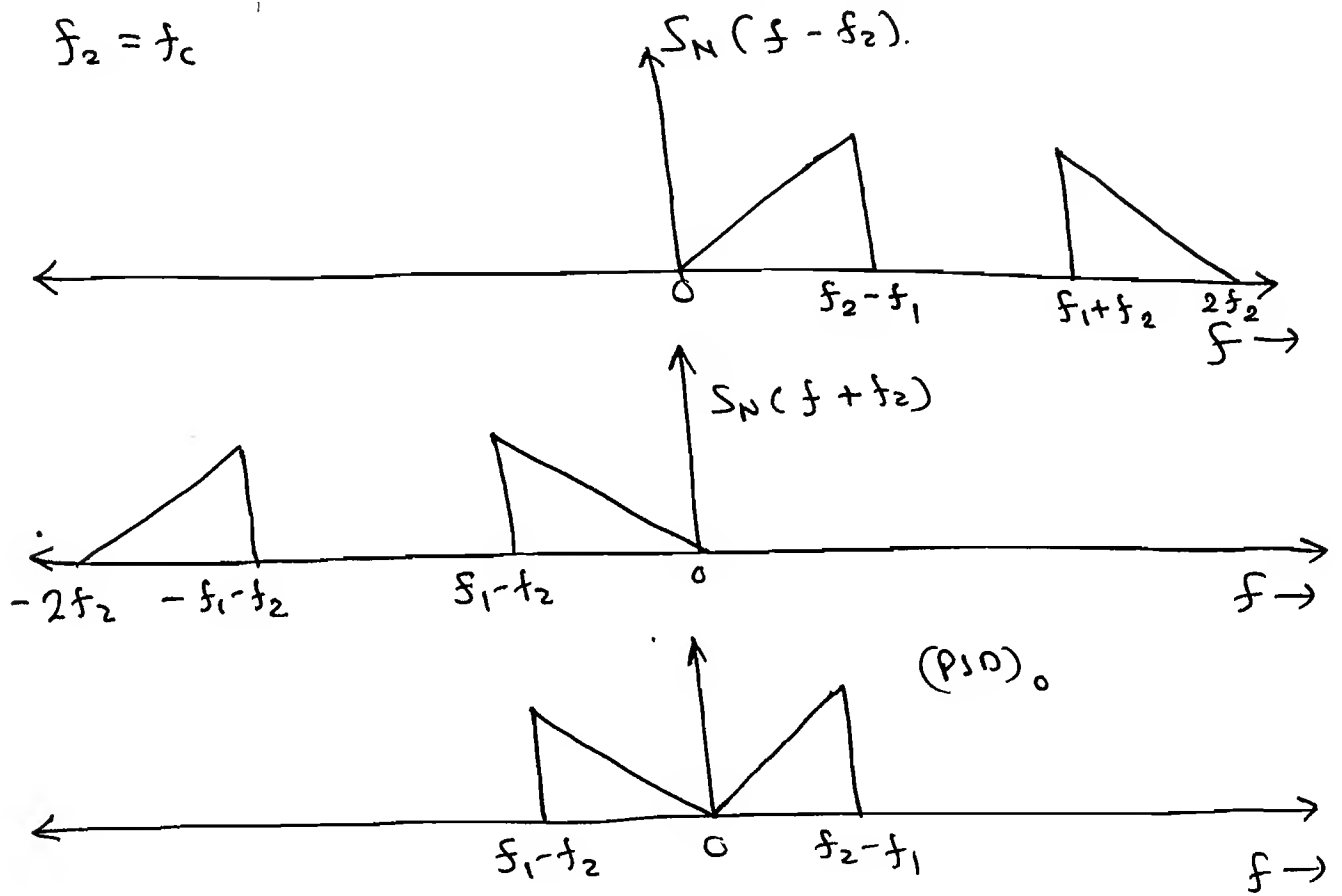
Soln:

- ①  $f_c = f_1$

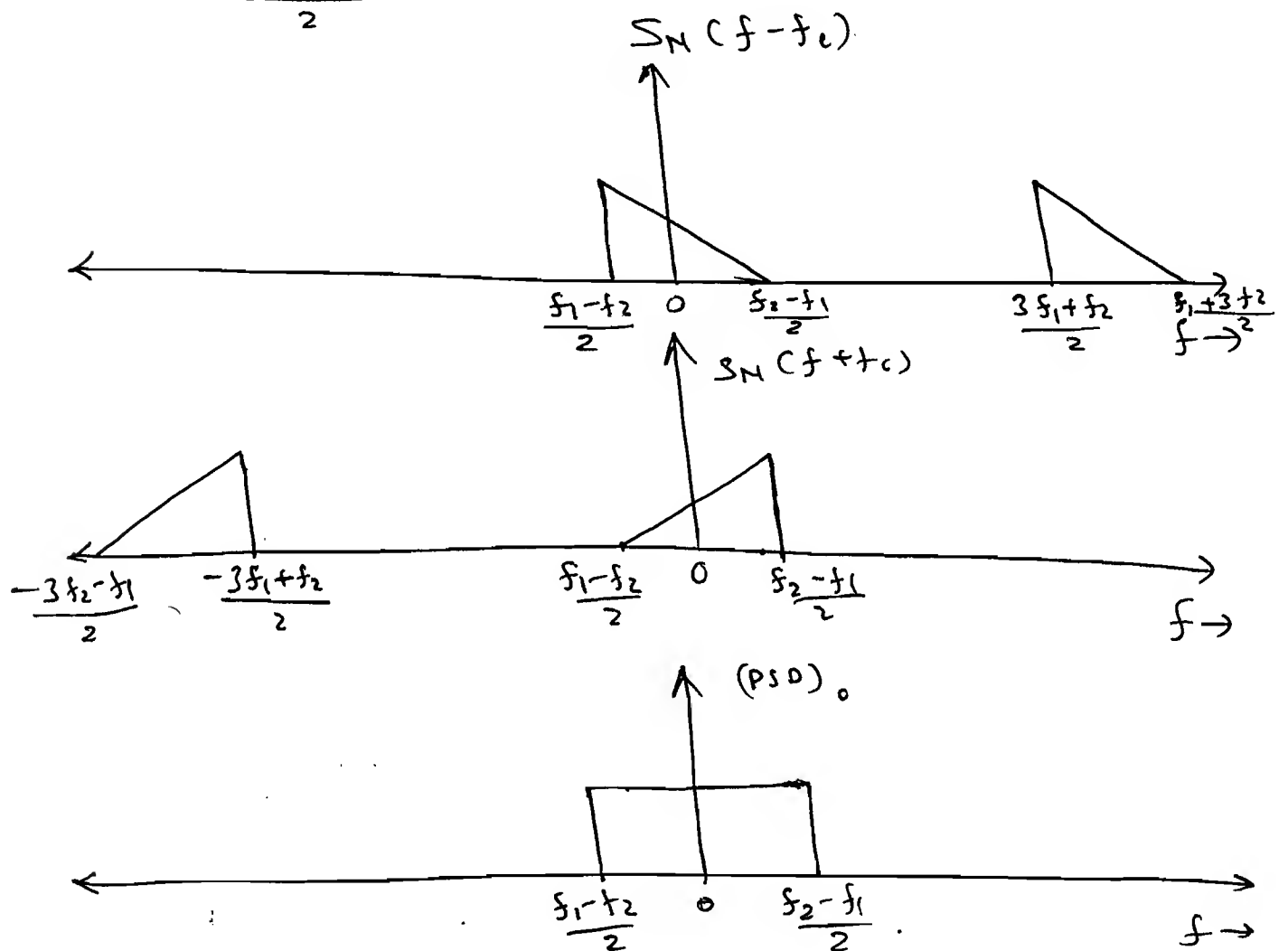
$$f_2 > f_1$$



②  $f_2 = f_c$



②  $f_c = \frac{f_1 + f_2}{2}$



Q PSD of noise is  $10^{-8} \left[ 1 - \frac{|f|}{10^8} \right]$  in

freq. range  $-10^8 \leq f \leq 10^8$  & '0' otherwise

The noise is passed through an ideal BPF having a centre freq. of 50 MHz. & a BW of 2 MHz. ① Sketch

the PSD at i/p & o/p at B.P.F.

② Find the noise power at o/p.

③ Sketch PSD of inphase component

when  $f_c = 50$  MHz.

Sol<sup>n</sup>: Let, PSD of noise  $S(f) = Y = 10^{-8} \left[ 1 - \frac{|f|}{10^8} \right]$

Let,  $S(f) = Y$   
 $f = X.$

$$\therefore Y = 10^{-8} \left[ 1 - \frac{|X|}{10^8} \right].$$

$$\therefore Y = 10^{-8} - 10^{-16} |X|.$$

$$\therefore 10^{-16} |X| + Y = 10^{-8}$$

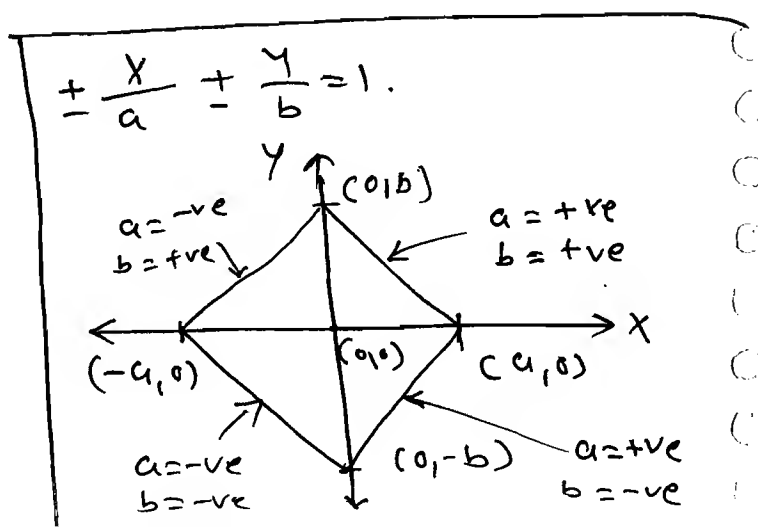
$$\therefore \frac{|X|}{10^8} + \frac{Y}{10^{-8}} = 1.$$

So, ①  $X = +ve.$

$$\therefore \frac{X}{10^8} + \frac{Y}{10^{-8}} = 1 \quad \text{--- ①}$$

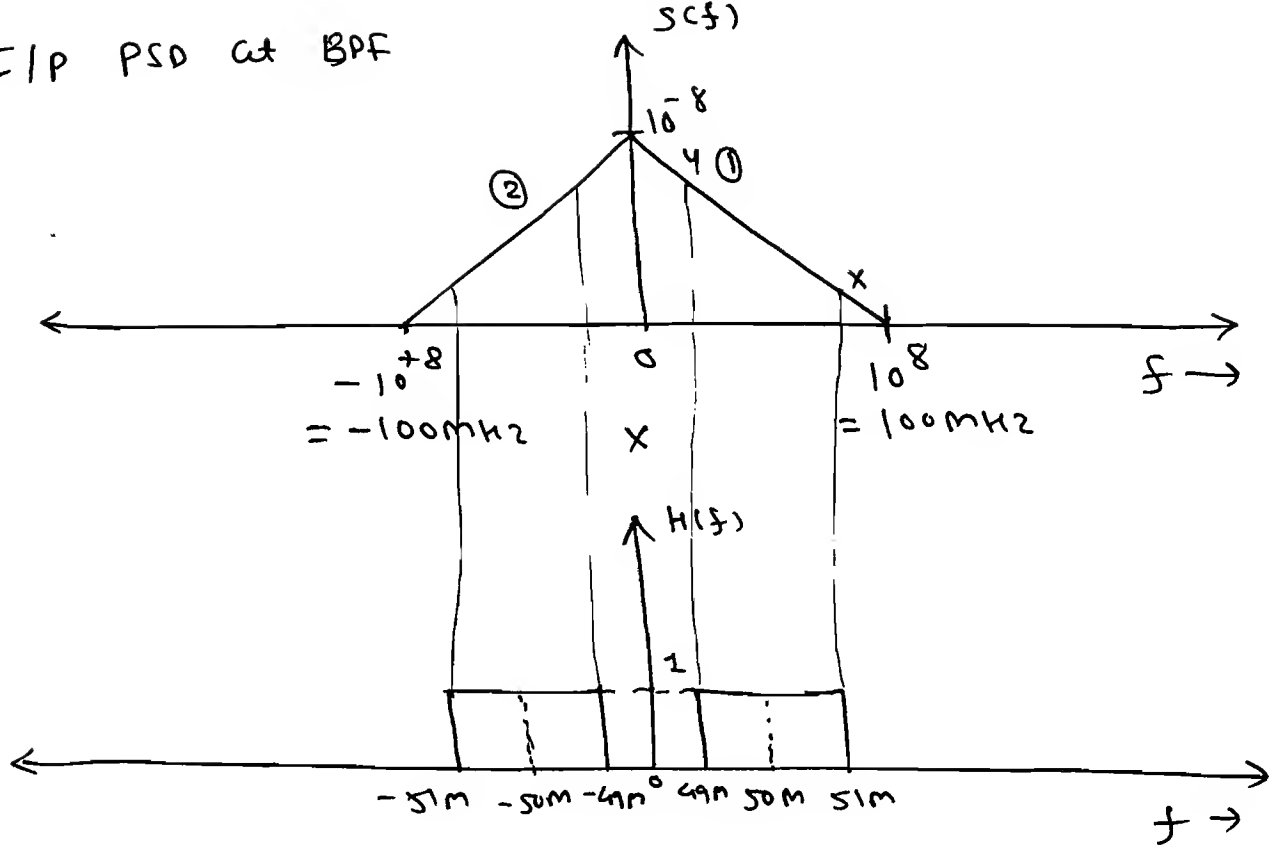
②  $X = -ve$

$$-\frac{X}{10^8} + \frac{Y}{10^{-8}} = 1 \quad \text{--- ②}$$

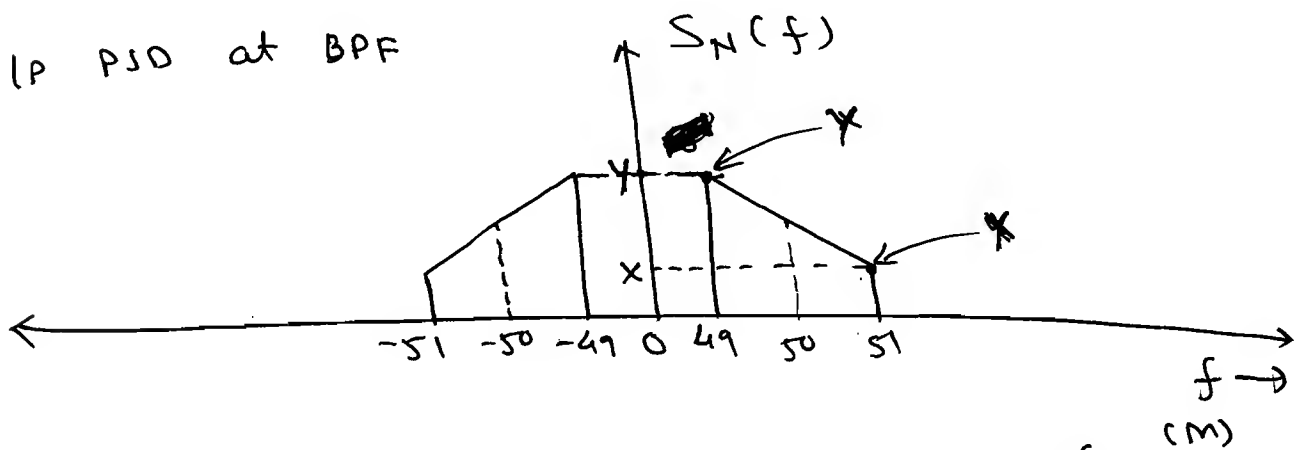




① I/P PSD at BPF



O/P PSD at BPF



$$\Rightarrow \text{at } f = 51 \text{ MHz} \quad S_N(f) = 10^{-8} \left[ 1 - \frac{51 \times 10^6}{100} \right]$$

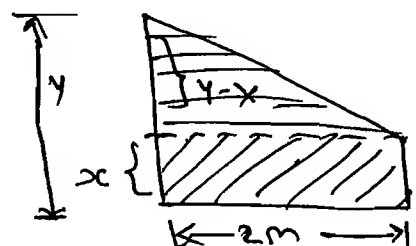
$$x = S_N(f) = 0.49 \times 10^{-8} \text{ W/Hz.}$$

$$\Rightarrow \text{at } f = 49 \text{ MHz} \quad S_N(f) = 10^{-8} \left[ 1 - \frac{49}{100} \right]$$

$$y = S_N(f) = 0.51 \times 10^{-8} \text{ W/Hz.}$$

$\Rightarrow$  Noise Power = Area under (PSD)

$$\text{Noise Power} = 2 \left[ (x \times 2 \text{ m}) + \frac{1}{2} (2 \text{ m} \times (y - x)) \right]$$



Noise power

$$= 2 \left[ (2 \times 10^{+6} \times 0.49 \times 10^{-8}) + \left( \frac{1}{2} \times 2 \times 10^6 \times 2 \times 10^{-10} \right) \right]$$

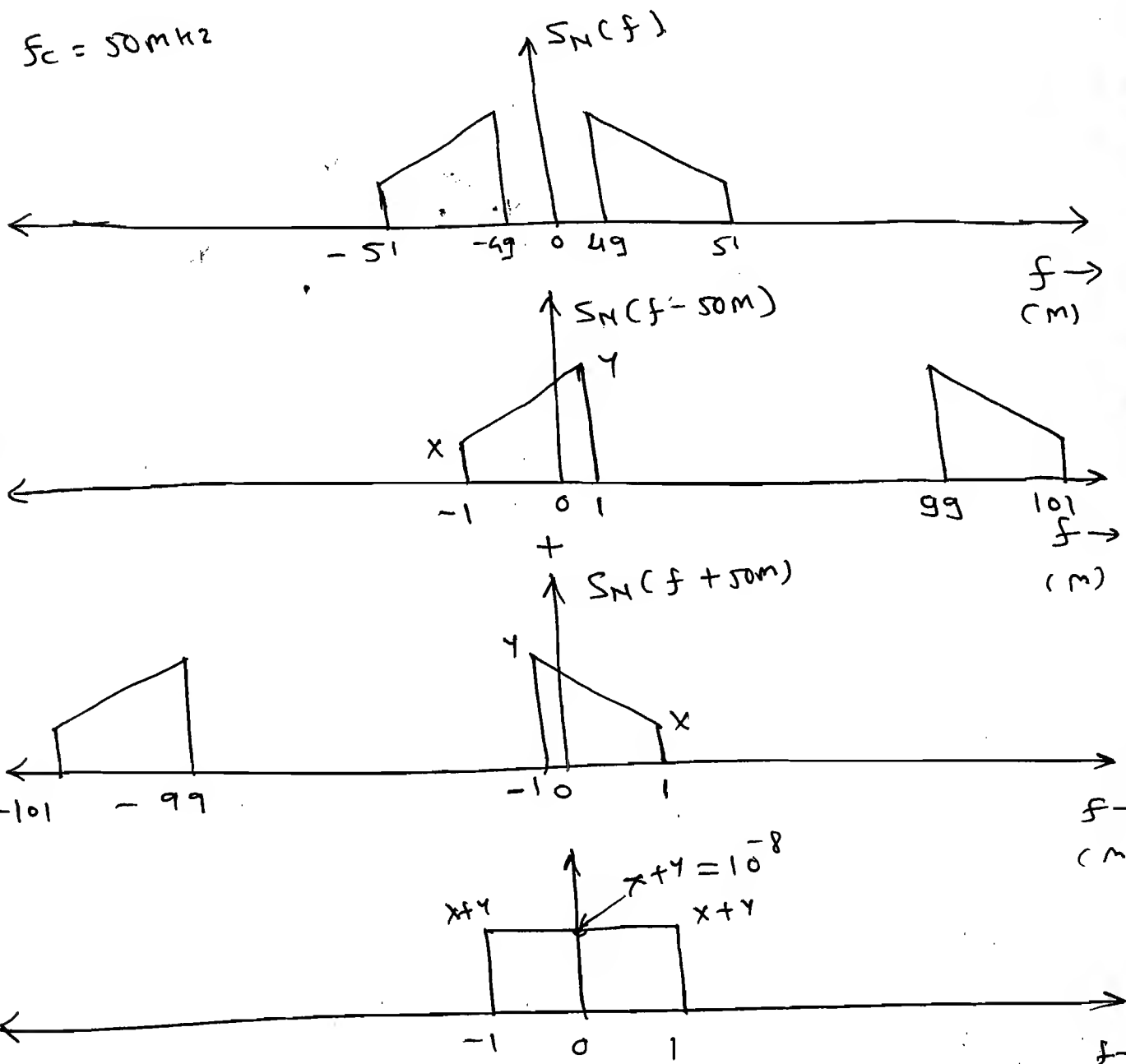
$$= 2 \left[ 9.8 \times 10^{-3} + 0.2 \times 10^{-3} \right]$$

$$= 2 \times 10 \times 10^{-3}$$

$$\therefore \text{Noise Power} = 20 \text{ mW}$$

③

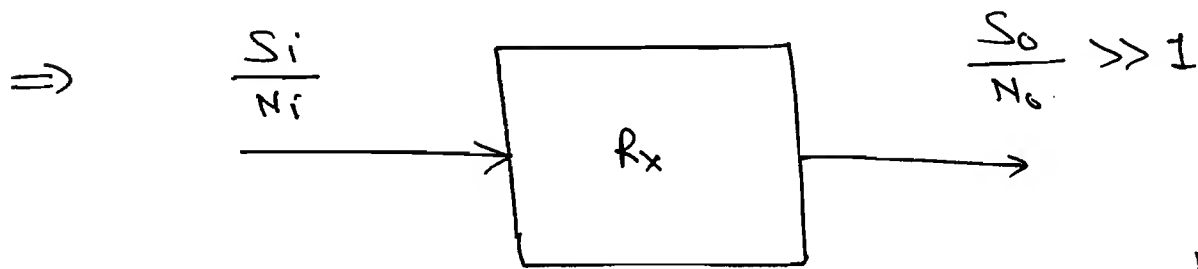
$$f_c = 50 \text{ MHz}$$



# ★ Noise Analysis of Analog

Communication:

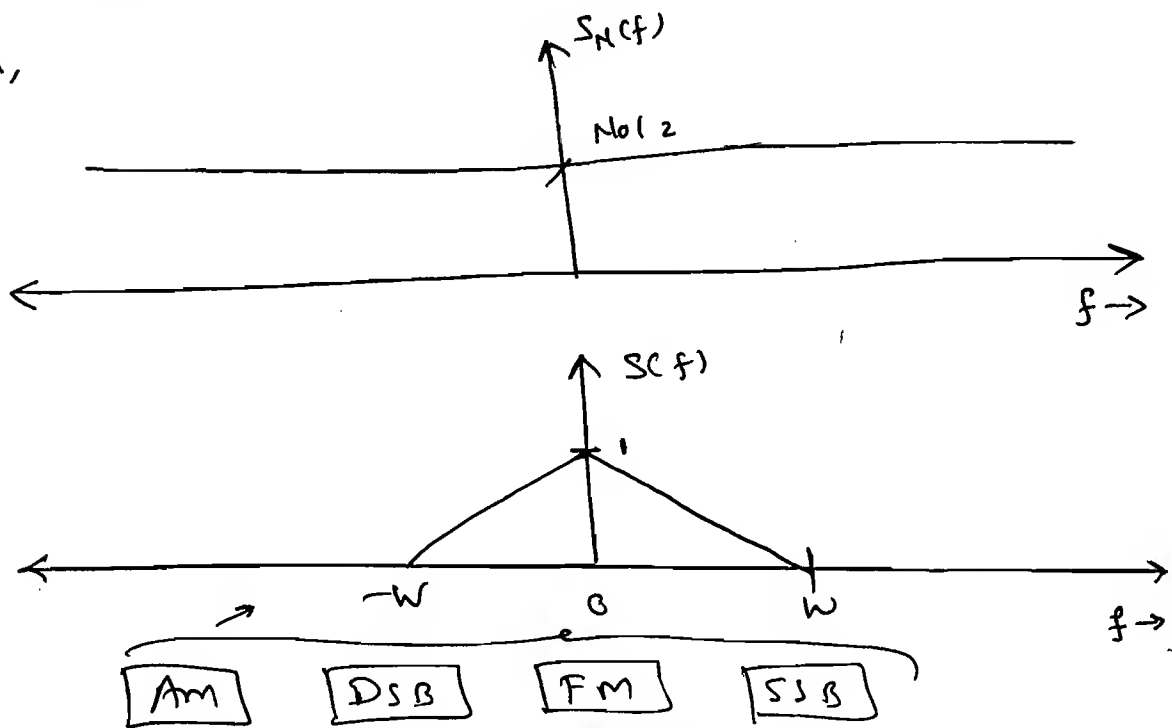
⇒ To determine the Performance of the modulation technique, Figure of Merit (FOM) is used.



⇒ Figure of Merit =  $\frac{S_o/N_o}{S_i/N_i} \gg 1$  ↓ H.B.

⇒ Noise Figure =  $\frac{\frac{S_i}{N_i}}{\frac{S_o}{N_o}} \ll 1$  ↓ H.B.

⇒ To determine  $n_i$  following procedure is used,



$n_i$  = noise Power in message BW  
= area

$$\therefore n_i = \left( \frac{N_0}{2} \times 2W \right)$$

$$\therefore \boxed{n_i = 'N_0 W' \text{ Watts.}} \quad \leftarrow H.B.$$

\* Figure of Merit of a DSB Receiver:

$$\Rightarrow \begin{array}{l} \text{dc} \longrightarrow \text{Power} \\ \text{ac} \longrightarrow (2m)^2 \end{array}$$

$$10 \cos 2\pi f_c t \longrightarrow \left( \frac{10}{\sqrt{2}} \right)^2 = 50W$$

$$\Rightarrow \boxed{\sqrt{K} (10 \cos 2\pi f_c t) \longrightarrow K^2 (50W).} \quad \leftarrow H.B.$$

→ Time domain eqn of DSB is,

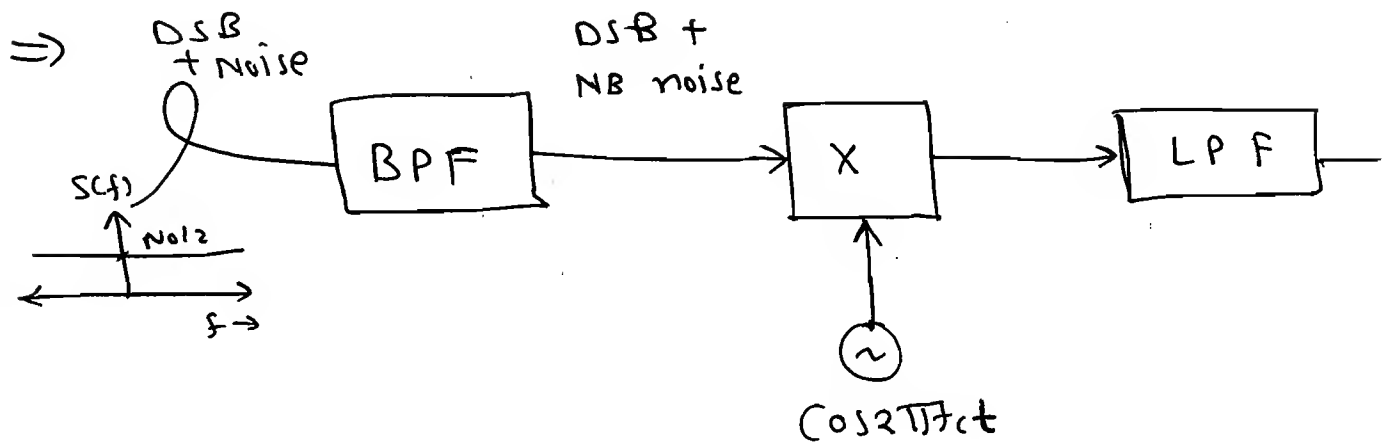
$$s(t) = A_c m(t) \cdot \cos 2\pi f_c t.$$

$$\therefore S_i = \left( \frac{A_c m(t)}{\sqrt{2}} \right)^2$$

$$S_i = \frac{A_c^2}{2} \cdot m^2(t), \quad n_i = 'N_0 W', \quad S_{ir}$$

let,  $P$  = Power of message signal

$$\therefore \boxed{(SNR)_i = \frac{S_i}{N_i} = \frac{A_c^2 \cdot P}{2 N_0 W}} \quad \leftarrow H.B. \quad S_i = \frac{A_c^2 \cdot P}{2}$$



⇒ The o/p of multiplier is,

$$[DSB + NB \text{ noise}] \cos 2\pi f_c t.$$

$$= [A_c \cdot m(t) \cdot \cos 2\pi f_c t + n_c(t) \cos 2\pi f_c t + n_s(t) \cdot \sin 2\pi f_c t] \cos 2\pi f_c t$$

$$= A_c \cdot m(t) \cdot \cos^2 2\pi f_c t + n_c(t) \cdot \cos^2 2\pi f_c t + n_s(t) \cdot \sin 2\pi f_c t \cdot \cos 2\pi f_c t$$

⇒ o/p of LPF is.

$$\frac{A_c \cdot m(t)}{2} + \frac{n_c(t)}{2} \rightarrow \text{noise.}$$

Signal

$$m(t) \rightarrow P$$

$$\frac{A_c}{2} \cdot m(t) \rightarrow \frac{A_c^2 \cdot P}{4}$$

$$\Rightarrow S_o = \left( \frac{A_c \cdot m(t)}{2 \sqrt{2}} \right)^2 = \frac{A_c^2 \cdot P}{2 \times 2} = \frac{A_c^2 \cdot P}{4}$$

$$n_c = \frac{n_s}{\sqrt{2}}$$

power

$$m(t) \rightarrow P$$

$$\frac{A_c}{2} \cdot m(t) \rightarrow \frac{A_c^2}{4} \cdot P$$

$$\Rightarrow n_c(t) \rightarrow (2 N_0 W).$$

$$\therefore \frac{1}{2} n_c(t) \rightarrow 2 N_0 W \times \frac{1}{4} = \frac{2 N_0 W}{4} = n_0$$

$$S_o, \quad S_o = \frac{A_c^2 \cdot P}{4}, \quad n_o = \frac{2 N_o W}{4}$$

$$\therefore \boxed{SNR_o = \frac{S_o}{N_o} = \frac{A_c^2 \cdot P}{2 N_o W}} \leftarrow \text{H.B.}$$

$$\Rightarrow \text{Figure of Merit} = \frac{(SNR)_o}{(SNR)_i}$$

$$\text{H.B.} \rightarrow \boxed{FOM = 1} \text{ for DSB Receiver}$$

\* Figure of Merit of a SSB Receiver.

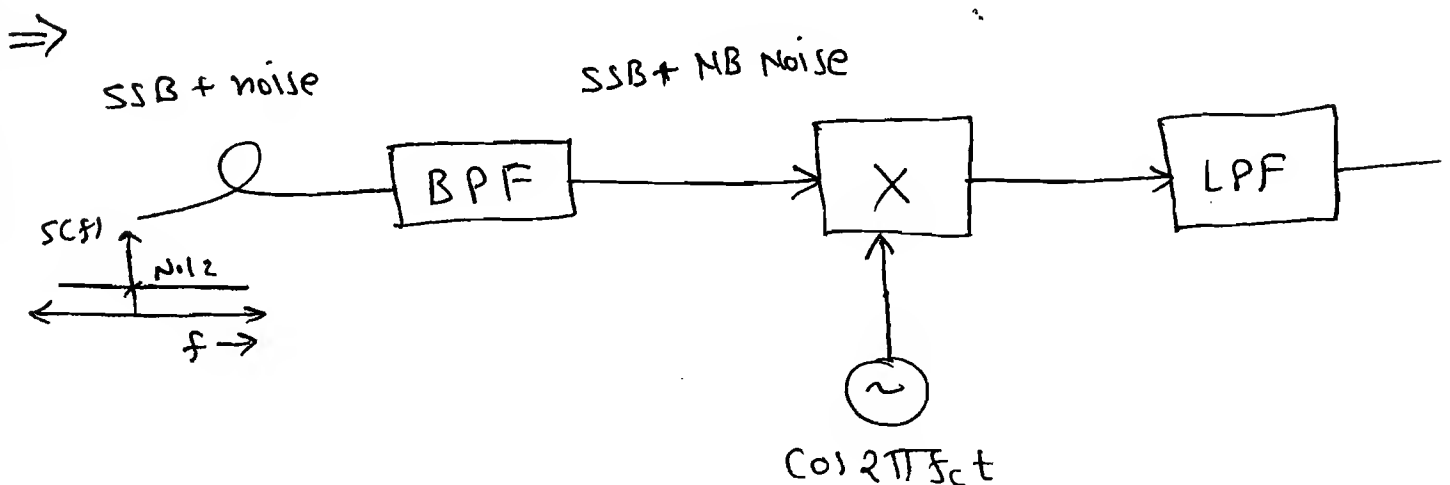
$$\Rightarrow (S_i)_{SSB} = 50\% \cdot (S_i)_{DSB}$$

$$\therefore (S_i)_{SSB} = \frac{1}{2} \cdot (S_i)_{DSB}$$

$$\therefore (S_i)_{SSB} = \frac{A_c^2 \cdot P}{4}$$

$$n_i = N_o \cdot W$$

$$\therefore \boxed{(SNR)_i = \frac{S_i}{n_i} = \frac{A_c^2 \cdot P}{4 N_o W}} \leftarrow \text{H.B.}$$



$\Rightarrow$  The op of multiplier is,  
 (SSB + NB noise)  $\cos 2\pi f_c t$ .

$$= \left[ \frac{A_c}{2} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t + n_c(t) \cos 2\pi f_c t + n_s(t) \sin 2\pi f_c t \right] \cos 2\pi f_c t$$

$$= \frac{A_c}{2} m(t) \cos^2 2\pi f_c t + \cancel{A_c \hat{m}(t) \sin 2\pi f_c t \cos 2\pi f_c t} + n_c(t) \cos^2 2\pi f_c t + \cancel{n_s(t) \sin 2\pi f_c t \cos 2\pi f_c t}$$

$\Rightarrow$  OIP of the LPF is,

$$\therefore = \underbrace{\frac{A_c}{2} m(t)}_{\text{signal}} + \underbrace{\frac{n_c(t)}{2}}_{\text{Noise}}$$

$$\Rightarrow m(t) \longrightarrow P$$

$$\rightarrow \frac{A_c}{4} m(t) \longrightarrow \frac{A_c^2}{16} P$$

$$\rightarrow \frac{n_c(t)}{2} \longrightarrow N_o \cdot W$$

$$\frac{1}{2} n_c(t) \longrightarrow \frac{1}{4} N_o \cdot W$$

$$\therefore S_o = \frac{A_c^2 P}{16}, \quad N_o = \frac{N_o \cdot W}{4}$$

$$\therefore (SNR)_o = \frac{S_o}{N_o} = \frac{\frac{A_c^2 P}{16}}{\frac{N_o \cdot W}{4}}$$

$$\therefore \boxed{(SNR)_0 = \frac{A_c^2 \cdot P}{4 N_0 W}} \leftarrow \text{H.B.}$$

$$\Rightarrow \text{Figure of Merit} = \frac{(SNR)_0}{(SNR)_0} = 1$$

$$\therefore \boxed{F_o M = 1} \text{ for SSB Receiver.}$$

\* Figure of Merit of AM:

$$\Rightarrow S(t) = A_c \cos 2\pi f_c t + A_c K_a \cdot m(t) \cdot \cos 2\pi f_c t.$$

$$\therefore S_i = \frac{A_c^2}{2} + \frac{A_c^2 \cdot K_a^2 \cdot m^2(t)}{2}$$

$m(t) \xrightarrow{\text{Power}} P$

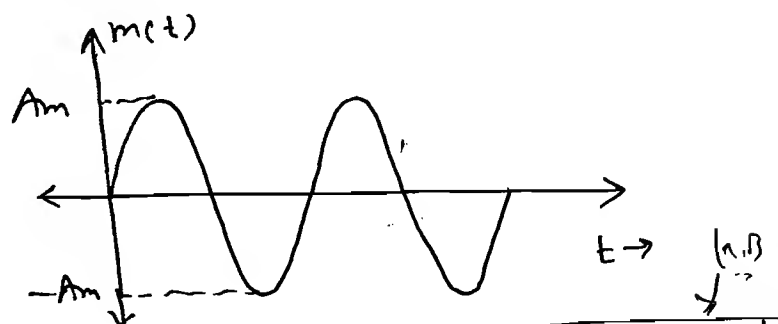
$$\therefore S_i = \frac{A_c^2}{2} + \frac{A_c^2 \cdot K_a^2 \cdot P}{2}$$

$$\therefore \boxed{S_i = \frac{A_c^2}{2} \left[ 1 + \frac{K_a^2 \cdot P}{2} \right]} \leftarrow \text{H.B.}$$

$\Rightarrow$  if  $m(t) = A_m \cos 2\pi f_m t = \text{sinusoidal signal.}$

$$r_{rms} = \frac{V_m}{\sqrt{2}} = \frac{A_m}{\sqrt{2}}$$

$$\therefore P = \frac{(r_{rms})^2}{R} = \frac{A_m^2}{2R}$$



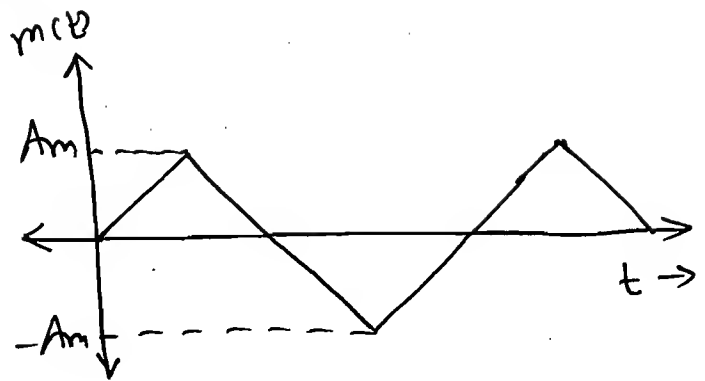
$$\text{let, } R = 1 \Omega \quad S_i = \frac{A_c^2}{2} \left[ 1 + \frac{K_a^2 A_m^2}{2} \right] = \frac{A_c^2}{2} \left[ 1 + \mu^2 \right]$$



$\Rightarrow$  if  $m(t) = \text{triangular wave}$ .

$$\Rightarrow r_{ms} = \frac{A_m}{\sqrt{3}}$$

$$\therefore P = \frac{A_m^2}{3}$$



$$\Rightarrow \therefore S_i = \frac{A_c^2}{2} \left[ 1 + \frac{K_a^2 A_m^2}{3} \right]$$

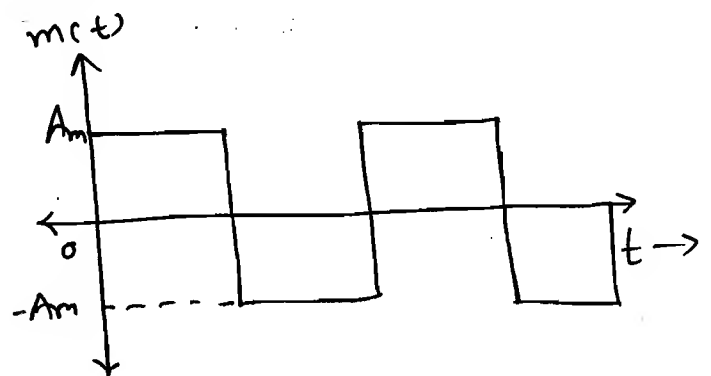
$$\text{as } \mu = K_a A_m$$

$$\therefore S_i = \frac{A_c^2}{2} \left[ 1 + \frac{\mu^2}{3} \right] \leftarrow \text{H.B.} =$$

$\Rightarrow$  if  $m(t) = \text{Square wave}$ .

$$\therefore r_{ms} = A_m$$

$$\therefore P = A_m^2$$



$$\therefore S_i = \frac{A_c^2}{2} [1 + \mu^2] \leftarrow \text{H.B.}$$

$$\Rightarrow S_i = \underbrace{\frac{A_c^2}{2}}_{\text{Carrier power}} + \underbrace{\frac{A_c^2 K_a^2 P}{2}}_{\text{sideband power}}$$

$$\therefore \eta = \frac{P_{SB}}{P_t}$$

$$\therefore \eta = \frac{P_{SB}}{P_t}$$

$$\therefore \eta = \frac{\frac{A_c^2 K_a^2 P}{2}}{\frac{A_c^2}{2} + \frac{A_c^2 K_a^2 P}{2}}$$

$$\therefore \eta = \frac{K_a^2 P}{1 + K_a^2 P} \leftarrow \text{H.B.}$$

$$\therefore \textcircled{1} \text{ sine} \Rightarrow P \rightarrow Am^2/2.$$

$$\therefore \eta = \frac{K_a^2 Am^2/2}{1 + K_a^2 \cdot Am^2/2}$$

$$\text{When } \underline{\underline{\mu=1}}$$

$$\boxed{\eta = 33.33 \%}$$

$$\therefore \eta = \frac{K_a^2 Am^2}{2 + K_a^2 \cdot Am^2}$$

$$\therefore \eta = \frac{\mu^2}{\mu^2 + 2} \leftarrow \text{H.B.}$$

$$\textcircled{2} \text{ Square wave} \Rightarrow P = Am^2.$$

$$\therefore \eta = \frac{K_a^2 Am^2}{1 + K_a^2 Am^2}$$

$$\text{When } \underline{\underline{\mu=1}}$$

$$\boxed{\eta = 50 \%}$$

$$\therefore \eta = \frac{\mu^2}{\mu^2 + 1} \leftarrow \text{H.B.}$$

$$\textcircled{3} \text{ Triangular wave} \Rightarrow P = Am^2/3.$$

$$\therefore \eta = \frac{K_a^2 \cdot Am^2/3}{1 + K_a^2 \cdot Am^2/3}$$

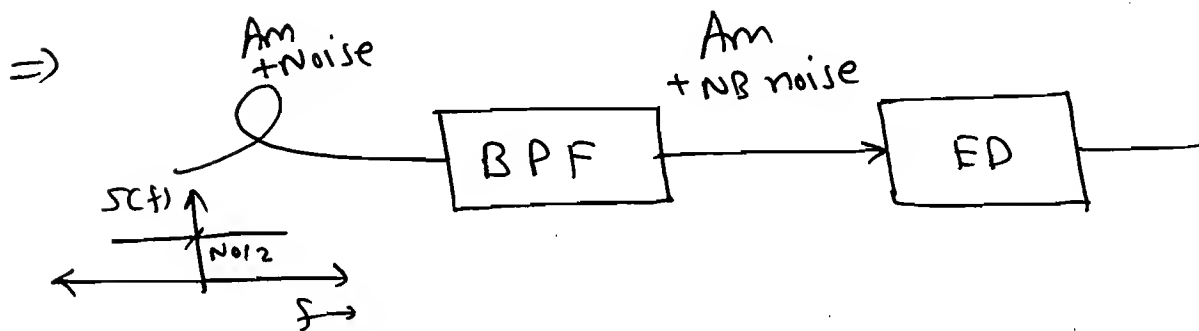
$$\text{When } \underline{\underline{\mu=1}}$$

$$\boxed{\eta = 25 \%}$$

$$\therefore \eta = \frac{\mu^2}{\mu^2 + 3} \leftarrow \text{H.B.}$$

$$\Rightarrow (SNR)_i = \frac{S_i}{n_i}$$

$$\therefore (SNR)_i = \frac{A_c^2 [1 + K_a^2 m^2(t)]}{2 N_{0W}} \leftarrow \underline{H.D}$$



$\Rightarrow$  O/P of BPF is.  
AM + NB noise.

$$= A_c [1 + K_a m(t)] \cos 2\pi f_c t + n_c(t) \cdot \cos 2\pi f_c t + n_q(t) \cdot \sin 2\pi f_c t$$

$$= [A_c [1 + K_a m(t)] + n_c(t)] \cos 2\pi f_c t + n_q(t) \sin 2\pi f_c t$$

Now, O/P of ED is

$$\sqrt{[A_c [1 + K_a m(t)] + n_c(t)]^2 + \underbrace{(n_q(t))^2}_{\approx \rightarrow \text{very small}}}$$

So, O/P of ED is

$$= A_c [1 + K_a m(t)] + n_c(t)$$

$$= \cancel{A_c} + \underbrace{A_c K_a m(t)}_{\text{signal}} + \underbrace{n_c(t)}_{\text{noise}}$$

$\Rightarrow$

$$m(t) \rightarrow P$$

$$A_c k_a m(t) \rightarrow A_c^2 \cdot k_a^2 \cdot P$$

$$\therefore S_o = A_c^2 \cdot k_a^2 \cdot P.$$

$$n_c(t) \rightarrow 2 N_o \omega.$$

$$\therefore N_o = 2 N_o \omega.$$

$$\therefore (SNR)_o = \frac{S_o}{N_o} = \frac{A_c^2 \cdot k_a^2 \cdot P}{2 N_o \omega}$$

$\leftarrow H.B.$

$$\therefore \text{Figure of Merit} = \frac{(SNR)_o}{(SNR)_i}$$

$$= \frac{\frac{A_c^2 \cdot k_a^2 \cdot P}{2 N_o \omega}}{\frac{A_c^2 [1 + k_a^2 m^2(t)]}{2 N_o \omega}}$$

$\leftarrow H.B.$

$$\therefore F_o M = \frac{k_a^2 \cdot P}{1 + k_a^2 \cdot P} = \eta < 1.$$

for AM.

\* FOM of FM:

$$\Rightarrow FOM = \frac{3 K_f^2 P}{\omega^2}$$

$$\therefore \text{for single tone } P = \frac{A_m^2}{2}$$

$$\therefore FOM = \frac{3}{2} \cdot \frac{K_f^2 \cdot A_m^2}{f_m^2}$$

$$\therefore FOM = \frac{3}{2} \cdot \left( \frac{K_f \cdot A_m}{f_m} \right)^2$$

$$\beta = \frac{K_f \cdot A_m}{f_m}$$

$$\therefore FOM = \frac{3}{2} \cdot \beta^2 \quad \leftarrow \text{H.B.}$$

$$\text{for } \beta = 10 \quad FOM = 150.$$

\* FOM of PM:

$$\Rightarrow FOM = K_p^2 \cdot P$$

for single tone modulation

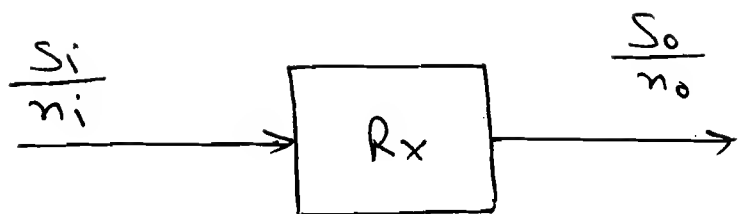
$$P = \frac{A_m^2}{2}$$

$$\therefore FOM = \frac{K_p^2 \cdot A_m^2}{2}$$

$$\therefore \beta = \Delta \phi = K_p \cdot A_m$$

$$\therefore FOM = \frac{\beta^2}{2} = \frac{\Delta \phi^2}{2} \quad \leftarrow \text{H.B.}$$

\*



$\Rightarrow$  For  $\left. \begin{array}{l} \text{DSB} \\ \text{SSB} \\ \text{VSB} \end{array} \right\} F_{om} = 1 \Rightarrow (SNR)_o = (SNR)_i$

$\Rightarrow$  For AM  $\rightarrow F_{om} = \frac{1}{3} \Rightarrow (SNR)_o = \frac{1}{3} (SNR)_i$   
when  $[m=1]$

$\Rightarrow$  For FM  $\rightarrow F_{om} = 150 \Rightarrow (SNR)_o = 150 (SNR)_i$   
when  $[\beta=10]$

$\Rightarrow$  For PM  $\rightarrow F_{om} = 50 \Rightarrow (SNR)_o = 50 (SNR)_i$   
[ $\beta=10$ ]

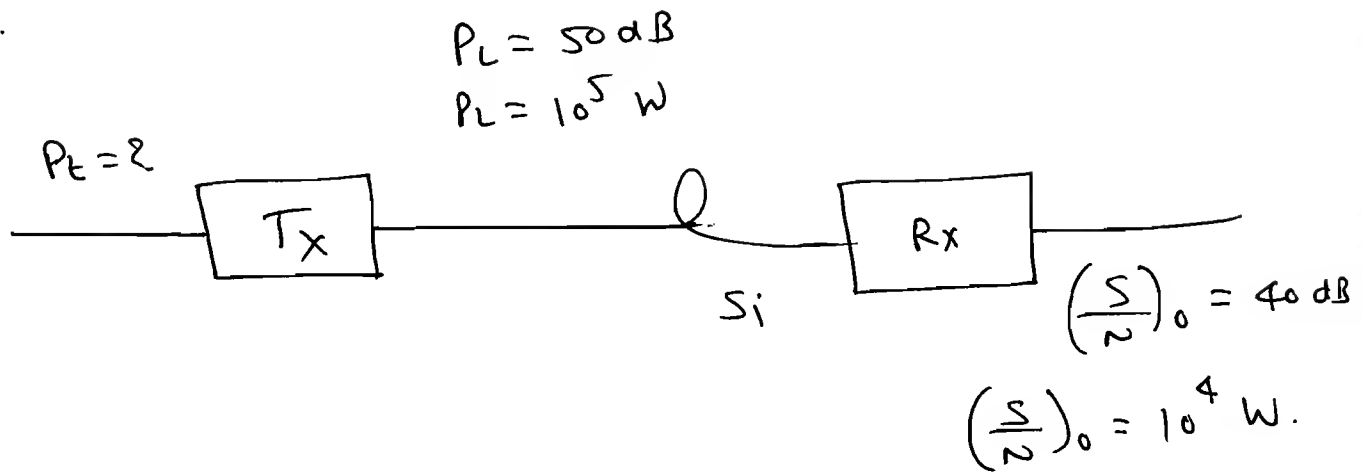
**Q** An Audio signal is Band limited to 15 kHz and transmitted through a channel after modulation. The signal is affected by white noise having a two sided PSD of  $10^{-10}$  watts/Hz. The Power loss in the channel is 50 dB. The SNR required at the output of the receiver is 40 dB. Determine the transmitted power when the modulation used is,

① DSB

② Am with  $\mu=1$

③ FM ( $B=5$ ).

Ans:



① DSB:

For DSB  $FOM = 1$ .

$$\therefore \frac{S_o}{N_o} = \frac{S_i}{n_i}$$

$$\frac{N_o}{2} = 10^{-10} \text{ watts/kHz}$$

$$\therefore S_i = n_i \times 10^4$$

$$n_i = N_o W = \frac{2 \times 10^{-10} \text{ watts/kHz}}{15 \times 10^3}$$

$$\therefore S_i = 2 \times 10^{-10} \times 15 \times 10^3 \times 10^4$$

$$\therefore \boxed{S_i = 30 \text{ m watts}}$$

$$\therefore (S_i)_{dB} = (P_t)_{dB} - (P_L)_{dB}$$

$$\therefore 20 \log (S_i)_{dB} = 20 \log (P_t)_{dB} - 20 \log (P_L)_{dB}$$

$$\therefore \boxed{S_i = \frac{P_t}{P_L}}$$

$$\Rightarrow P_t = S_i \times P_L.$$

$$\therefore P_t = 30 \times 10^{-3} \times 10^5$$

$$\boxed{P_t = 3 \text{ kW}}$$

② AM with  $\mu = 1$ .

$$\therefore Fom = \frac{\mu^2}{\mu^2 + 2}$$

$$Fom = \frac{1}{3}.$$

$$\therefore \frac{S_o}{n_o} = \frac{1}{3} \cdot \frac{S_i}{n_i}.$$

$$\therefore S_i = 3 \times n_i \times \frac{n_o}{S_o}.$$

$$= 3 \times 2 \times 10^6 \times 10^4.$$

$$= 3 \times 2 \times 10^{-10} \times 15 \times 10^3 \times 10^4.$$

$$\therefore \boxed{S_i = 90 \text{ mW.}}$$

$$\therefore P_t = S_i \cdot P_L.$$

$$\therefore P_t = 90 \times 10^{-3} \times 10^5$$

$$\boxed{P_t = 9 \text{ W.}}$$

③ FM [ $\beta = 5$ ].

$$\therefore Fom = \frac{3}{2} \cdot \beta^2 = 150 \cdot \frac{25}{2}.$$

$$\therefore \left(\frac{S}{n}\right)_o = \frac{150 \times}{4} \frac{S_i}{n_i}.$$



$$\therefore S_i = \cancel{\text{No. w}} \times \frac{150}{4} \times \cancel{10^4}$$

$$\therefore S_i = \cancel{10^{-10}} \times 2 \times \cancel{15} \times \cancel{10^3} \times \frac{25}{2} \times \cancel{10^4}$$

$$\therefore S_i = \frac{2}{25} \times 10^4 \times \text{No. w.}$$

$$= \frac{2}{25} \times 10^4 \times 2 \times 10^{-10} \times 15 \times 10^3$$

$$S_i = \frac{4}{5} \times 10^{-3}$$

$$\therefore S_i = 0.8 \text{ mW.}$$

$$\therefore P_t = S_i \times P_L$$

$$\therefore P_t = 0.8 \times 10^{-3} \times 10^5$$

$$\therefore \boxed{P_t = 80 \text{ W}}$$

$\Rightarrow$  FM require very less power compared to other modulation technique. But BW is very large compared to AM.

$$\textcircled{4} \text{ PM } [\beta = 5]$$

$$\therefore F_{om} = \beta^2 / 2 = \frac{25}{2}$$

$$(F_{om})_{pm} = \frac{1}{3} (F_{om})_{fm}$$

$$\therefore P_t = 3 \times 80$$

$$\therefore \boxed{P_t = 240 \text{ W}}$$

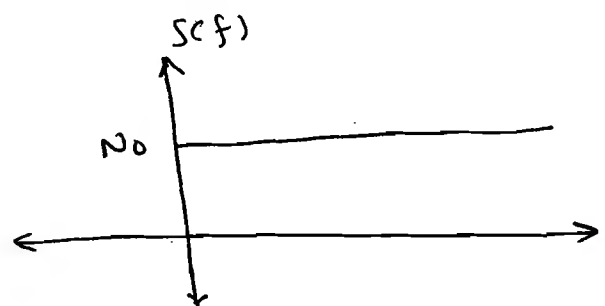
**Q-2**  $\pm$  MW Video signal Band limited to 100 MHz is transmitted through a channel the power loss in the channel is 40 dB. The signal is affected by white noise having single sided power spectral density of  $10^{-20}$  watts/Hz. Determine the (SNR)<sub>i</sub>.

Ans:  
5

$$P_t = 1 \text{ MW}, \quad W = f_m = 100 \times 10^6 \text{ Hz}$$

$$P_L = 40 \text{ dB}$$

$$P_L = 10^4 \text{ W.}$$



$$\therefore n_i = N_0 \cdot W$$

$$\therefore n_i = 10^8 \times 10^{-20}$$

$$\therefore n_i = 10^{-12} \text{ W.}$$

$$\therefore S_i = P_t / P_L$$

$$\therefore S_i = \frac{10^{-3}}{10^4} = 10^{-7}$$

$$\therefore (SNR)_i = \frac{S_i}{n_i} = \frac{10^{-7}}{10^{-12}}$$

$$\therefore (SNR)_i = 10^5$$

$$\therefore (SNR)_i \text{ dB} = 50 \text{ dB.}$$